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THEORY OF THE ABSORPTION OF SOUND BY COMPRESSIBLE WALLS WITH A NON-POROUS SURFACE-LAYER

BY

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2	Ir. C. W. Kosten and Prof. Dr. C. Zwikker	A method of measuring, and an apparatus for determining the elastic behaviour of elastic substances.	1937	n	n	n	a
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Summary

Sound absorbing materials with a non-porous surface-layer possess great advantages over the usuall well-known porous materials. These advantages are related mostly to their hygienic and decorative qualities.

This kind of material, therefore, is already used in practice, the physical basis of the absorption, however, remaining often unknown.

It appears from theory that a low specific weight and a small modulus of elasticity are conditional for obtaining a strong absorption. These conditions are satisfied by layers of sponge rubber. The circumstances leading to the most favourable results have been examined and turn out to be as follows: the pores should be preferably such that the air inside is to some extent free to vibrate as an independent body. Big holes in the layer as well as the application of the material at some distance from the rigid wall appear, under certain circumstances, to influence the sound absorbing qualities favourably.

1. *Introduction.* In principle absorption of sound by walls can be effected by any material, provided its acoustic impedance causes loss of sound energy. By this condition any absorption is out of the question if the impedance is a purely imaginary quantity (mass or compressible material without loss). The majority of the sound absorbing materials known up to now consist of very porous and mainly solid matter. Owing to their great importance in practice the theory of sound absorption by this kind of porous materials has been developed very thoroughly, for which the reader is referred to the publications on this subject by Rayleigh[1]), Zwikker[2]),

Kühl and Meyer [3]), Rettinger [4]), Cremer [5]) and Monna [6]). It may be observed here, that, occasionally, in these articles the possibility of resonance of the skeleton has been taken into account to a certain extent. This resonance is then supposed to express itself as an apparent increase of the density of the air. Needless to say the great variety of the materials (compare, for example, acoustic plaster and hair-felt) prevents such suppositions from possessing any general validity.

Materials with a non-porous surface can also absorb sound. It is, for example, a well-known fact that a wooden panelling or window-panes absorb sound by their vibrating in resonance. For this reason the region of maximum absorption is located at the lower frequencies, whereas porous materials absorb as a rule the higher frequencies best. In this respect, therefore, the materials vibrating in resonance possess an important advantage over the porous ones as, in practice, the absorption of the lower notes offers the greater trouble. As a secondary, though important advantage, we may point out here, that such materials with a non-porous surface can be painted and washed. In those cases, therefore, in which the use of porous materials must be rejected, non-porous materials can be applied, as, for example, in nursing homes, engine rooms, office buildings, etc. By a suitable construction, one can make still more of this panelling effect and Meyer [7]) and Lauffer [8]) succeeded in this way in manufacturing non-porous wall coverings, possessing rather satisfactory absorption characteristics. From their experiments the application of materials causing the losses of sound energy, at the back of the non-porous surface appeared to be indeed essential. One can, moreover, infer from these measurements, that on these lines the manufacturing of materials, which are wholly satisfactory in practice, is impossible. The region of absorption, namely, remains rather narrow in any case and the maximum value of the absorptionscoefficient amounted to 50%.

In the present article we shall treat the theory of sound absorption by compressible wall-coverings with non-porous coating layer, on as general lines as possible; based on these developments we shall further examine the possibilities of its application in practice. More in particular we shall pay attention to sponge rubber layers coated with a solid surface-layer.

2. *Acoustic impedance and absorption coefficient.* Let us put:

a = absorption coefficient, i.e. the energy of the reflected wave in terms of the energy of the incident wave,

ρc = wave-resistance of the air = specific mass × velocity of sound,

z = specific acoustic impedance = pressure divided by the velocity component at right angles to the plane under consideration,

α = angle of incidence of sound,

then, for any value of α, we have for the sound absorption the rigorous formula

$$a = 1 - \left| \frac{z \cos \alpha - \rho c}{z \cos \alpha + \rho c} \right|^2 \tag{1}$$

and, therefore, in the case of normal incidence:

$$a_0 = 1 - \left| \frac{z_0 - \rho c}{z_0 + \rho c} \right|^2 \tag{2}$$

z, and thereby a, is in general a function of α (apart from other quantities). The importance of a_0 and z_0 is mainly purely scientific. They can be determined by measuring-methods in which the normal incidence of sound is the only one applied, as, for example the well-known interference method. In practice, however, one has, preferably, to deal with $\bar{a}$, the average absorption coefficient for sound of all possible directions, which is calculated from a by means of the familiar formula

$$\bar{a} = \int_0^{\pi/2} a \sin 2\alpha \, d\alpha \tag{3}$$

The principal contribution to $\bar{a}$ is the one for $\alpha = 45°$, $\sin 2\alpha$ then reaching its maximum value. The value obtained by the reverberation method, if properly carried out, is about equal to $\bar{a}$.

The connection between a_0 and z_0 (equation (2)) seems a very simple one, but this is only apparent, as z_0 is a complex quantity.

A very convenient mode of representation is shown in fig. 1, in which the curves for constant a_0 are drawn in the complex z_0-plane *). These curves turn out to be circles with centre $z_0 = (2 - a)\rho c/a$ and radius $2\sqrt{1 - a} \, . \, \rho c/a$. The quantities r_0 and x_0 are defined by the equation $z_0 = r_0 + jx_0$.

The figure enables one to find immediately for any assigned value

*) K. Schuster und A. Hohberg, Ann. Physik **16**, 203, 1933.

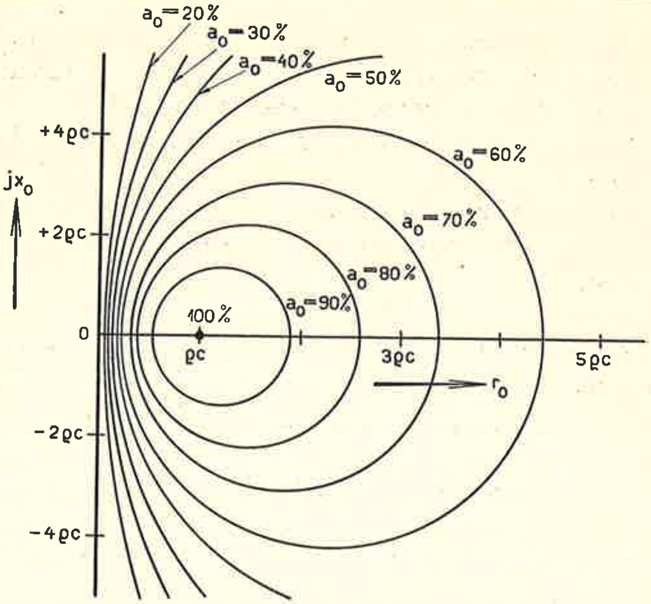


Fig. 1. The connection between the absorption coefficient a_0 and the specific acoustic impedance z_0 in the case of normal incidence of the sound.

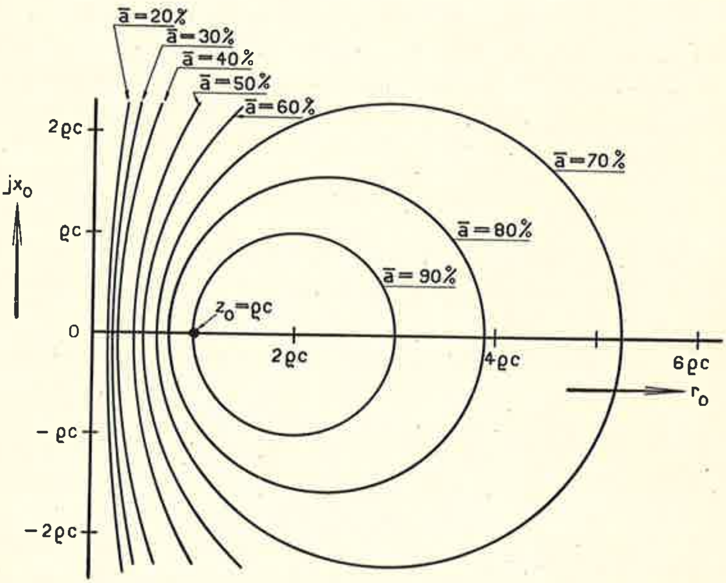


Fig. 2. The connection between the average absorption coefficient $\bar{a}$ and the acoustic impedance z for the case that z is independent of the angle of incidence α.

of z_0, the corresponding value of a_0, while, moreover, one can read from it at once how a_0 will be affected by any definite change of z_0. This mode of representation remains valid without any modification for the connection between a and $z \cos \alpha$ as follows directly from equations (1) and (2).

If z be independent of α (and this will frequently turn out to be very nearly true for compressible materials), so that $z = z_0$, then, for any given value of z_0, equation (1) determines a as a function of α, whereupon $\bar{a}$ is obtained by means of equation (3). This means that, for any assigned z_0, $\bar{a}$ has a well-defined value. It will be clear that we are now in a position to draw a diagram as in fig. 2, which now shows the curves of constant $\bar{a}$ in the complex z_0-plane. The maximum value for $\bar{a}$ amounts to 0.95, therefore less than 1. Practically speaking the curves for constant $\bar{a}$ cannot be distinguished from circles; they satisfy the following equation:

$$\bar{a} = 8\frac{\rho c r_0}{r_0^2 + x_0^2} - 8\left(\frac{\rho c r_0}{r_0^2 + x_0^2}\right)^2 \lg\left\{\frac{r_0^2 + x_0^2}{\rho^2 c^2} + 1 + \frac{2r_0}{\rho c}\right\} + \\ + 8\frac{r_0}{x_0}\left(\frac{\rho c x_0}{r_0^2 + x_0^2}\right)^2\left(\frac{r_0^2}{x_0^2} - 1\right) \operatorname{bg\,tg} \frac{x_0}{r_0 + \rho c} \tag{4}$$

From a comparison of fig. 1 and 2, it appears that averaging a can mean a considerable increase relatively to a_0. One must here bear in mind, however, that this is only true if z is independent of α.

In order to obtain some idea of the order of magnitude of the absorption coefficient attainable with compressible materials, one can proceed as follows:

For the specific impedance z we can write, without thereby restricting its general sense, the expression

$$z = \frac{1}{j\omega C}(1 + j \operatorname{tg} \delta), \tag{5}$$

where C and δ denote respectively the specific compliance and the angle of loss. Both these quantities may depend on the frequency; for the present, however, we shall consider them to be constant. We shall, moreover, restrict ourselves to the case of normal incidence.

After a few transformations equation (2) now yields

$$a_0 = \frac{4 \operatorname{tg} \delta}{q + \dfrac{1 + \operatorname{tg}^2 \delta}{q} + 2 \operatorname{tg} \delta} \tag{6}$$

where q is written for $\rho c \omega C$.

Fig. 3 represents the way in which a_0 depends on ω, C and $\operatorname{tg} \delta$. Practically speaking the absorption acquires already its maximum value for $\rho c \omega C \approx 1$; this maximum value is strongly dependent on $\operatorname{tg} \delta$. For maximum absorption at 800 Hz, we must have $C \approx 5 \times 10^{-6}\ \mathrm{dyn}^{-1}\ \mathrm{cm}^{-1}$.

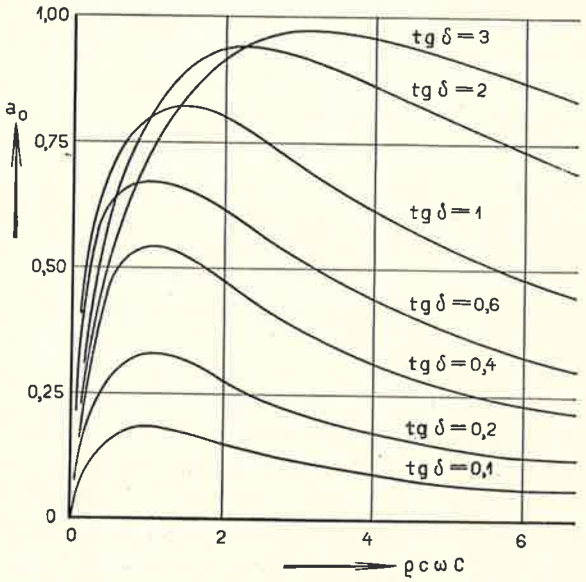


Fig. 3. Absorption by a compressible layer without mass.

Fig. 4 shows the impedance curves. Combining this figure with the circle-diagram of fig. 1 (added in broken lines), the occurrence of the tops will be at once clear.

If C and δ depend on the frequency, this means a transition from one curve in fig. 3 to another.

We will not extend these considerations to sound, incident in any arbitrary direction, as the problem is wholly a purely theoretical one.

Owing to the mass of the layer, C and δ will in reality be strongly dependent on the frequency. In order to take this influence of the mass duly into account, we must study the vibratory motion of the material in more detail. Our next point of discussion will therefore

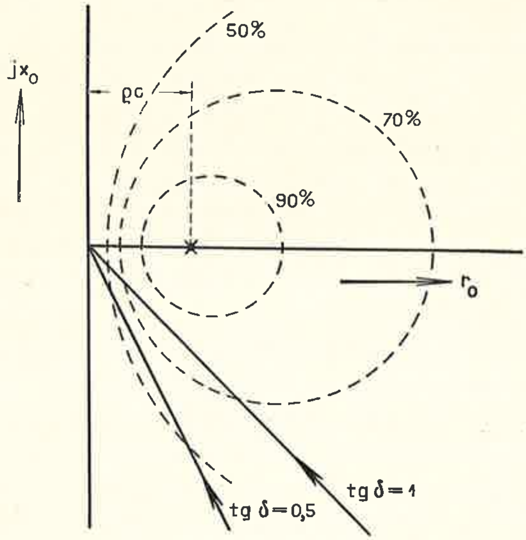
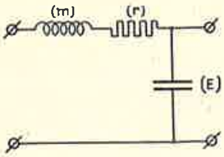
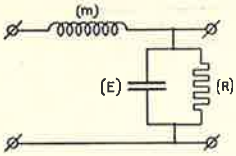
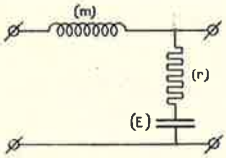
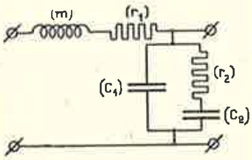


Fig. 4. Shape of impedance curves with increasing frequency.

be: what differential equations must be used as a basis for the necessary calculations.

3. *The differential equations of the motion in the absorbing medium.* We must mention first, that not one of the equations will be valid rigorously. In the schematic table further down four of the most simple cases are represented. Generally speaking the absorbing layer can be compared to a long electrical conductor with homogeneously distributed resistance, self-inductance and capacity. Possible schemes for an element of this conductor are drawn in column 2. Column 3 gives the fundamental differential equations for the corresponding system, column 4 the differential equation one can derive from these two equations. As transient phenomena are not taken into account, some of these equations hold only for solutions, that possess a cosine-like dependence on the time. Column 3 can be considered as a continuation of column 2, the differential equation of column 4, however, has been extended to three dimensions ($\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$). Column 1 gives the nature of the acoustic material for which the corresponding equations may hold.

Acoustic material	Electrical analogue	Fundamental relations	Differential equation for p
homogeneous porous		$-\frac{\partial p}{\partial t} = E\frac{\partial v}{\partial x}$ $-\frac{\partial p}{\partial x} = m\frac{\partial v}{\partial t} + rv$	$\Delta p = \frac{m}{E}\frac{\partial^2 p}{\partial t^2}(1 - j\,\mathrm{tg}\,\delta)$ $\mathrm{tg}\,\delta = r/\omega m$
compressible		$-\frac{\partial p}{\partial t} = \frac{E}{1+\left(\frac{E}{\omega R}\right)^2}\frac{\partial v}{\partial x} + \frac{1}{\frac{\omega R}{E}+\frac{E}{\omega R}}\frac{\partial^2 v}{\partial x\,\partial t}$ $-\frac{\partial p}{\partial x} = m\frac{\partial v}{\partial t}$	$\Delta p = \frac{m}{E}\frac{\partial^2 p}{\partial t^2}(1 - j\,\mathrm{tg}\,\delta)$ $\mathrm{tg}\,\delta = E/\omega R$
compressible		$-\frac{\partial p}{\partial t} = E\frac{\partial v}{\partial x} + r\frac{\partial^2 v}{\partial x\,\partial t}$ $-\frac{\partial p}{\partial x} = m\frac{\partial v}{\partial x}$	$\Delta p = \frac{m}{E(1+\mathrm{tg}^2\,\delta)}\frac{\partial^2 p}{\partial t^2}(1 - j\,\mathrm{tg}\,\delta)$ $\mathrm{tg}\,\delta = \omega r/E$
inhomogeneous porous		$-\frac{\partial p}{\partial t} = \frac{1 + j\,\mathrm{tg}\,\delta_2}{C_2 + C_1(1 + j\,\mathrm{tg}\,\delta_2)}\frac{\partial v}{\partial x}$ $-\frac{\partial p}{\partial x} = m\frac{\partial v}{\partial t}(1 - j\,\mathrm{tg}\,\delta_1)$	$\Delta p = \left\{C_1 m(1 - j\,\mathrm{tg}\,\delta_1) + C_2 m\frac{1 - j\,\mathrm{tg}\,\delta_1}{1 + j\,\mathrm{tg}\,\delta_2}\right\}\frac{\partial^2 p}{\partial t^2}$ $\mathrm{tg}\,\delta_1 = r_1/\omega m$; $\mathrm{tg}\,\delta_2 = \omega r_2 C_2$

The meaning of the various symbols is as follows:

p = overpressure of sound
v = vibrational velocity of the particles (therefore *not* the phase-velocity of sound)
R and r = the real parts of the acoustic impedance
E = modulus of elasticity
m = specific mass of moving medium
ω = angular frequency
t = time
C_1 and C_2 = compliances (dimensions as of $1/E$).

The case of homogeneous porous material offers no difficulties. The electrical arrangement as well as the equations are the expressions of the fact that we have here to deal with a compressible medium free from losses, capable of a vibration with friction relatively to its surroundings. That is why the mass is a complex quantity (r), the compliance not. In the case of compressible materials it is just the other way round. Since the moving particles are not coupled to non-moving surroundings the assumption of a mass free from losses is an obvious one, whereas the compressible medium is certainly not free from losses. We can suppose that in its electrical analogue this is represented by a resistance in parallel R or by a resistance in series r. For massive material both suppositions are wrong, as in this case according to experiment tg δ is often, to a great extent independent of the frequency. As, however, it will appear that for our present purpose porous compressible materials with a non-porous coating are the most suitable, the supposition of a resistance in parallel R is still of considerable importance. For the time being, we shall, therefore, accept the validity of this supposition. Thereby, the value of tg δ will at the same time agree with the results of earlier measurements [9]), namely tg $\delta \sim \omega^{-1}$ for acoustic frequencies. The mental picture of a resistance in series or in parallel for compressible media with losses is often translated mathematically by the conception of a complex E^*, which is then supposed to be equal to $E(1 + j \text{ tg } \delta)$ and $E/(1 - j \text{ tg } \delta)$ respectively, while tg δ is supposed to change in these expressions directly and inversely proportional to the frequency respectively. As for solid materials tg δ is independent of the frequency over a fairly extensive region, reminding one thereby of mechanical hysteresis, one might in this case use for the complex quantity E^* the expression $Ee^{j\delta}$, with constant δ.

For the majority of porous materials the above mentioned case of homogeneous porous matter will not give a sufficiently close approximation, owing to the fact that the resistance of the air, statically measured, is not the all important quantity. Part of the holes in the substance will have a somewhat isolated position, so that the air contents of these holes will not move during a *stationary* air resistance experiment. *Dynamically*, however, these "remote" holes come into action, because now the pressure in the holes has to vary owing to the travelling waves. Any widened part of a pore corroborates this effect. In the above schematic table the equations for inhomogeneous porous material of this kind are also given. In the present connection, however, we cannot enter further into this matter but must suffice with the preliminary statement that in our opinion, based on calculations concerning this case, it is this effect, that clarifies those differences between theory and experiment, that as a rule are „explained'' by an apparent increase of the density of the air [10]) [11]) [12]).

4. *Compressible layer of solid material.* As already said, we shall use the differential equations, valid for the system in which condensor and resistance are in parallel, namely

$$\left.\begin{aligned} \Delta p &= \frac{m}{E}\,\frac{\partial^2 p}{\partial t^2}\,(1 - j\ \mathrm{tg}\ \delta) \\ -\frac{\partial p}{\partial x} &= m\,\frac{\partial v_x}{\partial t};\ \text{cycl.} \\ \mathrm{tg}\ \partial &= E/\omega R \end{aligned}\right\} \qquad (7)$$

The results, however, are also valid without modification for matter with resistance in series or with hysteresis-losses, as regards the former

1°. if tg δ is interpreted as $\omega r/E$ and

2°. if one multiplies m with $\cos^2 \delta$,

as regards the latter

1°. if tg δ is taken to be independent of the frequency and

2°. if one multiplies m with $\cos \delta$.

Usually $\cos \delta = 1$ will be a more or less sufficient approximation, the only remaining difference between the above cases then being the different interpretation of tg δ. We may point out at once, that with porous (non-compressible) materials δ has a much higher value so that it is by no means allowed in that case to put $\cos \delta = 1$.

In the following calculations we shall in principle, proceed in the same way as Zwikker and Monna did. We suppose, therefore, that a plane wave falls on the material at an angle α, this wave is refracted by the material. Let us denote the angle of refraction by β. All waves present in the material can be composed to form one wave travelling onwards and one wave, travelling backwards.

The solutions which we are going to try are: for the onwards moving wave:

$$p_1 = P_1\, e^{j\omega\{t - (x \sin \beta + y(\cos \beta - jk))/c'\}} \qquad (8)$$

and for the backwards moving wave

$$p_2 = P_2 \, e^{j\omega\{t-(x \sin \beta - y(\cos \beta - jk))/c'\}} \tag{9}$$

Here the plane of the x, y-coordinates is taken parallel to the plane of incidence, the x-axis parallel and the y-axis at right angles to the surface of the material (see fig. 5).

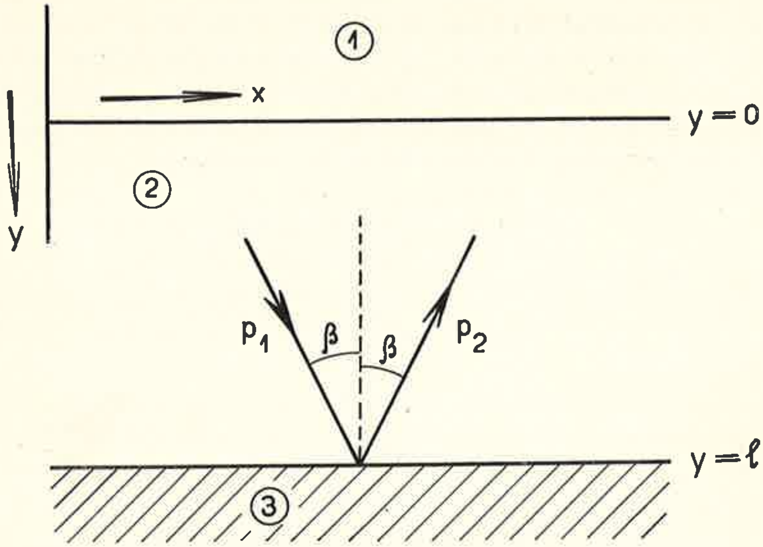


Fig. 5. Slanting incidence on a layer of finite thickness. 1. air, 2. absorbing layer, 3. reflecting wall.

In these formulae P_1 and P_2 are real, c' is the (real) velocity in the medium, k is the damping constant of the travelling wave.

Substitution of (8) and (9) in the first differential equation of (7) gives at once

$$\frac{\cos \beta - jk}{c'} = \sqrt{\frac{m}{E}\left(1 - j \operatorname{tg} \delta - \frac{E \sin^2 \alpha}{mc^2}\right)} \tag{10}$$

in which use has been made of the relation of refraction

$$\frac{\sin \alpha}{\sin \beta} = \frac{c}{c'} \tag{11}$$

Together with the second differential equation of (7) the boundary condition $v_y = 0$ for $y = l$ furnishes a relation between P_1 and P_2 namely:

$$P_1 \, e^{-j\omega l(\cos \beta - jk/c')} = P_2 \, e^{+j\omega l(\cos \beta - jk/c')} \tag{12}$$

For the acoustic impedance, the quantity we want to know

$$z = \frac{p}{v_y} = \left| \frac{p_1 + p_2}{v_{y_1} + v_{y_2}} \right|_{y=0} = \frac{P_1 + P_2}{P_1 - P_2} \frac{mc'}{\cos\beta - jk}$$

$$z = \sqrt{Em \Big/ \left\{1 - j\,\mathrm{tg}\,\delta - \frac{E \sin^2 \alpha}{mc^2}\right\}} \,.\, \coth j\omega l \sqrt{\frac{m}{E}\left\{1 - j\,\mathrm{tg}\,\delta - \frac{E \sin^2 \alpha}{mc^2}\right\}} \quad (13)$$

The way in which z depends on the angle of incidence is accounted for by the term $E \sin^2 \alpha/mc^2$. For all materials, at all likely to be used, the amount of this term varies from 0 to 0.02 ,so that as a very close approximation we can take z to be independent of the angle of incidence. Omitting, therefore, the term in α we have

$$z = z_0 = \sqrt{Em/(1 - j\,\mathrm{tg}\,\delta)} \,.\, \coth j\omega l \sqrt{\frac{m}{E}(1 - j\,\mathrm{tg}\,\delta)} \quad (14)$$

For a layer of infinite thickness coth.... = 1, so that then

$$z_\infty = \sqrt{Em/(1 - j\,\mathrm{tg}\,\delta)} \quad (15)$$

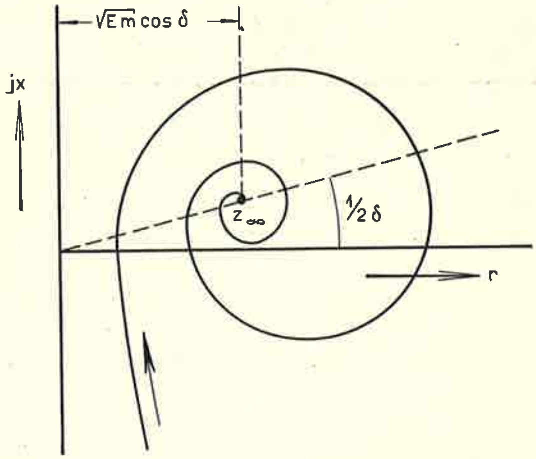


Fig. 6. Shape of the impedance-curve of a compressible wall. The arrow points in the direction in which z moves with increasing frequencies.

Fig. 6 shows the impedance-curve for z as a function of the frequency; z describes a spiral round the point z_∞. The fact ,that in the case of solid materials (wave-resistance from 100 to 5000 times ρc for air) the acoustic impedance measured at right angles to the surface describes indeed very beautifully a spiral curve when plotted against the frequency, was proved experimentally by H. B ö h m e [13]). The quantity $\sqrt{Em}$ determines directly whether a reasonable amount of absorption is to be expected, or not. The condition, namely, for absorption is, that the wave-resistance of the material shall be comparable with the wave-resistance ρc for air (see equation (2) or fig. 1) and $\sqrt{Em}$ measures fairly accurately the former resistance. By the condition mentioned, all solid materials are at once excluded and as the only likely materials remain those for with both E and m have extremely low values.

5. *Compressible porous material with mass-less non-porous coating; very narrow pores* (*quasi-solid*). This case shows no fundamental differences from what we have found for solid materials, for, indeed, on account of the pores being so very narrow, the air and the compressible material can only vibrate together. This means that the material is quasi-solid, so that equation (13) is still rigorously correct and equations (14) and (15) approximately so.

We shall now enter somewhat further into the question how to choose the material in order to obtain the most favourable results. In doing so we shall deal separately with the quantity z_∞ (namely the centre of the spiral) and with the influence of tg δ.

The value of δ lies, presumably, between 0° and 40° so that the position of z_∞ is at the most 20° from the real axis (see fig. 6), and as in the neighbourhood of this axis the absorption coefficient a_0 is insensitive to an angular turning and moreover, cos δ can be put approximately equal to 1, the decisive quantity in the present connection is $\sqrt{Em}$. In the case of air this is $\sqrt{1{,}4\, p_0\rho} = \rho c$ (p_0 = reading of the barometer); the direct measure for a_0 is $\sqrt{Em}$ expressed in ρc as unit (see fig. 1), that means $\sqrt{Em/1{,}4\, p_0\rho}$. The lowest value that can be reached by lowering E (less stiff material) is 1,4 p_0 for even if the material were infinitely soft, we have still to overcome the stiffness of the air 1,4 p_0. Though it is conceivable that we can reach p_0 itself (namely, if in the material the changes in the state of the air should take place isothermically instead of adiabatically) we shall for the present reckon with 1,4 p_0 as the lowest value. Now the value of the proper stiffness of those sponge rubber layers that have been investigated, happens to be so low, that the total stiffness E is already very nearly 1,4 p_0 so that in the case of this kind of materials, the remaining principal quantity is $\sqrt{m/\rho}$. The most suitable materials from the present point of view have turned out to be light sponge rubber layers with thicknesses of a few cm, the specific mass being from 0,05 to 0,1, and, thereby $z_\infty/\rho c$ from 6,5 to 9, from which one computes average coefficients of absorption of 62% to 52%. For layers about 10 cm thick an average absorption of 60% can be reached for frequencies higher than 75 Hz, while for frequencies over 250 Hz the absorption can even be regarded as independent of the frequency.

We shall now discuss the influence of tg δ. This quantity determines to a great extent the damping constant k of the travelling wave (see equation (10)). Physically speaking, the actual reaching of z_∞ would mean such a strong damping of the oncoming wave in the material that the development of a reflected wave in the absorbing medium is practically out of the question. A high value of tg δ means, therefore, that already after a few loops (i.e. after a few characteristic frequencies) point z_∞ is already reached.

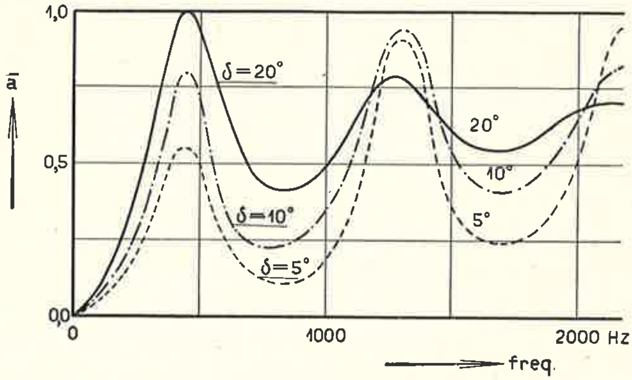


Fig. 7. Theoretical curves of sound absorption. Specific weight 0.05, thickness of layer 3 cm; very narrow pores. For an n-times thicker layer the frequency-scale must be stretched n-times.

For illustration, fig. 7 shows a few theoretical absorption curves, representing $\bar{a}$ as a function of the frequency, the specific weight being 0.05, the thickness of the layer 3 cm and the angle of loss δ 20°, 10° and 5°.

Fig. 8 shows the impedance curve corresponding to the $\delta = 20°$-curve. This is, however, not quite correct, as for constant δ the results must be slightly modified (see 5). For, if tg δ should actually be constant, this might mean a difference of a factor cos δ in m, so that, thereby, the absorption would be still slightly increased. We give up, however, deliberately any attempt at such a high precision, which in any case would be spurious, owing to our introduction of the approximation $E = 1{,}4\ p_0$. The curves of fig. 7 and fig. 8 must be considered as among the best solutions on the base of the theory here given. It is true that one might think of still further lowering the specific weight; this however, is not sure to be invariably accompanied by an

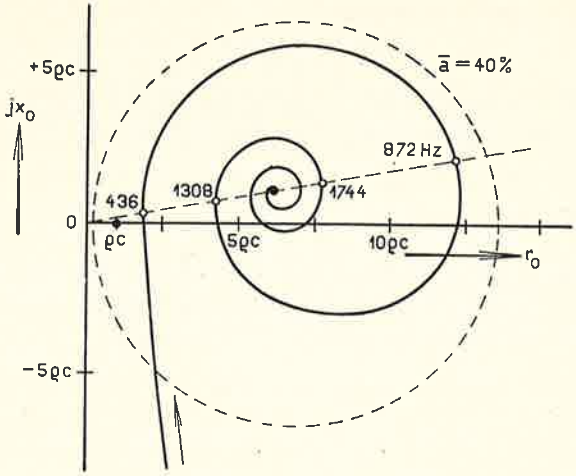


Fig. 8. Shape of impedance curve, specific weight 0.05, thickness 3 cm, $\delta = 20°$, very narrow pores.

increase of the absorption, as for such unsubstantial materials δ will decrease together with the specific weight. But this means that the limit of the absorption will only be reached at higher frequencies, a decided disadvantage, therefore. This is also the reason why in fig. 7 the curves for $\delta = 5°$ and $10°$ lie for lower frequencies at a considerable distance below the curve for $\delta = 20°$, whereas in the neighbourhood of 2000 Hz this difference has disappeared. At the end of this article we shall deal with the possibilities of still further improving the sound absorbing quality of the material.

6. *Compressible porous material with mass-less non-porous coating-layer; very wide pores.* In order to fix the ideas, one can think here of a light sponge rubber layer with very wide pores. The air and the rubber skeleton are understood to vibrate independently from each other. The characteristic frequencies of the vibrating skeleton are now considerably lower than in the former case, since, this time, it is only its proper stiffness that counts. As regards the amount of this difference, we estimate the frequencies to be reduced, 1,5 to 3 times,

according to the proper stiffness. The pressure of the air at any moment will invariably be the same everywhere in the material, so long as the thickness of the layer is very small compared with the wave-length in air; the air contents can then be regarded as a compressible medium without losses.

The total impedance becomes

$$z_{tot.} = z_{rubber} + z_{air} \tag{16}$$

The quantity z_{rubber} describes a complex impedance spiral, as in § 5, with low characteristic frequencies. For not too high frequencies we can write for z_{air}:

$$z_{air} = \frac{1}{j\omega C} = \frac{1{,}4\, p_0}{j\omega l} \tag{17}$$

where l denotes the thickness of the layer. For frequencies, comparable with the fundamental characteristic frequency of a layer of air, whose thickness is l, we have the correct formula

$$z_{air} = \frac{-j\rho c}{\operatorname{tg} \omega l/c} \tag{18}$$

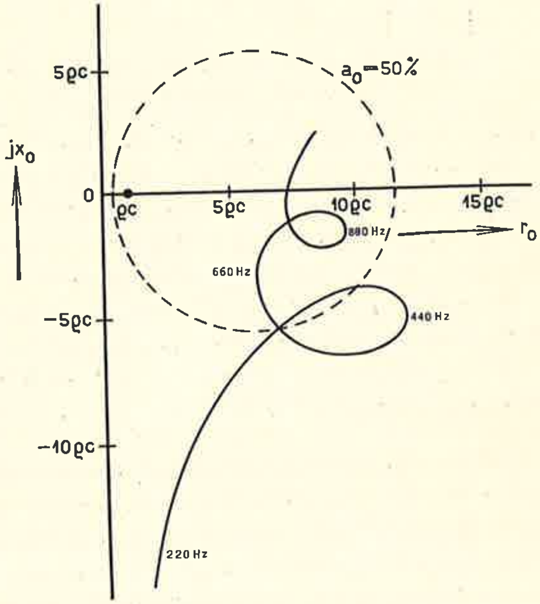


Fig. 9. Shape of impedance curve specific weight 0,06, thickness 3 cm $E_{skel.} = 0.3\, E_{air}$, very wide pores.

For illustration, fig. 9 shows the theoretical impedance curve for a material with wide pores and of which the specific weight is 0.06, $l = 3$ cm, while the modulus of elasticity of the skeleton is 0.3 times that of air ($1{,}4\ p_0$). In order to indicate roughly the value of the absorption coefficient to which one is led by these data, the circle for $a_0 = 50\%$ is added in the figure (dotted line). In our figure, the damping has been arbitrarily assumed, though fairly strong, because it is inconceivable that the vibrations of the air and of the skeleton are independent of each other to such an extent, that the characteristic vibrations are only slightly damped.

This time, we cannot draw directly the impedance curve of fig. 9 in the ,,circle'' diagram for the average absorption coefficient $\bar{a}$ with a view to finding $\bar{a}$, because now the total impedance shows an angular dependence, owing to the part z_{air} being dependent on the angle of incidence α. The impedance of the free air is now calculated by means of the differential equations for homogeneous porous material in a way completely analogous to that given in 4.

Neglecting the volume of the solid material, we find

$$z_{air} = \frac{\rho c\,(1 - j \operatorname{tg} \delta)}{\sqrt{1 - j \operatorname{tg} \delta - \sin^2 \alpha}} \coth j \frac{\omega l}{c} \sqrt{1 - j \operatorname{tg} \delta - \sin^2 \alpha} \qquad (19)$$

As the air is supposed to vibrate freely, the velocity of sound in this formula has already been put equal to c.

For $\delta = 0$ (19) becomes

$$z_{air} = \frac{-j\rho c}{\cos \alpha \operatorname{tg}\left((\omega l/c) \cos \alpha\right)} \qquad (20)$$

It follows from the above that for an assigned value of the angle of incidence α, the absorption coefficient decreases for low frequencies and increases for high frequencies. The decrease is due to the influence of $\cos \alpha$ on z_{air} (eq. 20), the increase to the fact, that for high frequencies z_{air} becomes small, whereas z_{rubber} is independent of α.

If now, we put the question whether suitable materials can be manufactured on this principle, the answer is in the affirmative. For a low value of E, namely, the wave-resistance $\sqrt{Em}$ of the compressible skeleton can be considerably lower than for the same material with very narrow pores. This would lead to very high values of the absorption coefficient, but for the fact that the part z_{air} has a high negative imaginary value. However, we have means at our disposal to minimize this imaginary part to any arbitrary extent. To this end the material is applied in the arrangement shown in fig. 10.

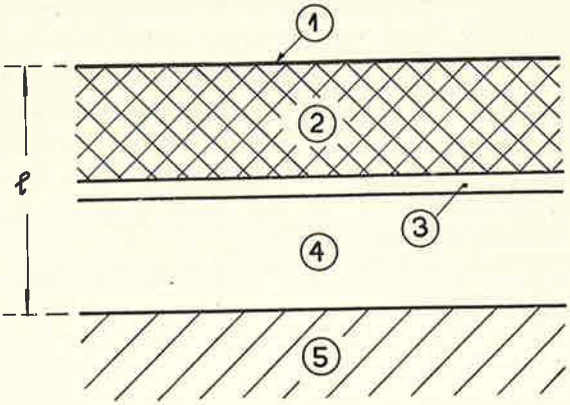


Fig. 10. Compressible layer with very wide pores on layer of air.
1. Coating layer, 2. Compressible layer, 3. Solid very porous layer,
4. Layer of air, 5. Rigid wall.

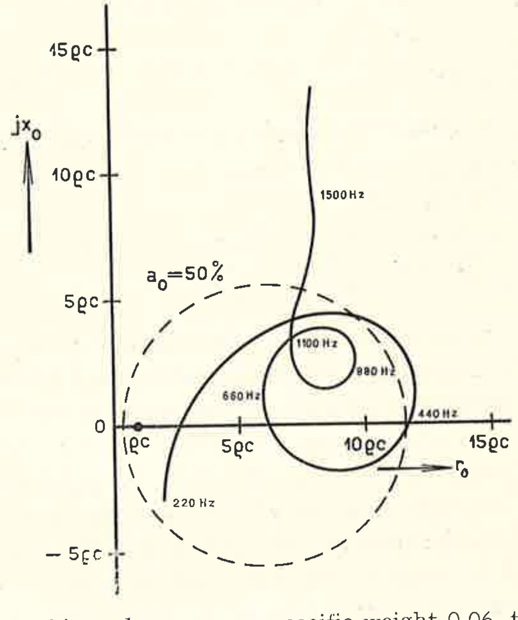


Fig. 11. Shape of impedance curve, specific weight 0.06, thickness 3 cm on layer of air of 7 cm (see fig. 10), $E_{skel.} = 0.3\ E_{air}$; very wide pores.

When compressed, the air of layer 2 escapes through layer 3 and reaches the layer of air 4. This is, therefore, equivalent to an extension of the layer of air, whereas the compressible skeleton remains unmodified. Fig. 9 shows the shape of the impedance curve in the absence of the extra layer of air, fig. 11 shows this curve for the same material applied according to fig. 10. It is therefore conceivable that in this way material can be manufactured, capable of absorbing, for frequencies above 200 Hz, 50% or more up to fairly high frequencies. The average absorption between 200 and 1600 Hz might be about 70%. An effect of this kind occurs in the case of an arrangement as shown in fig. 10, omitting however the intermediate solid layer 3, the only difference being that the values of the characteristic frequencies of the rubber skeleton are shifted.

The application on a layer of air as shown in fig. 10, gives results comparable with those of very thick compressible layers, the main difference being that in the former case the characteristic frequencies are higher.

7. *Compressible porous material with mass-less non-porous coating layer; fairly wide pores.* The pores of the material can be conceived to be of such a width, that the air-contents can neither be considered to vibrate freely, nor forced either. In this case calculations become rather too involved. What we can say for certain is, that now:

a) the air-contents will vibrate fairly freely for lower frequencies, for higher frequencies, on the contrary, fairly forced. The transition-frequencies between free and forced vibrations lie, therefore, right in the centre of the acoustically important frequency-region,

b) the damping is strong. It will show a maximum for the middle frequencies. For low frequencies we shall have to expect impedance curves similar to those of fig. 9. With increasing frequency the centre of the spiral will shift toward the right, because the modulus of elasticity increases. When the material is applied on a layer of air (fig. 10) we can expect for lower frequencies impedance curves as in fig. 11. There will be no motion in the positive imaginary direction for high frequencies, the centre of the spiral will move towards the right. For this reason the absorption will, presumably, decrease at a slower rate than in the case of fig. 11.

Generally speaking, the strong damping will be very advantageous, for a pronounced selective absorption at well defined frequencies is not to be desired.

8. *Influence of the mass of the coating layer.* Let us denote the mass of the coating layer per unit area by M. We can then write $j\omega M$ for the specific acoustic impedance. This value is independent of the angle of incidence. In order to obtain the total impedance in this case, the amount of $j\omega M$ must be added to the specific impedance of the compressible layer with mass-less coating. For low frequencies and small M we can neglect the term $j\omega M$ against the impedance of the compressible layer, so that the developments in the above §§ apply here without modification. For higher frequencies, however, $j\omega M$ becomes of importance; for with increasing frequencies the impedance curves move out of the complex plane in the positive imaginary direction, so that the absorption decreases accordingly. We can, therefore, speak of a kind of cut- off frequency, above which the absorption decreases quickly. One can use this effect for terminating the absorption-characteristic at a frequency above which absorption is in practice no longer necessary, for this is as a rule the case for frequencies higher than about 1500 Hz.

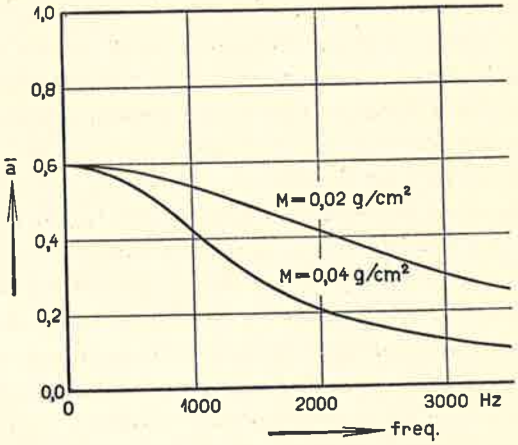


Fig. 12. Influence of a coating mass of 0.02 g/cm² and 0.04 g/cm² on compressible material with 60% average absorption.

Fig. 12 shows the effect of a coating layer with a mass of 0,02 and 0,04 g/cm² on material with 60% average absorption and of which the impedance is real. Generally speaking, the coating layer of a material with weak absorption may be heavier than of a material with strong absorption. In our opinion $M = 0{,}02$ or $0{,}03$ g/cm² is most likely to give satisfactory results *).

*) In practice this can very well be realised.

9. *Experimental results; possibilities of practical application.* In due time our experimental results will be published as a whole, but we can say now already that they corroborate the theory developed abovein its broad outline. Fig. 13 shows one of our results. The figure applies to the case of a coated sponge rubber layer of 3 cm thickness and a specific weight of 0.12; it represents the normal absorption coefficient a_0 plotted against the frequency. As is apparent from the figure we found for frequencies from $\pm$ 200 Hz to 1500 Hz, for a_0 a value of about 40%, which means an average absorption of about 55%. The value of M of the coating layer (triacel) was about 0.01 g/cm². Any cut-off effect at higher frequencies is, therefore, still out of the question.

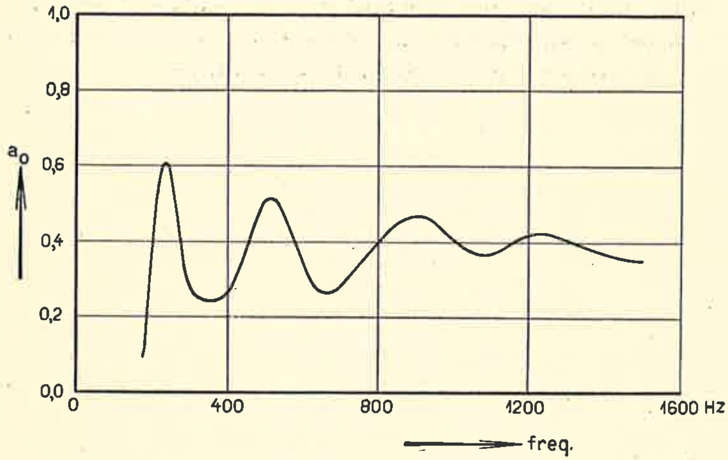


Fig. 13. Measured normal absorption of coated sponge rubber layer; specific weight 0.12, thickness 3 cm.

It will be obvious, that experiment must ultimately show the way to obtaining the most suitable material, for theory can, as a matter of course, do no more than give general indications. We will summarize here the possibilities of arriving at satisfactory practical applications. Theoretically speaking, a very low specific weight is to be desired and one must aim at material with pores of such a width that the air is sure to vibrate to some extent freely; this increases the losses, thereby decreasing the selective absorption. For the rest, selective absorption is not a serious objection, for we can counteract its effect by other means, namely by making the absorbing wall more or less inhomogeneous. This can be done, for example, by applying at regular intervals holes in the homogeneous layer. These holes can be vulcanized directly into the material, while, moreover, they would presumably give rise to extra losses, as they would facilitate the flow of the air from regions of high to regions of low pressure. For one and the same quantity of rubber this means, besides, a thicker layer, by which an effect as in fig. 10 would be obtained. The selective absorption can also be counteracted by an only partial fastening of the layer to its rigid support. The characteristic frequencies of the loose parts will differ from those of the fastened parts, and thereby the selective absorption is on the average, made to decrease. Still another possibility is the combination of layers of two materials with equal thicknesses but different specific weights. These must be chosen in such a way that characteristic frequencies are shifted relatively to each other which leads to the same effect.

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No.	Schrijver Schriftsteller Auteur Author	Titel. Titel. Titre. Title.	Jaar / Jahr Année / Year	Nederl.	Deutsch	Français	English
12	Dr. C. Koningsberger	De synthese van rubber.	1939	a	n	n	n
13	Ir. C. Kuijper	Het gebruik van luchtbanden in den landbouw.	1939	a	n	n	n
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