that was seen. There was a proper canal for the spinal cord, but it had no osseous covering. The deep groove bifurcated at the cranial extremity into grooves of half its size, which took a direction at right angles to that of the former. The left pneumogastric nerve was seen passing through the base of the cranium to the surface, where it appeared to have come from the membrane from which other nerves proceeded. After descending to the cervical region and giving off the recurrent, the principal branch was not continued to the lungs and œsophagus, but directly to the ganglion of the sympathetic in the upper part of the thorax, so that the sympathetic chain of ganglia in the thorax appeared to be simply a continuation of the pneumogastric. To compensate for this absence of nervous supply on the left side, the nervous plexuses on the roots of the lungs were found to be enormously increased on the opposite side. A large branch ascended from the solar plexus and united with the divisions of the right pneumogastric. The splanchnic on this side was large, and was composed of filaments from the upper thoracic ganglia, not merely from those below the sixth. The action of the heart and the functions of the liver, kidneys, and other organs must have continued during the uterine existence of the fœtus.

The author expects to be afforded further means of prosecuting his dissections of the nerves of acephalous monsters, in which case he will communicate the results of his examinations to the Royal Society.

II. "On the Conditions, Extent, and Realization of a Perfect Musical Scale on Instruments with Fixed Tones." By Alexander J. Ellis, B.A., F.C.P.S. Communicated by C. Wheatstone, Esq., F.R.S. Received January 7, 1864.

Euler*, perceiving that the relative pitches of all musical notes might be represented by $2^m . 3^n . 5^p$, formed different "genera musica" by allowing n and p to vary from 0 to fixed limits. His "genus diatonicum hodiernum" (*op. cit.* p. 135) limits n to 3 and p to 2, and consists of 12 tones. These tones and 12 others are contained in his "genus cujus exponens est $2^m . 3^7 . 5^2$," that is, which limits n to 7 and p to 2. He has further (*ib.* p. 161) given a scheme in which each manual of an instrument should represent two sounds, the primary belonging to the first 12 tones, and the secondary to the additional 12. He says (*ib.* p. 162), "Soni secundarii summo rigore ab iisdem clavibus edi nequeunt, quia vero tam parum a primariis discrepant, ad eos exprimendos hæ claves *sine sensibili harmoniæ jactura* tuto adhiberi possunt. Nam *etiamsi ab acutioribus auribus comma seu diaschisma,* quibus intervallis soni secundarii a primariis differunt, *distingui queat,* tamen quia soni secundarii cum primariis *neque*

* Tentamen Novæ Theoriæ Musicæ ex certissimis Harmoniæ principiis dilucide expositæ auctore Leonhardo *Eulero*. Petropoli, 1739.

in eadem consonantia neque in duarum consonantiarum successione misceri possunt, error etiam ab acutissimo auditu percipi non poterit." It will appear in the sequel that these assertions, when tested by experiments on instruments with fixed tones, are all incorrect.

The musical scale has formed the subject of many recent investigations*; but I have been unable to find a complete account of the necessary conditions to be fulfilled by a perfect scale, the least number of fixed tones required, and the practical means of producing them uncurtailed without inconvenience to the performer, although instruments which produce a *limited* number of just tones have been practically used by Gen. Perronet Thompson, Mr. Poole, Prof. Helmholtz, Prof. Wheatstone, myself, and others. This is therefore the subject of the present paper.

The following notation is employed. I have introduced it for the purpose of supplying a want which has been greatly felt by all writers on the theory of music. It is founded on the principle of retaining the whole of the usual notation unaltered, but restricting its signification so as to prevent ambiguity, and introducing the smallest possible number of additional signs to express the required shades of sound with mathematical accuracy, selecting such signs as are convenient for the printer, and harmonize with the ordinary notation of accidentals on the staff.

A letter, as C, called a *note*, will represent both a certain *tone* and its *pitch*, defined to be the number of *double vibrations* in one second, to which the tone is due. The letters D, E, F, G, A, B represent other tones and pitches, so that

$$8D = 9C,\ 4E = 5C,\ 3F = 4C,\ 2G = 3C,\ 3A = 5C,\ 8B = 15C.$$

Small and numbered letters will be so used that

$$c = \frac{1}{2}c^2 = \frac{1}{4}c^4 = \frac{1}{8}c^8 = \ldots,\ 2C = 4C^4 = 8C^8 = \ldots,$$

and similarly for other letters. The pitch of c is that of the "tenor or middle c," usually written on the leger line between the treble and bass staves; and the other letters are noted on the staff as usual in the scale of C major.

* *Gen. T. Perronet Thompson,* F.R.S., Instructions to my daughter for playing on the Enharmonic Guitar, 1829; Just Intonation, 6th ed. 1862. *H. W. Poole,* On a perfect musical Intonation, Silliman's American Journal of Science, 2nd ser. vol. ix. pp. 68 and 199. *W. S. B. Woolhouse,* Essay on Musical Intervals, 1835. *Prof. A. De Morgan,* Cambridge Philosophical Transactions, vol. x. p. 129. *M. Hauptmann,* Die Natur der Harmonik, Leipzig, 1853. *M. W. Drobisch,* Abhandlungen der Fürstlich Jablonowskischen Gesellschaft, 1846; Poggendorff's Annalen, vol. xc. *C. E. Naumann,* Ueber die verschiedenen Bestimmungen der Tonverhältnisse, Leipzig, 1858. *Prof. H. Helmholtz,* Lehre von den Tonempfindungen, Braunschweig, 1863. To this last writer we owe the first satisfactory theory of consonance and dissonance.

The following symbols always represent the fractions, and are called by the names written against them :

1. *Following a Note.*

♯ $= \dfrac{135}{128} =$ sharp, or greater limma.

♭ $= \dfrac{128}{135} =$ flat, or hypolimma.

× $=$ ♯ . ♯ $=$ double sharp ; ♭♭ $=$ ♭ . ♭ $=$ double flat.

2. *Preceding a Note.*

† $= \dfrac{81}{80} =$ acute, or comma.

‡ $= \dfrac{80}{81} =$ grave, or hypocomma.

ʒ $= \dfrac{63}{64} =$ septime (an inverted 2).

¶ $= \dfrac{32805}{32768} = 1{\cdot}001129150390625 =$ schisma.

‖♭ $= \dfrac{32768}{32805} = 0{\cdot}99887212315 =$ hyposchisma.

The name and pitch of the tones represented by any such notes as

‡*c*♯ $=$ grave *c* sharp $= \dfrac{80}{81} \cdot c \cdot \dfrac{135}{128} = \dfrac{25}{24}\, c,$

†*e*♭ $=$ acute *e* flat $= \dfrac{81}{80} \cdot e \cdot \dfrac{128}{135} = \dfrac{24}{25}\, e,$

and the ratio of their pitches to the corresponding notes in the scale of *C* major is therefore precisely indicated. In ordinary musical notation on the staff, it is only necessary to prefix the signs †, ‡, ʒ, ¶, ‖♭, to those already in use. These symbols suffice for writing any tone whose index is the product $2^m . 3^n . 5^p . 7^q$ (see Tables I. and III.). For equally tempered tones, when it is necessary to distinguish them, the sign ‖ is prefixed to the usual names, and read "equal." Since

$$\Vert g : c = \sqrt[12]{2^7} : 1 = 0{\cdot}998871384584 \times \tfrac{3}{2},$$

and

$$\Vert\flat\, g : c = 0{\cdot}99887212315 \times \tfrac{3}{2},$$

we may without sensible error consider ‖*g* $=$ ‖♭*g*, and hence represent the equally tempered scale

c, ‖*d*♭, ‖*d*, ‖*e*♭, ‖*e*, ‖*f*, ‖*f*♯, ‖*g*, ‖*a*♭, ‖*a*, ‖*b*♭, ‖*b* by

c, ¶⁵‡*d*♭, ‖♭²*d*, ¶³*e*♭, ‖♭⁴†*e*, ¶*f*, ‖♭⁶†*f*♯, ‖♭*g*, ¶⁴*a*♭, ‖♭³†*a*, ¶²*b*♭, ‖♭⁵†*b*.

In calculating relative pitches or intervals, and in all questions of temperament, it is most convenient to use *ordinary* logarithms to five places, because the actual pitches, and the length of the monochord (which is the reciprocal of the relative pitch), can be thus most easily found. In Table I. the principal intervals are given as fractions, logarithms, and *degrees*. If we call 0·00568 one *degree*, then 53 degrees $=0·30104=\log 2-0·00001$, and 31 degrees $=0·17608=\log \frac{3}{2}-0·00001$. If we moreover represent the addition and subtraction of 0·00035 (or one-sixteenth of a degree) by an acute or grave accent respectively, then 17′ degrees $=0·09691=\log \frac{5}{4}$, and 1‵ degree $=0·00533=\log \frac{81}{80}-0·00007$. Two numbers of degrees which differ by a single accent of the same kind, as 17′, 17″ represent notes whose real interval is a schisma (thus *e* has 17′ degrees; and *d*×, =¶*e*, has 17″ degrees), having a difference of logarithm $=0·00049$ or 0′ degrees $+0·00014$. By observing this, degrees may be very conveniently used for all calculation of intervals between tones of pitches represented by $2^m \,.\, 3^n \,.\, 5^p$. Table IV. contains a list of tones which differ from each other by a schisma, and will be useful hereafter.

The conditions of a perfect musical scale are *not* discovered by taking all the tones which can be expressed by one of Euler's "exponents," nor by forming all the tones which are consonant with a certain tone, and then all the tones consonant with these, as Drobisch has done. Such processes produce many useless, and omit many necessary tones. Since modern music depends on the relations of harmonies, and not on scales, it is necessary to find what consonant chords of three tones are most closely connected*.

Three tones whose pitches are as 4 : 5 : 6, or 10 : 12 : 15 form a major or minor consonant chord respectively. The same names are used when any one or more of the pitches is multiplied or divided by a power of 2, notwithstanding the dissonant effect in some cases. Thus, $C:E:G=$ 4 : 5 : 6 is a major, and *c* : †*e*♭ : *g* = 10 : 12 : 15 is a minor chord, and the same names are applied to $e:g^2:c^4=5:2\times 6:2^2\times 4$, and *G* : †*e*♭ : $c^2=$ $15:2\times 12:2^2\times 10$, although these chords are really dissonant (Helmholtz, *ib.* p. 333–4). I shall consequently use a group of capitals, as *C E G*, to represent a major chord, and a group of small letters, as *c* †*e*♭*g*, to represent a minor chord, irrespective of the octaves. The three notes in this order, being the first, third and fifth of the major or minor scale commencing with the first, are called the first, third and fifth of the chords respectively. Both chords contain a fifth, a major and a minor third. If the interval of the *fifth* is contained by the same tones in a major and minor chord, as

* There are consonant chords of four tones, such as *g b* d^2 ʒf^2, and these are insisted on by Poole (*loc. cit.*); but, though they are quite consonant and agreeable, and much pleasanter than the dissonant chords by which they are replaced, such as *g b* $d^2 f^2$, they do not form a part of modern music, for reasons clearly laid down by Helmholtz (*op. cit.* p. 295). Dissonant chords must always arise from the union of tones belonging to two consonant chords, or from the inversions of consonant chords; and therefore their tones are determined with those of the others.

C E G, c †e♭ g, or *A ‡C♯ E, a c e,* the chords are here termed *synonymous.* If the interval of the *major third* is contained by the same tones, as *C E G, a c e*; or *†E♭ G †B♭, c †e♭ g,* they are termed *relative.* If two chords, major or minor, have the fifth tone of the one the same as the first tone of the other, as *F A C, C E G*; *f †a♭ c, c †e♭ g*; *f †a♭ c, C E G*; *F A C, c †e♭ g,* they are here termed *dominative.* If a chain of three such dominative chords be formed (as *F A C, C E G, G B D,* or *f †a♭ c, c †e♭ g, g †b♭ d,* the minor and major chords being interchanged at pleasure), the first is called the *subdominant,* the second the *tonic,* and the third the *dominant.* Three such chords contain seven tones, and if such octaves of these tones are taken that all seven tones may lie within the compass of one octave they form a scale, of which 24 varieties can be formed by varying the major and minor chords, and beginning with the first of any one of the three chords. These scales include all the old ecclesiastical modes and several others. If all three chords are major and the scale begins on the first of the tonic chord, the result is the *major scale, C, D, E, F, G, A, B, c.* If all three chords are minor and the scale begins on the first of the tonic chord, the result is the *minor descending scale,* c^2, *†b♭, †a♭, g, f, †e♭, d, c.* If the first and second are minor, and the third major, or if the first and third are major and the second minor, we have the two usual *ascending minor scales, c, d, †e♭, f, g, †a♭, b,* c^2, or *c, d, †e♭, f, g, a, b,* c^2. Three major chords may therefore be considered to represent a major scale, but both major and minor chords are necessary for the various minor scales. If to each of three dominative major chords we form the relative and synonymous minor chords, the synonymous and relative majors of these, and the relative minor of this synonymous major, we shall have a group of 9 major and 9 minor chords, which I shall call the *key of the first of the tonic chord.* Thus the following is the

Key of C.

Relative Ma. (of Syn. Mi.).	Synonymous Minor.	Primary MAJOR.	Relative Minor.	Synon. Ma. (of Rel. Mi.).	(Sub-) Relative Minor.
†A♭ C †E♭	f †a♭ c	F A C	‡d f a	‡D ‡F♯ A	‡b ‡d ‡f♯
†E♭ G †B♭	c †e♭ g	C E G	a c e	A ‡C♯ E	‡f♯ a ‡c♯
†B♭ D †F	g †b♭ d	G B D	e g b	E ‡G♯ B	‡c♯ e ‡g♯

These chords contain 16 tones, which, when reduced to the compass of the same octave, form the *complex scale c, ‡c♯, ‡d, d, †e♭, e, f,* (*†f*), *‡f♯, g, ‡g♯, †a♭, a, †b♭,* (*‡b*), c^2, of which the acute fourth (*†f*), and the grave seventh (*‡b*), have been enclosed in parentheses, as being of rare occurrence. From this complex scale 54 scales of 7 tones each may be formed, similar to the 24 scales already named. A selection of 12 tones, such as *c, ‡c♯, d, †e♭, e, f, ‡f♯, g, †a♭, a, †b♭, b,* c^2 forms the so-called *chromatic scale,* which, however, has no proper existence except in equal temperament.

Now proceed to form a series of seven dominative major chords, as

E♭ ‡G B♭, B♭ ‡D F, F A C, C E G, G B D, D F♯ †A, †A C♯ †E, and form the five related chords of each as before. The result will be *five* keys, as those of *B♭, F, C, G, D,* such that the primary major scales of each will have either two major chords, or one major chord in common with the original primary major scale. I call these five keys the *postdominant, subdominant, tonic, dominant,* and *superdominant* keys, and the whole group of 21 major and 21 minor chords, with the 30 tones which they contain, I term the *system* of the first tone of the tonic chord of the original primary major scale, which tone may be called the *tonic of the system.*

A piece of music is written in a certain system, determined by the compass or quality of tone of the instruments or voices which have to perform it, and rarely exceeds that system*. It is only in the system that the true relation of the tones of a piece of music, the course and intention of the modulation, and the return to the original key or scale can be appreciated. I have not yet found these relations fully expressed in any theoretical work on music; but their full expression was necessary to the solution of the problem here proposed.

It will be found practically that only 11 systems are used in music. These are, in dominative order, the systems of *‡D♭, A♭, E♭, B♭, F, C, G, D, †A, †E, †B,* which contain the 11 keys of the same name, together with the 4 keys of *‡C♭, ‡G♭,* and *†F♯, †C♯*. In Table V., columns III. to VIII., the whole of the major and minor chords of these 15 keys are exhibited in dominative order§. This Table, therefore, furnishes the tones which must be contained in a perfect musical scale of fixed tones, or the *conditions* of the problem.

On examination it will be found that these six columns contain 72 different notes. Hence the *extent* of a perfect scale is fixed at 72 tones to the octave. It is therefore six times as extensive as the equally tempered scale. Some means of reducing this unwieldy extent is required. The most obvious is that proposed by Euler, in the passage already quoted, namely, the use of certain tones for others which differ from them by a comma or diaschisma. Such substitution *within the same chord* creates intolerable dissonance. But in melody and in successions of chords it might seem feasible. I have had a concertina tuned, so that the three chords of

* The use of the equally tempered scale has much diminished the feeling for the relations of the system, by confounding tones originally distinct, and has thus led to the confusion of the corresponding notes. Thus such a note as ‖*g♯* will have to be read as *‡g♯, g♯, †g♯; ‡a♭, a♭* or *†a♭*, according to the requirements of the system, for all six tones are represented by one on the equally tempered scale.

§ The Table of Key-relationships (*Tonartenverwandtschaften*) in Gottfried Weber's *Theorie der Tonsetzkunst* (3rd ed. 1830, vol. ii. p. 86), may be formed from Table V., by suppressing the signs †, ‡, supposing all the notes to represent tempered tones, contracting the names of the chords to their first notes, and extending the Table indefinitely in all directions.

the major scale of *D* are played as *G B D*, *D F*♯ †*A*, and *A* ‡*C*♯ *E*, instead of †*A C*♯ †*E*. The dominant chord is therefore too flat by a comma, and in passing from the chord of *A* to that of *D*, as in the ordinary cadence, the note *A* has to be changed into †*A*. If *A* is the highest or lowest note in the chords, the effect is decidedly bad. The flatness of the "leading note" ‡*C*♯, in place of *C*♯, although only a comma in extent, is felt as annoying in the succession ‡*c*♯, *d*. The result is such that it would not be worth while to invent new instruments with such a defect in common scales. On the same instrument I have the three chords of the major scale of *A* tuned as *D F*♯ †*A*, *A* ‡*C*♯ *E*, *E* ‡*G*♯ *B*, in which the subdominant chord is now a comma too sharp. As the subdominant is a much less important chord than the dominant, the effect is better, but trouble arises from having occasionally to alter the tonic note *A* itself. Even the dissonance of the dominant seventh, when played as *E* ‡*G*♯ *B d* is perceptibly harsher than the correct *E* ‡*G*♯ *B* ‡*d* (both forms lie on the instrument), although the added seventh *d* now forms a true minor third with the fifth *B*, whereas the correct note ‡*d* forms a dissonant Pythagorean minor third with the same note *B*. When, however, the first *E* is omitted, the chord of the diminished fifth ‡*G*♯ *B* ‡*d* is not so pleasant as ‡*G*♯ *B d*. Again, on the same instrument, instead of having ‡*D* ‡*F*♯ *A*, as the synonymous major of ‡*d f a* in the scale of *a* minor, I have only *D F*♯ †*A*, which is a comma too sharp. The rarity of the chord, however, renders the bad effect of less importance. Again, I am obliged to modulate from *D* major to ‡*d* minor instead of *d* minor. Even here the error of a comma is perceptible. The general result, therefore, is that *commatic substitution, even within the same melody or succession of chords, is inadmissible in just intonation.*

Professor Helmholtz (*op. cit.* pp. 433 & 484) has suggested what may be termed *schismatic substitution*, or the use of one note for another which only differs from it by a schisma, the eleventh part of a comma. Having one concertina tuned to equal temperament, and another to just intervals, the equation ‖*g* = ‖♭*g* has enabled me to test this suggestion by practice. I find that in slow chords, the altered fifth *c* ‖♭*g*, the altered major third ‖♭*g b*, and the altered minor third *e* ‖♭*g* are all decidedly, though only slightly, dissonant. In rapid chords the effect would be necessarily much less perceptible. Such chords as *C E* ‖♭*G*, *e* ‖♭*g b* are far superior either to the Pythagorean *C* †*E G*, †*e g* †*b* (of which I can produce the counterparts *F* †*A C*, *d f* †*a*), or the still worse tempered chords *C* ‖*E* ‖*G*, ‖*e* ‖*g* ‖*b*. If we modified Professor Helmholtz's suggestion, and, where practicable, used only *entire* chords which are too flat or too sharp by a schisma, so that the schismatic errors would only occur in harmonies where a note was prolonged from a chord to which it belonged into another for which it was too sharp or too flat by a schisma, then there could be no objection whatever to schismatic substitution, which would be quite inappreciable in melody.

Now schismatic substitution will materially reduce the number of different tones required. By referring to Table IV. it will be seen that all the tones in Table V., lines 1 to 8, throughout all the columns are exactly *one schisma flatter* than the corresponding tones in lines 10 to 17. Hence we only require the tones in lines 5 to 13 in order to reproduce the whole Table, with the help of schismatic substitution. It is, however, more convenient to use columns III., IV., lines 14 to 17, in place of columns I. and II., lines 5 to 8; and columns VII., VIII., lines 1 to 4, in place of columns IX. and X., lines 10 to 13. In this case only 48 tones will be required. If the schismatic substitution of ‡*f*, *a*♭, *c*♭ for *e*♯, *g*♯, †*b* were allowed, which would introduce three schismatic errors of no great importance, the number of tones would be reduced to 45, which is the lowest possible number of tones by which a complete scale can be played. All these tones are enumerated in Table III.

There are several ways of realizing such a scale in whole or in part*. The following appears to be the most feasible, as it would render the mere mechanism of playing a perfect scale on an organ or harmonium easier than that of playing the tempered scale on the same instruments.

On a board of manuals similar to that now in use for the organ, introduce two additional *red* manuals (of the same shape as the black, but with a serrated front edge to be recognizable by blind and colour-blind performers, as in some cases on General Perronet Thompson's organ) in the two gaps between *B* and *C*, and between *E* and *F*, so as to make 14 manuals in all. Let there be 16 stops worked as pedals with the foot, as in Mr. Poole's Euharmonic Organ (*loc. cit.* p. 209). Let one of these stops give the equally tempered tones to the manuals, so that any piece could be played in the tempered scale, and thus compared with the same piece when played with just intervals. Let the 15 other stops give the tones required for the 15 keys ‡*C*♭ to †*C*♯, as shown in Table II., and be numbered 7♭, 6♭ ... 1♭, natural, 1♯.... 7♯. When any pedal is put down, let the seven white manuals give the seven tones of the primary major scale of the corresponding key, and the seven coloured manuals give seven out of the nine other tones required to complete the key, omitting the acute fourth (which would be found in the key of the dominant) and the grave seventh (which would be found in the key of the subdominant). To the right of each white manual let there be its conjugate coloured manual, of such a value that, if the seven tones of the major scale be indicated by the numbers 1 to 7, the tones corresponding to the manuals in any key may be

Coloured..	‡1♯	‡2	†3♭	‡4♯	‡5♯	†6♭	†7♭
White....	1	2	3	4	5	6	7.

Table II. shows the tones associated with the manuals in each stop; capital letters indicate white manuals, small letters black, and small

* Singers and performers on bowed instruments and trombones can produce any scale whatever. Other instruments are more limited in range and would require special treatment, similar to the "crooks" of the horns and the various clarinets.

capitals red*. By this arrangement the fingering of every key would be the same. The performer would disregard the signature except as naming the pedal, and play as if the signature were natural. Table V. would inform him whether the accidentals belonged to the key, its dominant, or any other key; and if they indicated another key, he would change the pedal. It would be convenient to mark where a new pedal had to be used; but no change would be required in the established notation §.

Mr. Poole's organ, which suggested the above arrangement, has 11 stops, from 5♭ to 5♯, and only 12 manuals, which appear to be associated with the following tones on each stop:

Black . .		‡2		(‡2♯)			(4♯)		‡5♯		ƨ7♭	
White . .	1		2		3	4		5		6		7

The two manuals whose notes are put in parentheses are inadequately described. Mr. Poole's scale does not include the synonymous minor chords, which he plays by commatic substitution.

Another method of realizing such a scale is by additional manuals and additional boards of manuals. Thus three boards of manuals, each with 23 manuals, containing the tones in Table V. cols. III. to VIII., lines 4

* On examining Table II. it will be found that 10 different tones lie on each pair of manuals, so that there are only 70 different tones. The two missing tones are, necessarily, ††*f*♯ (the acute fourth of the key of †*C*♯), and ‡‡*b*♭ (the grave seventh of the key of ‡*C*♭); and to this extent the scheme is defective. It would probably be more convenient to the instrument-maker to use all the 70 tones in this arrangement than to take the inferior number 45 due to schismatic substitution. A full-sized harmonium at present employs from 48 to 60 vibrators to the octave, so that the mechanical difficulties to be overcome in introducing 70 are comparatively slight. By omitting the two very unusual keys of ‡*C*♭ and †*C*♯, the 8 tones denoted by ‡‡*d*♭, ‡*F*♭, ‡‡*f*, *a*♭♭ and †*D*♯, *g*×, †*B*♯, ††*b* in Table II. would be saved, and the number of vibrators required would be reduced to 62, nearly the same as that actually in use. As each new key introduces 4 additional tones, and the key of *C* has 14 tones, the number of vibrators required for any extent of scale is readily calculated. Thus for the 11 keys from 5 flats to 5 sharps, or ‡*D*♭, *A*♭, *E*♭, *B*♭, *F*, *C*, *G*, *D*, *A*, *E*, *B*, which is Mr. Poole's range, and is sufficiently extensive for almost all purposes, only $4\times10+14=54$ vibrators to the octave would be required, distributed over 11 stops (exclusive of the tempered notes); and such a number of vibrators and stops is in common use.

§ If in Table V. we reject the marks †, ‡, consider $16\,A=27\,C$, $64\,E=81\,C$, $128\,B=243\,C$, $\sharp=\frac{2187}{2048}$, $\flat=\frac{2048}{2187}$, leaving the value of the other letters unchanged, the Table will represent the Pythagorean relations expressed by the usual notation (which is quite unsuited to the equally tempered scale). The chords thus formed were too dissonant for the Greek or Arabic ear to endure, although Drobisch and Naumann (*loc. cit.* ad finem) desire this system to be acknowledged as "the sole, really sufficient acoustical foundation for the theory of music" (*als einzige, wahrhaft genügende akustische Grundlage der theoretisch-musikalischen Lehre*).

to 8, 7 to 11, and 10 to 14 respectively would be nearly complete. The manuals might be similar to those on General T. Perronet Thompson's Enharmonic Organ, which has 3 boards, with 20, 23 and 22 manuals respectively, and contains the chords in Table V. cols. III., lines 6 to 11; IV. 6 to 12; V., VI., VII., 5 to 12; VIII. and IX., 6 to 12 (four chords belonging to col. IX., lines 6 to 9, are not in the Table, but can be readily supplied, as well as the additional lines 0, —1, named below).

Euler's "genus cujus exponens est $2^m . 3^7 . 5^2$," as developed in his *Tentamen*, p. 161, must be considered as adapted for an instrument with two boards of ordinary manuals, such as some harmoniums are now constructed. His "soni primarii" would occupy the lower, and his "soni secundarii" the upper board. If to these we add their schismatic equivalents, inclosed in brackets, and distinguish white and black manuals by capital and small letters as in Table II., Euler's scheme will appear as follows, where the notation interprets his arithmetical expressions of pitch ("soni"), and not his notes ("signa sonora"), which are too vague.

Euler's Double Scheme.

Upper Board.

Schism. Equival...	[‡*C*,	‡*d*♭,	‡*D*,	*e*♭,	*F*♭,	‡*F*,	*g*♭,	‡*G*,	*a*♭,	*B*♭♭,	*b*♭,	*C*♭]
"Soni Secundarii"	*B*♯,	*c*♯,	*C*×,	*d*♯,	†*E*,	*E*♯,	†*f*♯,	*F*×,	*g*♯,	†*A*,	†*a*♯,	†*B*.

Lower Board.

"Soni Primarii" .	*C*,	‡*c*♯,	*D*,	‡*d*♯,	*E*,	*F*,	*f*♯,	*G*,	‡*g*♯,	*A*,	*a*♯,	*B*
Schism. Equival...	[‡*D*♭♭,	‡‡*d*♭,	*E*♭♭,	‡*e*♭,	‡*F*♭,	‡*G*♭♭,	‡*g*♭,	*A*♭♭,	‡*a*♭,	‡*B*♭♭,	‡*b*♭,	†*C*♭].

Although it is evident from his notation that Euler regarded schismatic equivalents as identities, he has not especially alluded to them. The above scheme would contain Table V. col. V., lines 0 to 14, and the major third †*F*♯ †*A*♯ in 15 (with the schismatic error of ¶*B*♭ ¶‡*D F* for *B*♭ ‡*D F*), col. VI. 1 to 15; VII. 9 to 24; VIII. 10 to 24; IX. 18 to 24; III. —1 to 5; IV. 0 to 6. It would be therefore nearly complete in major scales, but would have only ‡*d*, *a*, *e*, *b*, *f*♯, *c*♯, *g*♯ minor, and their comparatively useless schismatic equivalents. It would have no single complete key, and would therefore require many commatic substitutions in modulation, and the use of the Pythagorean major third in the major chords of the comparatively common minor scales of ‡*f*, ‡*c*, ‡*g*. If only the "soni primarii" of the lower board are used the substitutions become very harsh, as for example *A*♯ *D F*, *D F*♯ *A* for *B*♭ ‡*D F*, *D F*♯ †*A*.

Euler's "soni primarii" may be compared with Rameau's scale *, which was as follows,

C, ‡*c*♯, ‡*D*, †*e*♭, *E*, *F*, ‡*f*♯, *G*, ‡*g*♯, *A*, †*b*♭, *B*,

* Traité de l'Harmonie, 1721. The values of the tones are determined from his arithmetical expression of the intervals.

and therefore only contained the following perfect harmonies, and *two* perfect scales, *A* major and *a* minor :—

		—‡*D F*		
		F A C	‡*d f a*	‡*D* ‡*F*♯ *A*
†*E*♭ *G B*♭,	*c* †*e*♭ *g*	*C E G*	*a c e*	*A* ‡*C*♯ *E*
	g †*b*♭ —	*G B* —	*e g b*	*E* ‡*G*♯ *B*.

Prof. Helmholtz has tuned an harmonium with two boards of manuals, somewhat in Euler's manner, as follows :—

Helmholtz's Double Scheme.

Upper Board.

Schism. equiv.	[‡*C*,	*d*♭,	‡*D*,	*e*♭,	*F*♭,	*F*,	*g*♭,	‡*G*,	*a*♭,	*A*,	*b*♭,	*C*♭]
Tones tuned..	*B*♯,	†*c*♯,	*C*×,	*d*♯,	†*E*,	†*E*♯,	†*f*♯,	*F*×,	*g*♯,	*G*×,	†*a*♯,	†*B*.

Lower Board.

Tones tuned..	*C*,	*c*♯,	*D*,	‡*d*♯,	*E*,	*E*♯,	*f*♯,	*G*,	‡*g*♯,	†*A*,	*a*♯,	*B*
Schism. equiv.	[‡*D*♭♭,	‡*d*♭,	*E*♭♭,	‡*e*♭,	‡*F*♭,	‡*F*,	‡*g*♭,	*A*♭♭,	‡*a*♭,	*b*♭♭,	‡*b*♭,	‡*C*♭].

This scheme has nearly the same extent and the same defects as Euler's.

The concertina, invented by Prof. Wheatstone, F.R.S., has 14 manuals to the octave, which were originally tuned thus, as an extension of Euler's 12-tone scheme.

C, ‡*c*♯, *D*, ‡*d*♯, *E*, †*e*♭, *F*, *f*♯, *G*, ‡*g*♯, *A*, †*a*♭, *B*, †*b*♭.

It possessed the perfect major and minor scales of *C* and *E*. The harshness of the chords †*B*♭ *D F*, *D F*♯ *A*, for *B*♭ ‡*D F*, *D F*♯ †*A* has, however, led to the abandonment of this scheme, and to the introduction of a tempered scale. I have taken advantage of the 14 manuals to contrive 4 different methods of tuning, so that 4 concertinas would play in all the common major and minor scales. Two of these I have in use, and find them effective and very useful for experimental purposes. The following gives the arrangement of the manuals in each, together with the scales possessed by each instrument, major in capitals, and minor in small letters. Where commatic substitution makes the dominant chord too flat in major scales, parentheses ( ) are used ; where it makes the subdominant chord too sharp, brackets [ ] are used. Minor scales in brackets have only the subdominant tone too sharp.

The major chord *G B D* and the tone *C* being common to all four instruments, determine their relative pitch. The method of tuning these and all justly intoned or *teleon** instruments is very simple. *C* being tuned to any standard pitch, the fifths above and below it are tuned perfect. To any convenient tone thus formed, as *C* itself, form the major thirds above,

* A convenient name, formed from τέλεον διάστημα, a perfect interval.

as *E*, ‡*G*♯, ‡*B*♯, &c., and below as †*A*♭, †*F*♭, &c., and then the fifths above and below these tones. The names of the tones thus tuned are apparent from Table V. This tuning is much simpler than any system of temperament, and can be successfully conducted by ear only, taking care to avoid all beats in the middle octave c to c^2.

Scheme for Four Concertinas.

1. †*A*♭ *Concertina.*

Manuals.. *C b*♭, *D d*♭, *E* †*e*♭, *F* †*f*, *G g*♭, *A* †*a*♭, *B* †*b*♭,
Scales.... *D*♭, †*A*♭, †*E*♭, (†*B*♭), [*F*], *C*; [*b*♭], *f*, *c*.

2. †*B*♭ *Concertina.*

Manuals.. *C* †*c*, *D c*♯, †*E* †*e*♭, †*F f*♯, *G* †*g*, †*A* †*a*♭, *B* †*b*♭,
Scales.... †*E*♭, †*B*♭, †*F*, (†*C*), [*G*], *D*; [*c*], *g*, *d*.

3. *C Concertina.*

Manuals.. *C* ‡*c*♯, ‡*Dd*, *E* ‡*d*♯, *F f*♯. *G* ‡*g*♯, *A* †*a*, *B b*♭,
Scales.... *F*, *C*, *G*, (*D*), [*A*], *E*; [‡*d*], *a*, *e*.

4. *D Concertina.*

Manuals.. *C c*♯, *D* ‡*d*♯, *E* †*e*, *E*♯ *f*♯, *G g*♯, †*A a*♯, *B* †*b*,
Scales.... *G*, *D*, †*A*, [†*E*], [*B*], *F*♯; [*e*], *b*, *f*♯.

In Table III. the first column shows the number of degrees of any tone, two tones whose degrees differ by one-sixteenth being schismatic equivalents. The second column contains the notes of the tones. The third column contains the logarithm of the ratio of their pitch to that of c, whence the ratio itself, the absolute pitch, and the length of the monochord are readily found. In the fourth column E marks Euler's primary, and E^2 his secondary tones; H, H^2 the tones on Helmholtz's lower and upper board; T, the 40 tones of General T. Perronet Thompson's Enharmonic Organ; P, the 50 tones of Mr. Poole's Euharmonic Organ; t, the 72 tones of Table V., cols. III. to VIII.; s, the 24 tones out of these 72 which may be played as their adjoining schismatic substitutes without injuring the harmony; se, the 3 tones which, if played as their schismatic equivalents, would produce a slight but sensible error; t, not followed by either s or se, the 45 tones which form the minimum number of a justly intoned or teleon scale; et, the 12 tones of the equally tempered scale. The seven tones of the major scale of C are printed in capitals in the second column.

Table I.—Principal Musical Intervals.

Name.	Example.	Ratio.	or	Log.	Deg.
Unison	c : c	1 : 1	$1 : 1$	·00000	0
¶ Schisma	†B♯: c	32805 : 32768	$3^8 . 5 : 2^{15}$	·00049	0′
†‖♭ Diaschisma	c : B♯	2048 : 2025	$2^{11} : 3^4 . 5^2$	·00491	1‵‵
† Comma	†c : c	81 : 80	$3^4 : 2^4 . 5$	·00540	1‵
†¶ Pythagorean Comma §	††B♯: c	531441 : 52488	$3^{12} : 2^{19}$	·00589	1
††‖♭ Diesis	d♭ : ‡c♯	128 : 125	$2^7 : 5^3$	·01030	2‵‵‵
‡♯ Minor Semitone	‡c♯: c	25 : 24	$5^2 : 2^3 . 3$	·01773	3″
‖♭♯ Limma	c : †B	256 : 243	$2^8 : 3^5$	·02263	4
♯ Sharp, or Greater Limma	c♯: c	135 : 128	$3^3 . 5 : 2^7$	·02312	4′
†‖♭^{7}♯ Equal Semitone*....	‖c♯: c	$\sqrt[12]{2}$: 1	$*2^{94} : 3^{49} . 5^7$	·02509	$4\frac{7}{12}$
Greater Semitone	c : B	16 : 15	$2^4 : 3 . 5$	·02803	5‵
Greatest Semitone	†c♯: c	2187 : 2048	$3^7 : 2^{11}$	·02852	5
Greatest Limma	†d : c♯	27 : 25	$3^3 : 5^2$	·03343	6‵‵
Minor Tone	e : d	10 : 9	$2 . 5 : 3^2$	·04576	8′
Greater Tone.............	d : c	9 : 8	$3^2 : 2^3$	·05115	9
Extended Tone	g : ʓf	8 : 7	$2^3 : 7$	·05799	$10\frac{1}{4}$
Contracted 3rd	ʓf : d	7 : 6	$7 : 2 . 3$	·06695	$11\frac{4}{5}$
Pythagorean Minor 3rd ..	f : d	32 : 27	$2^5 : 3^3$	·07379	13
Minor 3rd	g : e	6 : 5	$2 . 3 : 5$	·07918	14‵
Major 3rd	e : c	5 : 4	$5 : 2^2$	·09691	17′
Pythagorean Major 3rd ..	†e : c	81 : 64	$3^4 : 2^6$	·10231	18
Fourth, or Perfect 4th....	f : c	4 : 3	$2^3 : 3$	·12494	22
False 4th	d : A	27 : 20	$3^2 : 2^2 . 5$	·13033	23‵
Contracted 5th	ʓf : b	7 : 5	$7 : 5$	·14613	$25\frac{5}{7}$
Diminished 5th	f : b	64 : 45	$2^6 : 3^2 . 5$	·15297	27‵
False 5th	a : d	40 : 27	$2^3 . 5 : 3^3$	·17070	30′
Equal 5th	‖g : c	$\sqrt[12]{2^7}$: 1	$*2^{14} : 3^7 . 5$	·17560	31‵
Fifth, or Perfect 5th	g : c	3 : 2	$3 : 2$	·17609	31
Pythagorean Minor 6th ..	c : †E	128 : 81	$2^7 : 3^4$	·19872	35
Minor 6th	c : E	8 : 5	$2^3 : 5$	·20461	36‵
Major 6th	a : c	5 : 3	$5 : 3$	·22185	39′
Pythagorean Major 6th ..	†a : c	54 : 32	$2 . 3^3 : 2^5$	·22724	40
Diminished 7th..........	f : ‡G♯	128 : 75	$2^7 : 3 . 5^2$	·23215	41‵‵
Extended 6th	d : ʓF	12 : 7	$2^2 . 3 : 7$	·23408	$41\frac{3}{14}$
Perfect 7th	ʓf : G	7 : 4	$7 : 2^2$	·24304	$42\frac{4}{5}$
Minor 7th	f : G	16 : 9	$2^4 : 3^2$	·24988	44
Acute Minor 7th	†b♭ : c	9 : 5	$3^2 : 5$	·25527	45‵
Major 7th	b : c	15 : 8	$3 . 5 : 2^3$	·27300	48′
Octave	c : C	2 : 1	$2 : 1$	·30103	53

§ Hence the symbol ¶ for Pythagoras, with the † (comma) prefixed.
* Approximately.

Table II.—Stops on a Justly Intoned, or *Teleon* Organ or Harmonium.

[Capitals represent *white*, small capitals *red*, small letters *black* manuals: a prefixed asterisk (*) marks the key-note of each stop.]

Name of Stop.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Tempered	C	c♯	D	d♯	E	D♯	F	f♯	G	g♯	A	a♯	B	A♯
7 flats	*‡C♭	‡‡c	‡D♭	‡‡d♭	‡E♭	E♭♭	‡F♭	‡‡f	‡G♭	‡‡g	‡A♭	a♭♭	‡B♭	B♭♭
6 flats	‡C♭	‡‡c	‡D♭	‡‡d	‡E♭	E♭♭	‡F	f♭	*‡G♭	‡‡g	A♭	‡a♭	‡B♭	B♭♭
5 flats	‡C	c♭	*‡D♭	‡‡d	E♭	‡E♭	‡F	f♭	‡G♭	‡‡g	A♭	‡a	‡B♭	B♭♭
4 flats	‡C	c♭	‡D♭	‡‡d	E♭	‡E	‡F	f♭	‡G	g♭	*A♭	‡a	B♭	‡B♭
3 flats	‡C	c♭	‡D	d♭	*E♭	‡E	F	‡f	‡G	g♭	A♭	‡a	B♭	‡B
2 flats	C	‡c	‡D	d♭	E♭	‡E	F	‡f♯	‡G	g♭	A	†a♭	*B♭	‡B
1 flat	C	‡c♯	‡D	d♭	E	†E♭	*F	‡f♯	G	‡g	A	†a♭	B♭	‡B
Natural	*C	‡c♯	D	‡d	E	†E♭	F	‡f♯	G	‡g♯	A	†a♭	B	†B♭
1 sharp	C	‡c♯	D	‡d♯	E	†E♭	F♯	†f	*G	‡g♯	†A	a	B	†B♭
2 sharps........	C♯	†c	*D	‡d♯	†E	E	F♯	†f	G	‡g♯	†A	a♯	B	†B♭
3 sharps........	C♯	†c	D	‡d♯	†E	E♯	F♯	†f	G♯	†g	*†A	a♯	†B	B
4 sharps........	C♯	†c	D♯	†d	*†E	E♯	†F♯	f♯	G♯	†g	†A	a♯	†B	B♯
5 sharps........	†C♯	c♯	D♯	†d	†E	E♯	†F♯	f×	G♯	†g	†A♯	††a	*†B	B♯
6 sharps........	†C♯	c×	D♯	†d	†E♯	††E	*†F♯	f×	†G♯	g♯	†A♯	††a	†B	B♯
7 sharps........	*†C♯	c×	†D♯	d♯	†E♯	††E	†F♯	f×	†G♯	g×	†A♯	††a	†B♯	††B

Table III.

General List of Musical Tones.

Deg.	Note.	Log.	Remarks.	Deg.	Note.	Log.	Remarks.
0	C	·00000	E, H, T, P, t, et.	19‵	††e	·10770	t.
0′	†b♯	·00049	t, s.	20″	‡‡f	·11365	t.
1‵	†c	·00540	T, t.	20‴	‡e♯	·11464	T.
2‴	‡‡c♯	·01233	T.	$20\frac{4}{5}$	ʒf	·11810	P.
$2\frac{7}{9}$	ʒ‡d♭	·01579	P.	21′	‡f	·11954	T, P, t.
3′	‡‡d♭	·01724	t, s.	21″	e♯	·12003	E², H, P, t, se.
3″	‡c♯	·01773	E, T, P, t.	22	F	·12494	E, T, P, t.
4	‡d♭	·02263	P, t, s.	22′	†e♯	·12543	H², t, s.
4′	c♯	·02312	E², H, T, P, t.	$22\frac{1}{12}$	‖f	·12543	et = †e♯.
$4\frac{5}{12}$	‖c♯	·02509	et.	23‵	†f	·13033	T, t.
5‵	d♭	·02803	T, t.	$23\frac{7}{9}$	ʒ‡g♭	·14073	P.
5	†c♯	·02852	H², P, t, s.	25″	‡f♯	·14267	T, P, t.
7″	‡‡d	·04036	t.	26	‡g♭	·14757	P, t, s.
$7\frac{7}{9}$	ʒd	·04431	P.	26′	f♯	·14806	E, H, T, P, t.
8′	‡d	·04576	T, P, t.	$26\frac{1}{2}$	‖f♯	·15051	et.
8″	c×	·04625	E², H², t, s.	27‵	g♭	·15297	T, t.
$8\frac{5}{6}$	‖d	·05017	et.	27	†f♯	·15346	E², H², P, t, s.
9‵	e♭♭	·05066	t, s.	28‵	††f♯	·15886	t.
9	D	·05115	E, H, T, P, t.	29″	‡‡g	·16530	t.
10‵	†d	·05655	t.	29‴	‡f×	·16579	T.
11‴	‡‡d♯	·06349	T.	$29\frac{4}{5}$	ʒg	·16925	P.
$11\frac{7}{9}$	ʒe♭	·06695	P.	30′	‡g	·17070	T, P, t.
12′	‡e♭	·06839	t, s.	30″	f×	·17119	E², H², P, t, s.
12″	‡d♯	·06888	E, H, T, P, t.	31‵	a♭♭	·17560	t, s.
13	e♭	·07379	T, P, t.	$30\frac{11}{12}$	‖g	·17560	et = a♭♭.
13′	d♯	·07428	E², H², P, t, s.	31	G	·17609	E, H, T, P, t.
$13\frac{1}{4}$	‖d♯	·07526	et.	32‵	†g	·18149	t.
14‵	†e♭	·07918	T, t.	33‴	‡‡g♯	·18843	T.
14	†d♯	·07967	t, s.	$33\frac{7}{9}$	ʒa♭	·19189	P.
16″	‡e	·09151	T, P, t.	34′	‡a♭	·19333	t, s.
$16\frac{4}{5}$	ʒ†e	·09547	P.	34″	‡g♯	·19382	E, H, T, P, t.
17	‡f♭	·09642	t, s.	35	a♭	·19873	T, P, t.
17′	E	·09691	E, H, T, P, t.	35′	g♯	·19922	E², H², P, t, se.
$17\frac{2}{3}$	‖e	·10034	et.	$35\frac{1}{3}$	‖g♯	·20068	et.
1‵	f♭	·10181	t, s.	36‵	†a♭	·20412	T, t.
18	†e	·10231	E², H², T, P, t.	36	†g♯	·20461	t, s.

Table III. (*continued*).

Deg.	Note.	Log.	Remarks.	Deg.	Note.	Log.	Remarks.
38″	‡a	·21645	T, P, t.	45‵	†b♭	·25527	T, t.
38⅘	ʒ†a	·22040	P.	45$\frac{7}{9}$	ʒ‡c♭	·26567	P.
39′	A	·22185	E, T, P, t.	47″	‡b	·26761	T, P, t.
39″	g×	·22234	H², t, s.	48	‡c♭	·27251	t, s.
39¾	‖a	·22577	et.	48′	B	·27300	E, H, T, P, t.
40‵	b♭♭	·22675	t, s.	48$\frac{7}{12}$	‖b	·27594	et.
40	†a	·22724	E², H, T, P, t.	49‵	c♭	·27791	t.
41‵	††a	·23264	t.	49	†b	·27840	E², H², P, t, se.
42″	‡‡b♭	·23908	t.	50‵	††b	·28380	t.
42‴	‡a♯	·23958	T.	51″	‡‡c	·29024	t.
42⅘	ʒb♭	·24304	P.	51‴	‡b♯	·29073	T.
43′	‡b♭	·24448	P, t, s.	51⅕	ʒc	·29419	P.
43″	a♯	·24497	E, H, T, P, t.	52′	‡c	·29563	T, P, t.
44	b♭	·24988	T, P, t.	52″	b♯	·29612	E², H², P, t, s.
44′	†a♯	·25037	E², H², P, t, s.				
44⅙	‖a♯	·25086	et.				

January 28, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read :—

I. "On the Osteology of the genus *Glyptodon*." By Thomas Henry Huxley, F.R.S. Received December 30, 1863.

In 1862 the author communicated to the Royal Society an account of the more remarkable features of the skeleton of a specimen of the extinct genus *Glyptodon* which had been recently added to the Museum of the Royal College of Surgeons ; and he then promised to give a full description of the skeleton, illustrated by appropriate figures, in a memoir to be presented in due time to the Royal Society. The present communication consists of Part I., and Sections 1 and 2 of Part II., of the promised memoir. Part I. contains the history of the discovery and determination of the remains of the Hoplophoridæ, or animals allied to, or identical with *Glyptodon clavipes.* Part II. is destined to comprehend the description of the skeleton of *Glyptodon clavipes* (Owen)—*Hyplophorus Selloi*? (Lund); and the Sections 1 and 2 now given contain descriptions of the skull and the vertebral column.

The preliminary notice already published in the Proceedings (Dec. 18, 1862, vol. xii. p. 316) will serve as an abstract.

[*To face* p. 108.

	MAJ… I.	*Minor.* VIII.	MAJOR. IX.	*Minor.* X.	
1	C♭♭ E♭♭	‡‡d♭ ‡‡f			1
2	G♭♭ B♭	‡a♭ ‡‡c			2
3	D♭♭ F♭	‡e♭ ‡‡g			3
4	†A♭♭ C♭	‡b♭ ‡‡d			4
5	†E♭♭ G♭	‡f ‡a			5
6	†B♭♭ D♭	‡c ‡e			6
7	†F♭ †A♭	‡g ‡b			7
8	†C♭ †E♭	‡d ‡f♯			8
9		♯ a ‡c♯			9
10		♯ e ‡g♯	‡C♯ ‡E♯ ‡G♯	‡a♯ ‡c♯ ‡e♯	10
11		♯ b ‡d♯	‡G♯ ‡B♯ ‡D♯	‡e♯ ‡g♯ ‡b♯	11
12		♯ f♯ a♯	‡D♯ ‡F× A♯	‡b♯ ‡d♯ ‡f×	12
13		♯ c♯ e♯	A♯ ‡C× E♯	‡f× a♯ ‡c×	13
14		♯ g♯ b♯	E♯ ‡G× B♯	‡c× e♯ ‡g×	14
15		♯ d♯ f×	B♯ ‡D× F×	‡g× b♯ ‡d×	15
16		× †a♯ c×	F× A× C×	‡d× f× a×	16
17		× †e♯ g×	C× E× G×	a× c× e×	17
	I.	VIII.	IX.	X.	

Table IV.—Schismatic Equivalents.

¶c = †B♯	Whence	and
¶d = †c×	¶ ‡c♭ = b	ɓ†c♯ = d♭
¶e = d×	¶‡‡d♭ = ‡c♯	ɓ c× = ‡d
¶f = †e♯	¶ ‡d♭ = c♯	ɓ d♯ = e♭
¶g = †f×	¶ e♭♭ = d	ɓ†d♯ = †e♭
¶a = g×	¶ ‡e♭ = ‡d♯	ɓ e♯ = ‡f
¶b = †a×	¶ ‡f♭ = e	ɓ†e♯ = f
	¶ f♭ = †e	ɓ†f♯ = g♭
ɓc = ‡d♭♭	¶ ‡g♭ = f♯	ɓ f× = ‡g
ɓd = e♭♭	¶ a♭♭ = g	ɓ g♯ = a♭
ɓe = ‡f♭	¶ ‡a♭ = ‡g♭	ɓ†g♯ = †a♭
ɓf = ‡g♭♭	¶ ‡b♭ = a♯	ɓ g× = a
ɓg = a♭♭	¶ b♭♭ = †a	ɓ†a♯ = b♭
ɓa = ‡b♭♭		ɓ†b = c♭
ɓb = ‡c♭		ɓ b♯ = ‡c
		ɓ†b♯ = c

Table V.—Related Systems.

	Major. I.	*Minor.* II.	Major. III.	*Minor.* IV.	Major. V.	*Minor.* VI.	Major. VII.	*Minor.* VIII.	Major. IX.	*Minor.* X.	
1	C♭♭ E♭♭ G♭♭	a♭♭ c♭♭ e♭♭	A♭♭ ‡C♭ E♭♭	‡f♭ a♭♭ ‡c♭	‡F♭ ‡A♭ ‡C♭	‡‡d♭ ‡f♭ ‡a♭	‡‡D♭ ‡‡F ‡A♭	‡‡b♭ ‡‡d♭ ‡‡f			1
2	G♭♭ B♭♭ D♭♭	e♭♭ g♭♭ b♭♭	E♭♭ ‡G♭ B♭♭	‡c♭ e♭♭ ‡g♭	‡C♭ ‡E♭ ‡G♭	‡a♭ ‡c♭ ‡e♭	‡A♭ ‡‡C ‡E♭	‡‡f ‡a♭ ‡‡c			2
3	D♭♭ F♭ †A♭♭	b♭♭ d♭♭ f♭	B♭♭ ‡D♭ F♭	‡g♭ b♭♭ ‡d♭	‡G♭ ‡B♭ ‡D♭	‡e♭ ‡g♭ ‡b♭	‡E♭ ‡‡G ‡B♭	‡‡c ‡e♭ ‡‡g			3
4	†A♭♭ C♭ †E♭♭	f♭ †a♭♭ e♭	F♭ A♭ C♭	‡d♭ f♭ a♭	‡D♭ ‡F A♭	‡b♭ ‡d♭ ‡f	‡B♭ ‡‡D ‡F	‡‡g ‡b♭ ‡‡d			4
5	†E♭♭ G♭ †B♭♭	c♭ †e♭♭ g♭	C♭ E♭ G♭	a♭ c♭ e♭	A♭ ‡C E♭	‡f a♭ ‡c	‡F ‡A ‡C	‡‡d ‡f ‡a			5
6	†B♭♭ D♭ †F♭	g♭ †b♭♭ d♭	G♭ B♭ D♭	e♭ g♭ b♭	E♭ ‡G B♭	‡c e♭ ‡g	‡C ‡E ‡G	‡a ‡c ‡e			6
7	†F♭ †A♭ †C♭	d♭ †f♭ †a♭	D♭ F †A♭	b♭ d♭ f	B♭ ‡D F	‡g b♭ ‡d	‡G ‡B ‡D	‡e ‡g ‡b			7
8	†C♭ †E♭ †G♭	†a♭ †c♭ †e♭	†A♭ C †E♭	f †a♭ c	F A C	‡d f a	‡D ‡F♯ A	‡b ‡d ‡f♯			8
9			†E♭ G †B♭	c †e♭ g	C E G	a c e	A ‡C♯ E	‡f♯ a ‡c♯			9
10			†B♭ D †F	g †b♭ d	G B D	e g b	E ‡G♯ B	‡c♯ e ‡g♯	‡C♯ ‡E♯ ‡G♯	‡a♯ ‡c♯ ‡e♯	10
11			†F †A †C	d †f †a	D F♯ †A	b d f♯	B ‡D♯ F♯	‡g♯ b ‡d♯	‡G♯ ‡B♯ ‡D♯	‡c♯ ‡g♯ ‡b♯	11
12			†C †E †G	†a †c †e	†A C♯ †E	f♯ †a c♯	F♯ A♯ C♯	‡d♯ f♯ a♯	‡D♯ ‡F× A♯	‡b♯ ‡d♯ ‡f×	12
13			†G †B †D	†e †g †b	†E G♯ †B	c♯ †e g♯	C♯ E♯ G♯	a♯ c♯ e♯	A♯ ‡C× E♯	‡f× a♯ ‡c×	13
14			†D †F♯ ††A	†b †d †f♯	†B D♯ †F♯	g♯ †b d♯	G♯ B♯ D♯	e♯ g♯ b♯	E♯ ‡G× B♯	‡c× e♯ ‡g×	14
15			††A †C♯ ††E	†f♯ ††a †c♯	†F♯ †A♯ †C♯	d♯ †f♯ †a♯	D♯ F× †A♯	b♯ d♯ f×	B♯ ‡D× F×	‡g× b♯ ‡d×	15
16			††E †G♯ ††B	†c♯ ††e †g♯	†C♯ †E♯ †G♯	†a♯ †c♯ †e♯	†A♯ C× †E♯	f× †a♯ c×	F× A× C×	‡d× f× a×	16
17			††B †D♯ ††F♯	†g♯ ††b †d♯	†G♯ †B♯ †D♯	‡e♯ †g♯ †b♯	†E♯ G× †B♯	c× †e♯ g×	C× E× G×	a× c× e×	17
	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	