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A NEW METHOD FOR MEASURING SOUND ABSORPTION

by C. W. KOSTEN

Laboratorium voor Technische Physica der Technische Hogeschool, Delft

Summary

The electric impedance of a loudspeaker depends upon the acoustical load and may be used for measuring this load. The relation between the electric impedance of an electrodynamic loudspeaker and the acoustic impedance in front of the loudspeaker is given and discussed in detail. The consequences of the flexibility of the cone are studied. Simple formulas and graphs are given, connecting the electrical behaviour and the absorption coëfficient corresponding to the load on the loudspeaker. The method seems to permit absorption measurements at low frequencies (e.g. 50–500 Hz).

§ 1. *Introduction*. In general the electrical alternating current resistance (impedance) of a sound source (telephone, electrodynamic loudspeaker, quartz oscillator etc.) depends more or less on the acoustical load, i.e. on the acoustic impedance of the radiating surface. This is a well-known fact [1]). Whether or not this dependence may be used for computing the acoustic impedance from measurements of the electric impedance of the source, depends again upon the simplicity of the relation between the electric and the acoustic impedance and upon the relative magnitude of this acoustical reaction. The relation between both impedances referred to is in all cases a simple inverse one (see below) and the decisive question is that of the relative magnitude of the effect. The reaction on a telephone in resonance is large. This apparatus, therefore, can be used with advantage [2]). The telephone is, however, only sensitive in resonance. Hence, in order to enable us to measure at various frequencies a telephone with adjustable resonance frequency or a set of telephones with various resonance frequencies must be used.

This is no doubt a serious drawback of using a telephone. It would take us too far to discuss here the features of all possible systems. We confine ourselves to the mere statement that the electrostatic reaction (in principle the change in the impedance of a condensor loudspeaker, caused by acoustical load) is of a much too small order to be of much use, and to the description in the following §§ of the method based on the reaction on a normal moving coil electrodynamic loudspeaker.

It will be hardly necessary to stress here the practical importance of the measurement of the acoustic impedance. On the one hand the acoustic impedance z is connected by a simple formula [3]) with the absorption coëfficient a, viz.

$$a = 1 - \left| \frac{z - \varrho c}{z + \varrho c} \right|^2,$$

where ϱ = density of air, c = sound velocity in air. Therefore, when z is known, a may be computed or read directly from a graph. On the other hand the way in which z depends upon frequency gives us some information about the absorption mechanism to which the absorption of the material under test is due.

The way in which the investigations are carried out is in principle that of an ordinary interferometer. Loudspeaker and absorbing sample are placed at the two ends of an iron tube. Only normally incident sound is involved, so the use of the computed absorption coëfficient is confined to cases of normal incidence.

The method can only be used with rather low frequencies. The mechanical impedance of the loudspeaker cone itself above resonance (50 à 100 Hz) increases approximately proportional to frequency ($j\omega m$). At high frequencies (say above 1000 Hz) this impedance of the cone will in general be high in comparison with ordinary acoustical loads, resulting in a relative low sensitivity to acoustical load variations at high frequencies. As a rule ordinary loudspeakers seem to allow measurements with sufficient accuracy up to 700 à 1000 Hz.

§ 2. *Relation between electrical and acoustical impedance.* Let

e = the complex a.c. tension at the terminals of the coil,

i = the complex alternating current through the coil,

Z = complex electric impedance of the coil when it is not allowed to vibrate (loudspeaker with fixed cone),

Z_{tot} = idem with vibrating coil,
B = magnetic induction ($= \mu_0 H$) in the airspace of the (permanent) magnet,
l = total length of the thread which forms the coil,
v = complex "a.c." velocity of the coil,
Z_{mech} = complex mechanical impedance of the coil due to the stiffness, mass and losses in the cone material as well as to the total acoustic load on the cone (dimension: force/velocity).

Then two equations may be obtained, an electrical and a mechanical one, namely

$$e = iZ + Blv, \tag{1}$$

$$0 = -Bli + Z_{mech}\, v. \tag{2}$$

In (1) Blv stands for the e.m.f. of induction in the coil due to the velocity v of the coil. The term Bli in (2) is the Lorentz force on the coil due to the current i.

Eliminating v from (1) and (2) and replacing e/i by the total impedance Z_{tot}, we obtain

$$Z_{tot} = Z + \frac{(Bl)^2}{Z_{mech}\, v}. \tag{3}$$

According to this well-known relation the total impedance consists of two parts, the normal impedance Z to which is added a part due to the motion. Indeed this second part vanishes when Z_{mech} is increased to infinity. This part, therefore, is generally called *the motional impedance.*

The motional impedance may be found as a function of frequency by measuring both Z_{tot} and Z as a function of frequency and subtracting Z from Z_{tot}. At any frequency Z_{mech} can be calculated by dividing the motional impedance by the constant $(Bl)^2$ and inverting finally. Assuming that the cone moves as a whole with the same velocity, we may write

$$Z_{mech} = j\omega m + r + \frac{1}{j\omega c} + Sz = Z_{cone} + Sz.$$

The terms $j\omega m$, r and $1/j\omega c$ form together the mechanical impedance of the cone itself. The term Sz would be easily understandable if the vibrating surface were a plane piston of surface S, loaded with an acoustic impedance z, constant over the surface S. The consequences of the fact that the behaviour of the loudspeaker in experiment differs widely from that of a piston will be considered later on.

We conclude that the specific acoustic impedance z can be calculated from the electric impedance Z_{tot}, provided Z, $(Bl)^2/Z_{cone}$ and $(Bl)^2/S$ are known.

These constants of the apparatus can be measured once for all by carrying out measurements with the cone: *a*) absolutely fixed, *b*) absolutely free (acoustical load zero), *c*) loaded with a known acoustical load, e.g. the wave resistance ϱc of air (100% absorbing sample). It will be understood that we ought to know the constants as functions of the frequency. The constants will be slightly dependent on temperature, humidity and other quantities; exact data concerning these points have not yet been obtained, but it seems that a slight dependence is not a serious objection.

§ 3. *The coördinates in the acoustical z-plane.* Let p_i and p_r respectively be the complex sound pressure of the incident and reflected plane wave. Then

$$a = 1 - \left| \frac{p_r}{p_i} \right|^2 = 1 - \left| \frac{z - \varrho c}{z + \varrho c} \right|^2.$$

Replacing the complex quantity z by $x + jy$ and keeping a constant, this equation yields the analytic expression of the line in the z-plane, each point of which will correspond to the same a-value. These lines a = constant are circles, forming together a set of circles (see fig. 1). Orthogonal to these circles other circles can be found, also forming a set and having a simple physical meaning. They are circles of constant phase difference Δ between $z - \varrho c$ and $z + \varrho c$ or between p_r and p_i (see fig. 1). $+ \varrho c$ and $- \varrho c$ are points of all Δ-circles, the basic points of the set.

When x and y are given, a and Δ may be computed. The reverse is also true. a and Δ may be used as coördinates in the z-plane as well as x and y; and, since a and Δ have a simple physical meaning, they are often to be preferred. An obvious example is the interferometer. Let the impedance of the sample be $z (= p/v$ at the surface of the sample). At a distance l before the sample the value of the acoustic impedance generally will be different. It is evident at once, however, that the absorptioncoëfficient of the sample is independent of l, i.e. on proceeding from the sample inwards in the tube, z will describe a circle $a =$ constant in the z-plane. It even is not at all difficult to find the connection between l and the corresponding point of

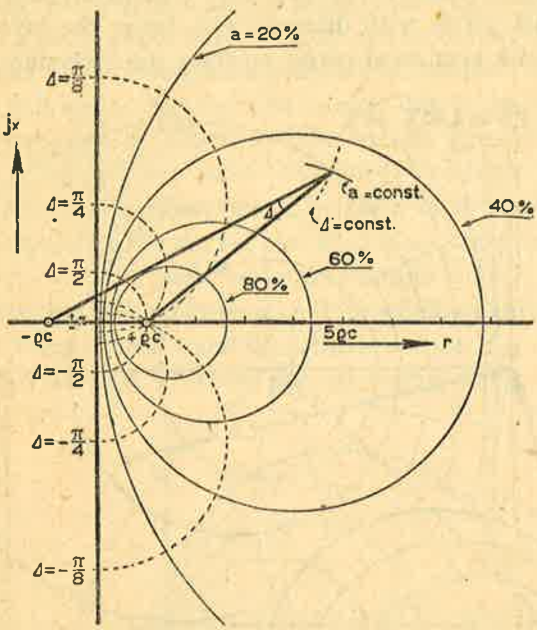

Fig. 1. Circles a = constant and Δ = constant in the complex z-plane.

the circle, for on proceeding inwards the phase of p_i and p_r respectively increases (or decreases) linearly with l, 180° for each half wave length $\lambda/2$. The argument Δ of p_r/p_i hence varies proportional to l, 360° for each half wave length. Proceeding over $\lambda/2$ means travelling along a complete a-circle of the z-plane.

§ 4. *The relation between motional and acoustical impedance.* This relation in an analytical form is

$$Z_{mot} = \frac{(Bl)^2}{Z_{cone} + zS}.$$

In order to avoid numerical complex calculations, which are very troublesome, it is worth while to study this relation. Since the loudspeaker is often used above resonance, Z_{cone} is chosen in fig. 2 with a positive imaginary part. At resonance this part equals zero, beneath resonance it will be negative. To Z_{cone} must be added zS, as is symbolised by the circle diagram with ϱcS as a basic point. Now inverting $Z_{cone} + zS$ with an arbitrary unit of inversion it is easily seen that the circle with diameter OA, being the inversion of the 0%-line of the acoustical plane, encloses the inversion points of all

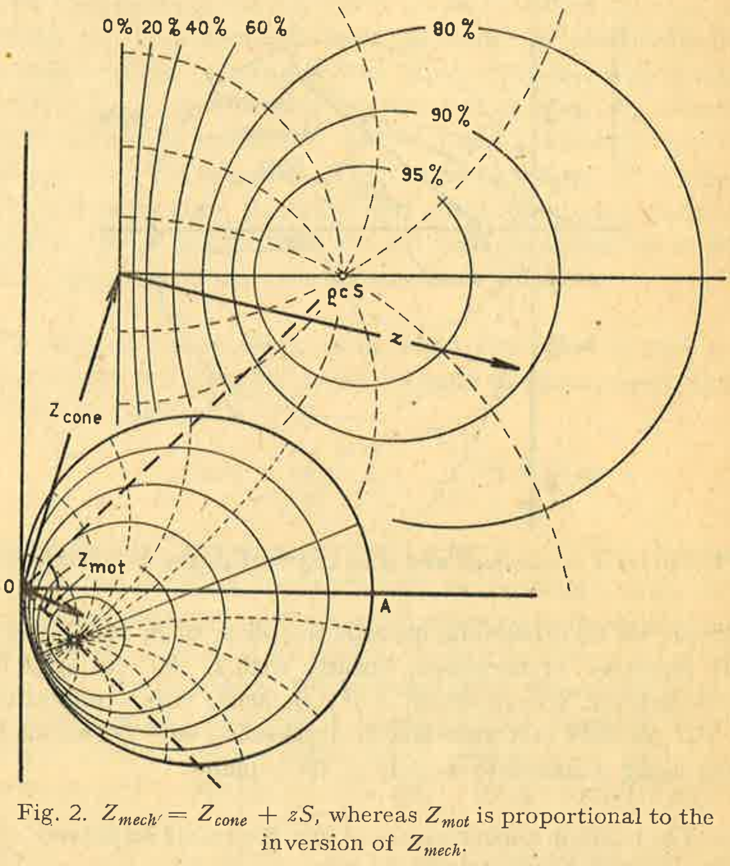

Fig. 2. $Z_{mech'} = Z_{cone} + zS$, whereas Z_{mot} is proportional to the inversion of Z_{mech}.

possible values of $Z_{cone} + zS$, since all values are lying to the right of the 0%-line. Furthermore the two orthogonal sets of circles of the acoustical plane are inverted into identical orthogonal sets *); for, the basis points of the Δ-circles after inversion remain points of the

*) I am indebted to Mr. D. H. Bekkering for this suggestion.

inverted Δ-circles. This Δ-set, therefore, remains unaltered, aside from a translation, rotation over a certain angle and magnification with an adequate factor. Moreover, circles with larger diameter than OA are absent.

Now a stage is reached, where experimental control is very easy, since circles a = constant in the z-plane can be obtained by varying the column of air between loudspeaker and sample. The electrical impedance will hence also describe circles, due to the variation in Z_{mot}, namely the inverse a-circles of the z-plane. For different samples different circles will be found, the circle OA corresponding to 0%, the point $\varrho c'$ to 100% absorption. The point of inversion O can be found as the point of the largest electrical absorption circle situated to the extreme left. It must be borne in mind that Z_{mot} cannot be measured directly. We measure Z_{tot}, from which quantity the impedance with fixed cone Z must be substracted (see fig. 3). With various samples we find therefore various circles of fig. 3, the point

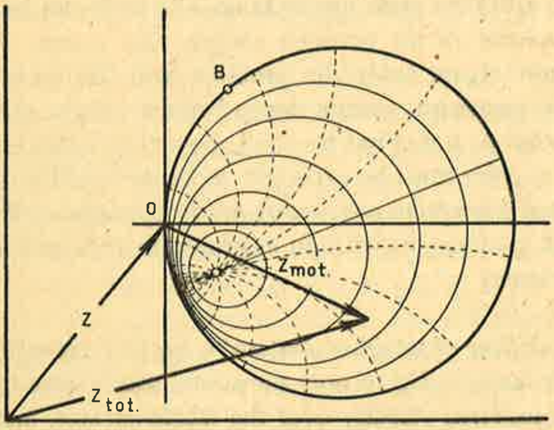

Fig. 3. $Z_{tot} = Z + Z_{mot}$.

at the extreme left of the largest being the inversion centre (impedance Z). This point corresponds to the point $z = \infty$ in the z-plane. This can be effected by fixing the cone rigidly to the magnet. Another method is to actually make the acoustical load infinite, i.e. the length of the column of air equal to $\lambda/2$ or a multiple of it, and closing the interferometer with a perfectly reflecting sample. Now, since the moving cone is not plane, it needs to be explained what is meant by „the length of the column of air". As long as the wave length λ is very large in comparison with the diameter of the tube, the air in front and in the neighbourhood of the cone can be treated as an incompressible fluid, the sound pressure being independent of the details of the sound source, but only dependent on the total volume of air between sample and source and the volume displacement of the source. Therefore, it must be assumed that the cone is acoustically fixed if the volume between the rigid end of the tube and the cone be $\lambda/2$ times the cross sectional surface of the tube. Now it turned out that the electrical impedance with acoustically fixed cone does not yield the point at the extreme left of the largest circle but a point near B in fig. 3. This was predicted and can easily be explained from the fact that the loudspeaker does not fit in an airtight manner into the tube (because it must be movable), or even if it did so, from the fact that the relative velocity distribution along the conic surface depends upon the acoustical load. To what point the inversion point is shifted by these effects cannot be predicted, since all quantitative data are lacking. The shift can be computed from the volume of air between sample and source (see below), but no doubt errors enter the problem and the question arises, whether the numerical results depend much on the choice of the inversion point or not. First we shall show that indeed the shift of the inversion point must be expected, secondly it will be shown that the computed *absorption coëfficients are independent of the inversion point,* which provides us with an easy means of measuring absorption coëfficients.

§ 5. *The shifting of the inversion centre due to the compliance of the cone and air leakage.* Up to now the sound source was treated as a piston with uniform velocity over the whole surface, moving in an airtight manner in the tube. We want now to get rid of these restrictions. The equation

$$e = Zi + Blv$$

remains valid with a normal loudspeaker, provided that with v is meant the velocity of the coil. The equation

$$0 = -Bli + v(Z_{cone} + zS)$$

must be reconsidered. It is the equation of motion of the cone. Bli is the correct expression for the Lorentz force. If Z_{cone} be defined as the ratio of the force at the coil and the coil velocity in the case of no acoustical load in front of the loudspeaker, vZ_{cone} denotes the force needed to maintain the velocity v in the absence of frontal acoustical load. vzS should be the expression for the extra force necessary to overcome (counteract) the acoustical load. It remains a question, however, what must be taken for v and S since the velocity varies along the surface of the cone, and the vibrating surface S is such a vague quantity. We proceed now to give an expression for this extra force, complying with the features of the cone. It will be assumed that the wave length $\lambda \gg$ diameter of the tube, so that p may be taken constant over the surface of the cone. Using polar coördinates ϱ, θ to denote a point on the cone, we can write for the velocity of the cone at ϱ, θ

$$v_{\rho,\theta} = v \,.\, \xi(\varrho, \theta) - p \,.\, \eta(\varrho, \theta),$$

where ξ and η are, if necessary, complex functions of ϱ and θ, ξ being dimensionless, η having the dimension of an acoustical admittance.

According to this equation the velocity at any point $v_{\rho,\theta}$ is assumed to be a linear function of the coil velocity v and the acoustical counterpressure p. Terms in v^2, p^2, vp etc. may be omitted at first as terms of second order. The meaning of the linear terms in v or p is evident. $v \,.\, \xi(\varrho, \theta)$ is the velocity of the point ϱ, θ of the cone in the absence of acoustical load ($p = 0$). This partial velocity will be proportional to v. In quite a similar way $p \,.\, \eta(\varrho, \theta)$ is the velocity of the point ϱ, θ of the cone with $v = 0$, i.e. the velocity at ϱ, θ *with fixed coil* under the influence of the pressure p in front of the cone. This second partial velocity, therefore, is a measure of the compliance of the cone *with fixed coil* against the sound pressure. Averaging over the cross-sectional surface of the tube we get

$$\bar{v} = v\bar{\xi} - p\bar{\eta},$$

in which $\bar{\xi}$ and $\bar{\eta}$ have to be looked upon as constants of the loudspeaker. Now $\bar{v} = p/z$, because $\lambda \gg$ the diameter of the tube and, therefore, only the average velocity over the surface is of importance for determining p. Combining these equations there results

$$p = v \frac{\bar{\xi}}{\bar{\eta} + 1/z}. \tag{4}$$

The total sound force transmitted over the whole cross-section is p times the cross-section. Only part of this force, however, will be felt by the coil. In this connection the cone may be compared with a plate, supported at the circumference and the centre, which is loaded homogeneously. In this case the centre is loaded by one third of the total load. Whether in our case this fraction equals also 1/3 or not does not matter. In any case, however, this force will be proportional to p. The equation of motion of the cone therefore may be written in the form

$$0 = -Bli + \left(Z_{cone} + \frac{A_1}{\bar{\eta} + 1/z}\right)v,$$

where A_1 is an unknown complex constant. Combining this equation of motion with the electrical equation yields

$$Z_{tot} = Z + (Bl)^2 \Big/ \left[Z_{cone} + \frac{A_1}{\bar{\eta} + 1/z}\right],$$

an expression differing essentially from the corresponding previous one. Introducing new complex constants A_i, this latter equation may be changed into

$$Z_{tot} = Z + A_2 + (Bl)^2 A_3/(z + A_4) = A_5 + A_6/(z + A_7) \quad (5)$$

Conclusions: *a*) the inverse relation between Z_{tot} and z remains valid;

b) the real constant $(Bl)^2$ becomes complex, i.e. the motional impedance graph is rotated over an unknown angle;

c) the inversion centre is no longer the point at the extreme left of the largest circle of the electrical impedance diagram, but is that point that corresponds with $z = \infty$. Therefore, and this conclusion is most important, the centre of inversion cannot be obtained be fixing the coil rigidly to the magnet, but only by making the acoustical load infinite. Acoustical fixing does not mean v = zero; on the contrary, the velocity v of the coil, when the acoustical load is infinite, can be computed from equation (4) by taking $z = \infty$.

d) the derivation is quite general and includes leakage through narrow slits between loudspeaker and interferometer tube.

§ 6. *The absorption coëfficient independent of the inversion centre.* Let the circles 1′, 2′ and $\varrho c'$ in fig. 4 be obtained in the electrical impedance plane (Z_{tot}) with samples of resp. 0%, a% and 100% absorption. The inversion point (corresponding to $z = \infty$) will be unknown, but in any case will be a point of circle 1′. 1′, 2′ and $\varrho c'$ are three circles of a set; the centres of all circles of this set are lying on the straight line $\overline{\varrho c' . M_2' . M_1'}$. In order to carry out the inversion we choose an arbitrary point O on 1′ as the centre of inversion and take as unit of inversion the distance $\overline{O . \varrho c'}$. The motional impedance is to be taken on the turned system OXY, thus taking into account the argument of the complex constant A_6 of equation (5). The inverted point of $\varrho c'$ has a positive ordinate y, because the arguments of $\varrho c'$ and its inversion are opposite in sign. For reasons of simplicity this inversion of the sign of all arguments is omitted, so each point lies on the same straight line through O with its own inverted point, $\varrho c'$ even being identical with its inversion ϱc.

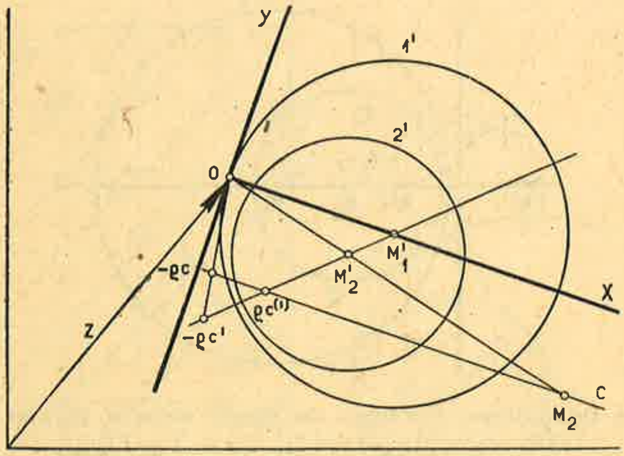

Fig. 4. Circles of 0%, a% and 100% absorption in the electrical Z_{tot}-plane and the proof that a is independent of the centre of inversion 0 on circle 1′.

The inversion of circle 1′ is a straight line 1 parallel to the Y-axis, which line is not shown in fig. 4. ϱc ($= \varrho c'$) and 1 determine together the new set of circles, the line of centres of the a-circles being the line $\overline{\varrho c . C}$ perpendicular to 1. E.g. the centre of the inverted circle 2′, called 2, must be M_2, since this centre lies on $\overline{\varrho c . C}$ and on $\overline{OM_2'}$ (for reasons of symmetry). (M_2 is the centre of the inverted circle 2, but not the inverted centre of the circle 2′

(M_2')). Therefore, 2 will be that circle of the new set of which the centre is M_2. Now it is easily seen that the distance $\overline{\varrho c \,.\, M_2}$ is independent of the choise of O, for

$$\overline{\varrho c \,.\, M_2} : \overline{\varrho c' \,.\, M_2'} = \overline{OM_1'} : \overline{M_2'M_1'}\,, \tag{6}$$

in which equation the last three quantities are independent of O and therefore, the first must be so too. Along the same lines it can be shown that the distance $\overline{-\varrho c \,.\, \varrho c}$ is independent of O, for $-\varrho c$ will be the point of intersection of the lines $\overline{O \,.\, -\varrho c'}$ and $\overline{\varrho c \,.\, C}$, and

$$\overline{-\varrho c \,.\, \varrho c} : \overline{O\,M_1'} = \overline{-\varrho c' \,.\, \varrho c'} : \overline{-\varrho c' \,.\, M_1'}\,, \tag{7}$$

in which equation again all quantities must be independent of O, because the last three are so.

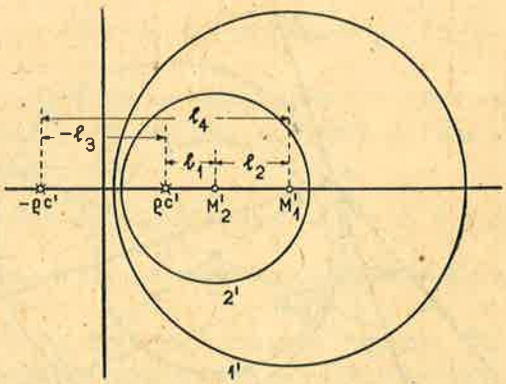

Fig. 5. Dependence of a upon the double ratio of M_2' and $-\varrho c'$ with respect to $\varrho c'$ and M_1'. $1/a = 1 - l_1 l_4 / l_2 l_3$.

The absorption coëfficient belonging to 2, and therefore to 2', may be computed with the aid of the equation of § 1, from which the following remarkably simple relation may be derived:

$$1/a = \overline{-\varrho c \,.\, M_2} : \overline{-\varrho c \,.\, \varrho c}.$$

This relation again, together with (6) and (7), yields

$$1/a = \frac{\overline{\varrho c' \,.\, M_2'}}{\overline{M_2'M_1'}} : \frac{\overline{-\varrho c' \,.\, \varrho c'}}{\overline{-\varrho c' \,.\, M_1'}} + 1.$$

None of these quantities which determine a depend upon the chosen point O. The inversion, even with respect to an arbitrary chosen centre of inversion, can be omitted, for in the last equation only quantities of the electrical impedance diagram enter.

a depends upon the double ratio of M_2' and $-\varrho c'$ with respect to $\varrho c'$ and M_1'. In simplified notation (see fig. 5)

$$\frac{1}{a} = 1 - \frac{l_1}{l_2} : \frac{l_3}{l_4},$$

the minus sign arising from the fact that all distances are counted positive from the left to the right in fig. 5, negative in opposite direction. This relation can be represented by fig. 6. Since l_4 is essentially larger than $-l_3$, only values of $-l_3/l_4$ smaller than unity have a physical meaning.

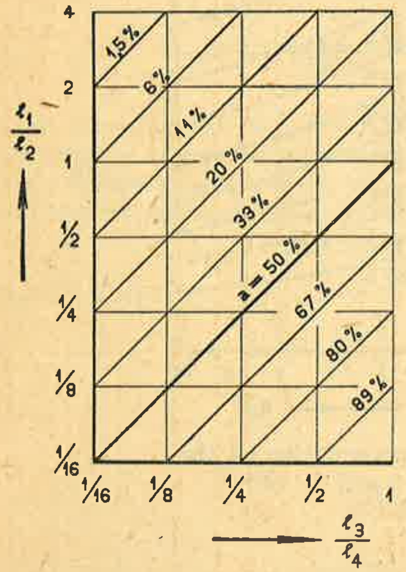

Fig. 6. Graphical representation of the relation between a and the ratios l_1/l_2 and l_3/l_4.

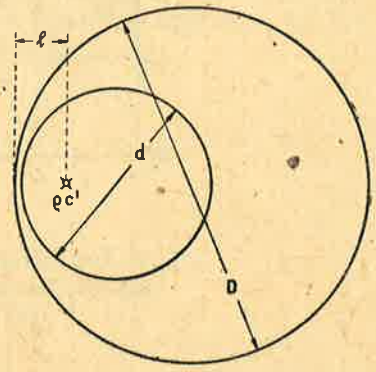

Fig. 7. Dependence of a upon the ratios D/l and d/D in the electrical impedancediagram (see fig. 8).

Fig. 6 can only be used when the point $-\varrho c'$ in the electrical impedance diagram has been constructed. It seems more practical to represent a as a function of the ratios d/D and D/l (see fig. 7).