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ABSORPTION OF SOUND BY POROUS MATERIALS II

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Zusammenfassung

Theoretisch wird gezeigt, dass die üblichen Materialkonstanten Strömungswiderstand und Porosität nicht genügen zur Beschreibung des akustischen Verhaltens von porösen Schallschluckmaterialien, sondern dass eine dritte Konstante, der sogenannte Strukturfaktor, unbedingt in die Differenzialgleichungen eingeht. Der Strukturfaktor wird hervorgerufen von zwei verschiedenen Effekten, die beide meistens in gewissem Masse bei jedem porösen Material vorhanden sind. Der eine Effekt rührt von der Richtungsverteilung der Poren her, der andere von etwaigen Höhlungen, die am stationären Strömungsversuch durch ihre besondere Lage fast nicht beteiligt sind, sondern sich bei akustischen Frequenzen bemerkbar machen durch eine Herabsetzung der Luftsteife.

Die Übereinstimmung zwischen Theorie und Experiment wird durch Einführung dieses Strukturfaktors wesentlich verbessert.

§ 8. *Introduction.* In a previous article [1]) we have compared the values of the acoustic absorption coefficient, measured by us on a few porous materials, with the value for that same quantity following theoretically from the assumption that the acoustic behaviour of any material can be completely described with the aid of two characteristic constants, namely the porosity and the resistance to a steady current of air. It turned out from this comparison, that the experimental and theoretical values did not agree, so that certainly more than two characteristic constants are needed for the complete description of the acoustic behaviour.

Continuing our former investigation, we have now measured at normal incidence the acoustic impedance of a few materials and have compared the values so obtained with the value, that follows from certain theoretical assumptions.

§ 8. *Theoretical basis.* We are in the first place concerned with the connection, between the local excess of pressure in the material, p, and the component in the direction of the propagation of the sound, of the velocity of the air present in the cavities.

This connection is deduced from the equation of continuity, together with an equation of motion.

In the case of a plane wave the first equation is:

$$-\frac{\partial \rho}{\partial t} = \rho \frac{\partial u}{\partial x},$$

ρ denoting the density of the air and u the x-component of the velocity of the air — averaged over all the air present in the cavities.

Introducing as the connection between the excess of pressure and the compression $\delta\rho/\rho$ the equation

$$p = K \frac{\delta \rho}{\rho},$$

by which, therefore, K is defined as the modulus of compression of the air, the equation of continuity becomes

$$-\frac{\partial p}{\partial t} = K \frac{\partial u}{\partial x} \tag{1}$$

For the equation of motion one has repeatedly introduced [2])

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t} + \sigma u, \tag{2}$$

in which σ denotes the resistance to a flow of air, offered by 1 cm^3 of the porous material.

Further development of the theory from this base leads to a calculated acoustic wave impedance $Z_w = p/u$, depending on only one characteristic quantity of the material, namely, the specific resistance of the air σ. But in order to obtain the absorption-coefficient the conditions, prevailing at the boundary plane between the air and the porous material, must be introduced, by which the cavity-factor h comes into play. For, the continuity of the flow of air requires that at that plane the boundary condition $v = h\,u$ shall be satisfied, v representing the velocity in the free air.

However, at an early date already, another equation of motion was proposed [3]) [4]), namely, in our present notation:

$$-\frac{\partial p}{\partial x} = k \rho \frac{\partial u}{\partial t} + \sigma u. \tag{3}$$

The physical meaning of the new quantity k has been interpreted in various ways. According to Kühl and Meyer [3]) $k\rho$ represents the total mass vibrating in 1 cm^3, so that one must eventually consider also parts of the rigid skeleton material. Zwikker [4]) interpretes k as a round about factor. If the pores of the material were all of them making an angle θ with the x-axis, one would have to put k equal to $1/\cos^2\theta$. For in that case the pressure gradient in one such pore is given by $-(\partial p/\partial x)\cos\theta$, while the x-component of the velocity is the true velocity multiplied by $\cos\theta$, so that Newton's law can now be written in the form:

$$-\frac{\partial p}{\partial x}\cos\theta = \frac{\rho}{\cos\theta}\cdot\frac{\partial u}{\partial t}.$$

We observe here that k can be interpreted in still a third way [5]). If, namely, one imagines the pores to be such, that the air can only flow through a few of them, which are normal to the surface, while, communicating with these pores, there are lateral cavities in which the air remains practically at rest, one can for a moment suppose these cavities to be closed altogether. The mass force per cm^3 will then be $\rho\,\partial u/\partial t$, u denoting the velocity in the pores. Now, imagine these lateral cavities to be opened again: the velocity of the air in the pores will then remain the same, whereas the value of u in the formula will be changed, as this quantity represents the average velocity of all the air, contained in the material. If u be decreased by a factor k, one can put matters right again by writing $k\,\rho\,\partial u/\partial t$ for the mass-force, k denoting this time the ratio between the volume of all the pores (cavities included) and the volume of those pores, that run straight through the material.

The interpretation of k as a round about factor and its interpretation as a lateral-cavity-factor are both closely connected with the inner constitution of the material; for that reason we shall call it the structure-factor (Fig. 1). Generally speaking, the contribution to k, due to rigid vibrating mass will be small [6]) compared with the influence on that quantity, due to the structure. As a rule the round-about effect and the lateral-cavity-effect combine and one is often not able to decide which interpretation would be the right one. Consider for example a grainy or fibrous material.

Already in his article, mentioned above, Zwikker [4]) made an estimate of the structure-factor and arrived at the value 3. Our estimate of k as a cavity-factor for the case of a close packing of spherical particles yielded also a value close to 3. The values found experimentally appeared to lie also in the neighbourhood of that same number, though in some cases much higher values were found. In our opinion it is impossible to assign to k one and the same value for all materials; it characterises, beside the porosity and the resistance to the air, a third fundamental property of the material.

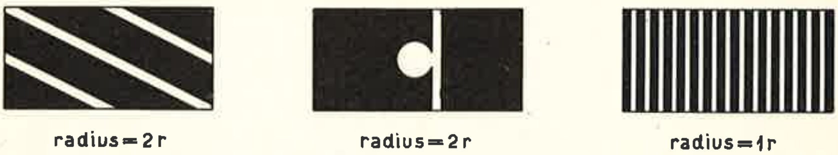

Fig. 1. Three models of materials having the same cavity factor and also the same resistance σ. The structure factor k, however, is different. For the first and for the second model k amounts to 4, for the third model to 1. If one imagines that in the third case fibres are present in the pores, vibrating together with the air, the structure factor becomes higher, leaving h and σ constant to a first approximation.

Fig. 1 shows, side by side, porous materials with three kinds of pores, but with the same cavity-factor and the same resistance to the air. In specimen 1*a* all the pores are inclined at an angle of 60° to the normal to the surface; let the radius of these pores be $2r$. In 1*b* the pores are perpendicular to the surface and their radius is also $2r$; besides, the number and size of the lateral cavities are such that the volume of the straight running pores is only $\frac{1}{4}$ of the total volume of air. Specimen 1*c* has only pores at a right angle to the surface and their radius is r. On computing, these specimina turn out to possess indeed the same σ and the same h, whereas the values of the constitution factor k are 4, 4 and 1 respectively.

§ 9. *Calculation of the impedance.* Starting from equations (1) and (3) the complex ratio p/u is calculated as follows:

From (1) we have, written symbolically,

$$p/u = -K \frac{\partial/\partial x}{\partial/\partial t} \tag{4}$$

Differentiating (1) with respect to x and (3) with respect to t and combining gives

$$\frac{\partial^2 u}{\partial x^2} = k \frac{\rho}{K} \frac{\partial^2 u}{\partial t^2} + \frac{\sigma}{K} \cdot \frac{\partial u}{\partial t}.$$

For the angular frequency ω we have

$$\partial/\partial t = j\omega$$

and, therefore,

$$\left(\frac{\partial}{\partial x}\right)^2 = -k\frac{\rho}{K}\omega^2 + j\omega\frac{\sigma}{K} = -k\frac{\rho\omega^2}{K}\left(1 - j\frac{\sigma}{k\rho\omega}\right).$$

For a wave propagated in the positive x-direction the imaginary part of $\partial/\partial x$ must be negative, so that

$$\frac{\partial}{\partial x} = -j\omega\sqrt{k\frac{\rho}{K}} \cdot \sqrt{1 - j\frac{\sigma}{k\rho\omega}}. \tag{5}$$

Therefore, from (4):

$$Z_w = p/u = \sqrt{k\rho K} \,.\, \sqrt{1 - j\frac{\sigma}{k\rho\omega}}, \tag{6}$$

while the expression for the entrance impedances for an infinitely thick layer of the material becomes

$$Z_\infty = \frac{p}{v} = \frac{1}{h}\sqrt{k\rho K}\sqrt{1 - j\frac{\sigma}{k\rho\omega}}. \tag{7}$$

For a layer of thickness l on the material applied on a rigid background, the general formula holds:

$$Z_l = Z_\infty \coth\left(-l\frac{\partial}{\partial x}\right),$$

so that

$$Z_l = \frac{1}{h}\sqrt{k\rho K}\sqrt{1 - j\frac{\sigma}{k\rho\omega}} \,.\, \coth j\omega l\sqrt{k\frac{\rho}{K}} \,.\, \sqrt{1 - j\frac{\sigma}{k\rho\omega}} \tag{8}$$

In our further discussion we will consider in particular the cases for which either $\sigma/k\rho\omega \gg 1$, or $\sigma/k\rho\omega \ll 1$; the treatment of the intermediate cases for which $\sigma/k\rho\omega$ is of the order of magnitude 1 is much more difficult. This same quantity, however, decides the question whether it is allowed to take the resistance σ as relating to a steady laminar flow of air according to Poiseuille's law or whether one must take Helmholtz' vicosity-waves into account [7]). In the first case the resistance of a circular pore is:

$$\sigma_P = \frac{8\eta}{r^2},$$

equal to the value measured on a steady flow; in the second case the resistance is

$$\sigma_H = \frac{1}{r}\sqrt{2\eta\rho\omega}$$

which differs from the value measured on a steady flow.

Considering the case $\sigma/k\rho\omega \gg 1$ first we see that by neglecting 1 against $\sigma/k\rho\omega$ the quantity k disappears from the formula for Z_l; this was indeed to be expected, as in (3) k refers only to the inertia term, which was neglected in the end. In this case, therefore, the entrance impedance can be completely calculated with the aid of the two characteristic constants h and σ.

In the other extreme case we have:

$$\sqrt{1 - j\frac{\sigma}{k\rho\omega}} \approx 1 - j\frac{\sigma}{2k\rho\omega}. \tag{9}$$

The imaginary part of $\partial/\partial x$ is then $-j\omega\sqrt{k\rho/K}$.

As we know, this part is always equal to $-j\,\omega/c$, c representing the velocity of propagation within the material, so that

$$c^2 = \frac{K}{k\rho}. \tag{10}$$

Inserting (9) and (10) in (8), we obtain for Z_l the formula:

$$Z_l = \frac{k}{h}\cdot\rho c\left(1 - j\frac{\sigma}{2k\rho\omega}\right)\coth j\frac{\omega l}{c}\left(1 - j\frac{\sigma}{2k\rho\omega}\right) \tag{11}$$

The fivefold occurrence of the quantity k leads to thé following consequences:

1°. the velocity of the sound has been reduced in the ratio as 1 to $\sqrt{k}$,

2°. thereby, the resonance frequencies of the coth have been lowered in the ratio as 1 to $\sqrt{k}$.

3°. the loop-width of the spiral, which represents the coth in the complex plane, being determined by the ratio of the real to the imaginary part of the argument, has increased,

4°. the material behaves as a material of which the cavity-factor, is k times smaller.

5°. the losses (represented by $\sigma/k\rho\omega$) are k times smaller.

§ 10. *Measurements.* In performing our measurements of the impedance, we have used the interferometer method with normal incidence. The actually measured quantities are the ratio between the maximum- and the minimum pressure in the field of sound and the location of the minima with respect to the reflecting surface;

from these quantities one can calculate the impedance by well-known methods [8]).

As an example of the results obtained fig. 2 shows the impedance values for a porous plaster, known as Sorbolite.

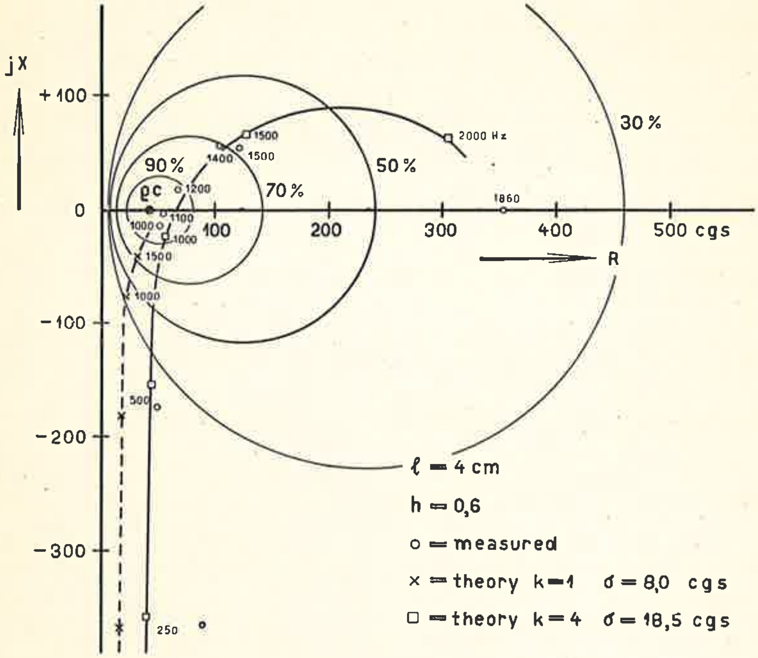

Fig. 2. Comparison of the acoustical impedance of a porous plaster (thickness 4 cm, cavity factor 0,6 σ = 8,0 cgs for a steady flow) with theoretical expectations. The circles are contours of constant absorption coefficient.

Its thickness was 4 cm, h, the cavity-factor was 0,6 and the specific resistance to the air was 8 dyne sec/cm^4. The dotted line represents what one would expect from theory, using these values for h and σ and taking k equal to 1.

The deviation of the theoretical curve from the measured values is in the direction, which could be foreseen as a result of putting k equal to 1. Putting k equal to 4 would bring about a satisfactory agreement between theory and experiment, provided the value of σ is changed from 8 to 18,5. Now in our case the quantity $\sigma/k\rho\omega$ for 800 Hz amounts to 0,3. As this value is not much higher than 1, it is not allowed to suppose that the value of σ as measured from steady current experiments may be taken as applicable. The dynamical value is in accordance with Helmholtz' theory of the viscosity waves higher and a correction factor 2 as demanded by experiment is well within the range of expectations. In a following paper we hope to enter further into the problem of the connection between the dynamical and the steady flow resistance.

From our measurements we found that other specimina of acoustic plaster and fibrous material behaved in the same way. In measuring the acoustic impedance of a layer of plaster, 16 cm thick, we were even able, while the frequency was varied between the limits put by our apparatus (160—1600 Hz) to describe two complete loops of the spiral.

We can trace this same effect e.g. in the measurements by Harmans [9]). Though he states that the general shape of his impedance curve for cotton wool agrees with the theoretical one, his measured points enable one all the same to draw the conclusion, that the velocity of sound in the cotton wool can only have amounted to 0,6 of the normal one. For this reason we must put the structure-factor k for cotton wool equal to about 3.

In a following article we intend to communicate the results of our measurements on specimina of known structure, for which, therefore, an estimate of the value of k can be made beforehand.

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