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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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[FOURTH SERIES.]

JANUARY 1873.

1. *On Manometric Flames. By* Dr. Rudolph König (*of Paris*)*.

[With Two Plates.]

IN the beginning of 1862 I invented a new method of observation, which had for its object to make apparent the sounding air-waves, or, what is the same thing, the changing density of the atmosphere while penetrated by sounding vibrations, or while itself in a state of vibration, in the same way as acoustic experiments were able to show clearly the vibration of bodies which produce the vibration of the atmosphere.

The first apparatus founded on this method was shown in the London Exhibition of 1862; and since that period I have invented a whole series of apparatus on the same principles: a short description of some has appeared in Poggendorff's *Annalen* for 1864; and others are briefly sketched in my Catalogue of 1865.

The following pages are designed to explain all these apparatus, as well those which have been added since the publication of my Catalogue, as also the experiments in connexion with them.

The small instrument, on the use of which my method is founded, and to which I have given the name of *Manometric Capsule*, consists of a cavity in a wooden plate, whose orifice is closed by a thin membrane. Illuminating gas may be introduced into this cavity through a pipe—a second pipe, terminating in a gas-burner, giving means for exit and ignition.

Now, if the air before the membrane be rendered suddenly of

* Translation, communicated by the Author, from Poggendorff's *Annalen*, vol. cxlvi. p. 161.

a greater density, the membrane will of course be driven and thus expel the gas and cause the flame to rise quic on the contrary, the air be suddenly rarefied, the m becomes drawn outwards, the space within moment creased, the gas expanded, and the flame lowered.

A membrane is known to possess, like every oth body, only a definite series of notes; and thus we should that the manometric capsule would only show an eff the note acting upon it agreed with one of the not membrane.

But this is not the case; for besides the vibration body makes under the influence of its elasticity, an whatever can be forced upon it if only the active force greater than the resistance which it can offer.

For example, let us take a long thin string, tuned to th mental note of 100 vibrations, and place its centre in f nexion with the prong of a strong massive tuning-fork o brations; it will then clearly move to and fro 110 times i with the vibrations of the tuning-fork, although in ac with its nature it could only execute 100, 200, 300, & tions. In point of fact it does not truly vibrate, but mechanically drawn to and fro. This is also the case manometric capsule, as it is so constructed that the re offered to the condensation and rarefaction of the atm must be considered very trifling, indeed almost nil. (the same capsule is thus equally effective for every nc different capsules, whose membranes have *not* been t unison, nevertheless give the same results under the i of the same note.

If out of several capsules which are fed by the same ga voir you set *one* in activity, the flames in all the other in motion. Thus, if the membrane be pressed into the the pressure will not only drive the flame higher from t pipe, but will also spread its influence through the entra to the general reservoir, and thence to the other capsu flames of which become prolonged, although in a less Of course, a pressure in the contrary direction prod opposite effect. If, therefore, several capsules are to ployed at the same time, this mutual influence must be a

I at first sought to attain this end by placing betweer servoir and the capsules long thin india-rubber tubes; did not act quite satisfactorily.

I attained my object, however, by the use of access sules, through which I permitted the gas to pass befor ducted it into the manometric capsule; they are construc the others, each consisting of a cavity closed by a thin me

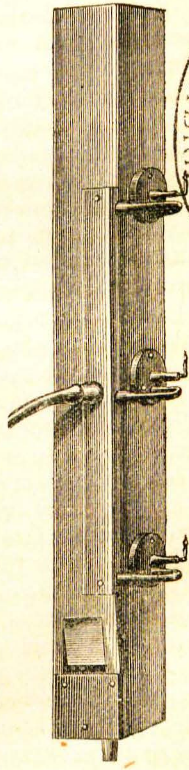
If the pressure derived from the manometric capsule pass through the entrance-tube towards the gas-reservoir, it will be annulled when entering the accessory capsule by the yielding of the membrane.

Practice shows that we may put into the strongest motion one of several flames isolated in the foregoing manner without in any way affecting the rest.

Proof of the different condition of the Air in the Nodes and Ventral Segments of a sounding Air-column.

In order to show the changing density of the air in the nodes and its fixed condition in the ventral segments of a sounding air-column generally, I make use of an open organ-pipe, which is so constructed that either its fundamental tone or its first harmonic or overtone, the octave, can be sounded at will (fig. 1). At the node of the fundamental and the two nodes of the octave are three orifices in one side of the pipe; over these three manometric capsules are so placed that the orifices are exactly closed by the membranes, being of the same diameter; a common reservoir, provided with accessory capsules, feeds the three flames, the length of which can be regulated by cocks.

Fig. 1.



If, now, we give to the three flames an equal height of 15 to 20 millimetres, and sound the octave, then the two exterior flames will be put into such violent motion that they will appear prolonged, narrow, quite blue, and without illuminating power, on account of the considerable amount of air which they draw with them in their flickering up and down, whilst the middle flame will remain almost still and bright, being placed at the centre of a ventral segment, where the air is only gliding to and fro.

At the sounding of the fundamental the middle flame is at the node, and therefore violently agitated; the two exterior ones, which are then between the node and the centres of the ventral segments at the ends of the pipe, show only a weaker motion. As in this case it is only a question of different intensity of motion in the individual flames, it is better here to make use of smaller flames, when the middle one becomes quite blue, while the exterior ones remain bright. If we give the

flames the length of only 8 to 10 millims., on sounding the fun
damental the middle flame will be extinguished, on sounding th
octave the exterior ones will disappear.

These experiments may also be made with a closed orgar
pipe which can be sounded on its fundamental and its first ove
tone. One of the flames must then be at the end of the pip
where the node of the fundamental, as well as one of the node
of the overtone, are found.

If the flame be shortened, on sounding the fundamental th
end flame will be the first to go out, and then the middle on
because the latter is nearer to the node than to the ventr
segment in the mouth of the pipe.

But on sounding the first overtone, the 12th of the fun
damental, the middle flame remains unchanged, while the tw
exterior ones become extinguished.

Comparison and Combination of several Tones.

These experiments have only shown the general working
whole series of consecutive vibrations; if, however, we allow th
flame to be reflected by a rotating mirror, we see all phases
their motion side by side, and we can then not only examine th
number of vibrations and the ratios of different tones, but als
observe the images made by the combination of several tones.

The apparatus which serves for these investigations consist
of a set of organ-pipes, each of which is provided at the node o
its fundamental with a manometric capsule. This can be con
nected by means of an india-rubber tube with gas-burners, whic
are placed on a special stand (fig. 2).

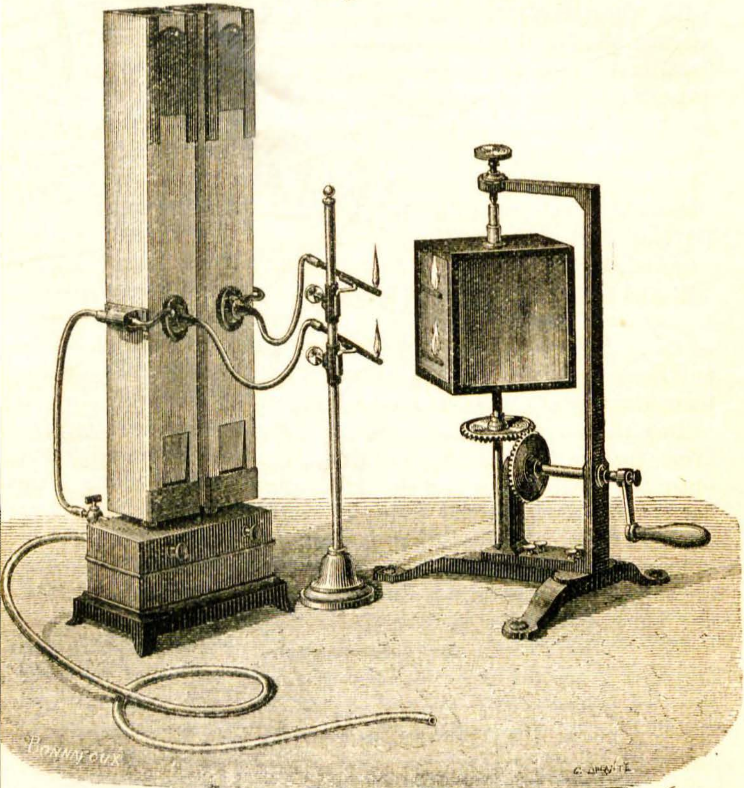
Before these gas-burners there is placed a revolving mirro
made of four glass plates coated with platinum. The platinu
surfaces are turned outwards, in order to avoid the confusin
double images of the common mirrors, which is caused by re
flection from the two surfaces of the glass plates. A small wind
chest, for the reception of two organ-pipes, has two mouth
pieces, the larger of which serves to conduct the air from
bellows. Through the smaller one the gas is conducted to a com
mon receiver, provided with two cocks, which are joined by mean
of india-rubber tubing to the capsules of the organ-pipes.

The reflection of a flame at rest shows in the rotating mirro
a band of light of the width of the height of the flame. If we
however, sound the organ-pipe in connexion with it, ther
appears in place of the band of light a series of regularly con
secutive flame-pictures, the tips of which are bent in a directio
opposite to that in which the mirror is moving.

If we place two burners in such a position that their reflectio
give two bands of light, one above another, and connect the

with two organ-pipes which together give the interval of the octave, the series corresponding to the higher tone gives double

Fig. 2.



the number of flames that the other one does, by which the vibrations are shown to be in the ratio of 1 : 2 (Pl. I. fig. 3). If we take organ-pipes of other intervals, we get with the fifth three flames above two, with the fourth four above three, and so on.

The rapidity of the motion of the flames allows their reflections in the mirror to be very sharply defined; but as they are of very short duration, it would be difficult in this experiment to observe trifling deviations from the purity of the intervals; for although in point of fact it is easy to recognize that in one of the series there are almost always two flames when there is one in the other, yet it would be difficult to discover that about in the one series coincide with about 101 in the other. exact observations can be made with the greatest facility ever, if we make the two capsules of the two corresponding organ pipes act on the same flame.

If we sound two organ-pipes, exactly tuned to an octave while the gas streams from their two capsules into the same burner, the flame has the appearance of containing within it a smaller one without motion. By the slightest discord, however, the latter becomes flickering, and lengthens and shortens periodically within the greater one. Each of these double movements composed of ascending and descending shows a fluctuation, either the deviation of the upper tone by a double vibration, or of the lower by a single vibration from the pure interval of the octave.

The fifth (2 : 3) shows three, the fourth (3 : 4) four, the third (4 : 5) five points of flame one above another, whose mutual position remains unchanged with the perfect purity of the interval; on the contrary, any deviation from this causes an up-and-down movement among them of each single point, which takes the appearance of a waving motion.

In all these intervals it is easy so to arrange the length of the flame that all the points may remain clearly bright and appear separated from each other by blue non-luminous parts of the flame. If, however, the ratios of vibration of the two tones becomes more complicated, it is often difficult to observe them exactly; but even in this case the flame shows plainly whether the interval be pure or out of tune, as we have but to see whether the flame is at rest or in motion.

This property of the manometric flame, of showing the least deviation from the purity of the interval, makes it in many cases exceedingly useful in tuning, as it is not necessary that the two notes which are to be brought into tune should be produced by organ-pipes provided with capsules: the notes of any instrument may be used, if they are produced before two resonators in relation with them, which act on two manometric capsules whose gas-pipes end in the same burner.

The ratio 1 : 2 is the most convenient, on account of its easy examination; so that if we want to tune a series of tuning-forks to the same note, it is better to choose the fork for comparison an octave lower or higher.

If we wish to observe the whole process of vibration in the above-mentioned flames on which two notes act at the same time, we must again employ the rotating mirror. The pure octave shows in it a series of flames, in which a shorter always follows a longer, and the shorter ones have all, like the longer ones, equal heights (fig. 4, Pl. I.). If any *beats* occur, the summits of the smaller as well as of the larger flames move up and down. However, these motions are opposed; so that in those positions where the long flames are at their longest, the short flames are at their shortest, and *vice versâ*.

In fig. 4, Pl. I. the picture of the seventh (8 : 15, or 8 : 16 − 1) shows this process, although in a very short period. The fifth (2 : 3) shows a period of three, the fourth (3 : 4) of four, the third (4 : 5) of five, and the second (8 : 9) of nine in the range of the increasing and then decreasing flames.

If the proportion is not of the form $n : n \pm 1$, then there takes place in the whole period not only a rise and fall of the flame-summits, but the curve connecting them shows as many elevations and depressions as the difference between the two ratios. For example, see the picture of the sixth (3 : 5) (fig. 4, Pl. I.).

The more complicated the interval of the two notes, the more carefully we must bring it into perfect purity of tune, until no further movement whatever can be discerned in the flame, because otherwise the recurring periods of the flame-pictures in the mirror suffer continual change by the change of phase, and in that case it becomes difficult to recognize them. But this exact tune becomes still more imperative if we wish to combine more than two notes while making them act on one flame. It will be remarked, besides, in these experiments, how difficult it is to retain absolutely constant notes with organ-pipes, even when we make use of a well-regulated bellows.

Coexistence of two Tones in the same Air-column.

The investigation of the combination of two related tones in one flame-picture is especially useful, because it teaches us from the flame-picture of a combination of tones, of which the components are unknown, to find the single tones of which it is composed.

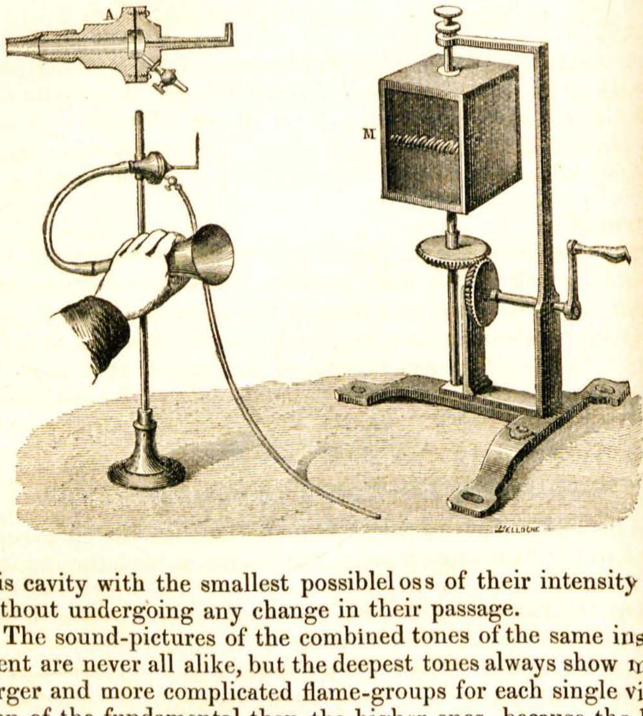
A passage to the trial of such a combination of tones as, *e. g.*, each sound offers, is the combination formed by a fundamental with a known overtone in the same air-column. Very suitable for such an experiment is the above described closed organ-pipe with three flames, since the node of the fundamental as well as one node of the first overtone are situated at their ends.

If we blow the fundamental (1) very gently, the flame-picture in the mirror shows the vibrations of this tone; if now we blow the overtone (3) strongly, each single vibration will be replaced by three flames. With a rather weaker blast both tones are produced together, and we always see three flame-summits over every fundamental flame (fig. 5, Plate I.). Therefore several tones present at the same time in the air-column give exactly the same flame-picture as the combination of the same tones when each is produced by its own particular organ-pipe.

Representation of Sounds.

The apparatus which is used for the representation of sound consists simply of a manometric capsule, before the membrane of which there is a small cavity terminating in a short tube (fig. 6). The sounds to be represented must be conducted into

Fig. 6.



this cavity with the smallest possible loss of their intensity and without undergoing any change in their passage.

The sound-pictures of the combined tones of the same instru-ment are never all alike, but the deepest tones always show much larger and more complicated flame-groups for each single vibra-tion of the fundamental than the higher ones, because the hig harmonic tones, which are to be heard in the sound of the deepe tones of the instrument, disappear more and more as the funda-mental ascends. Thus the higher the tone the smaller in com-parison are the dimensions of the means which produce it. Th vibrations of all resounding instruments, however, take a simple form if the dimensions of the latter are very small, because th different bodies lose their capacity of forming subdivisions in vibration, by which the accessory tones, if not exclusively, ye in many cases are chiefly produced.

A second reason, however, and that a very potent one, is this If the tones are produced not so much by the elastic vibration of a body as rather by gusts of air, as in the siren and pandea pipes, the upper notes which are contained in the sound of a lower note have so high a place in the scale for a high note that they produce no effect, either on the ear or on an artificial membrane.

The lowest note of the violin, for example, is *g* (192 vibrations), and its 8th harmonic $\overline{\overline{\overline{g}}}$ (1536 vibrations) is within the range of the instrument. It is produced on the G-string by a length of 4, and on the E-string by about $13\frac{1}{3}$ centimetres. Nevertheless, if we take this very $\overline{\overline{\overline{g}}}$ as fundamental tone, the length of string of its eighth harmonic on the E-string would be about 17 millimetres; and besides, with 12,288 vibrations, it would be already nearly two octaves above the highest notes used in music, which sufficiently explains why it is not heard in the sound of $\overline{\overline{\overline{g}}}$.

My success was but partial in the representation of violin sounds, owing to the high position of the notes of the instrument, since, with the exception of the notes from *g* to $\bar{c}$ on the G-string, I obtained only the fundamental vibrations for all the rest. In my endeavours to conduct the notes as loud as possible to the membrane I tried two methods. First, I connected the interior air of the violin with the small apparatus, by means of an india-rubber tube which I introduced into one of the *f*-shaped apertures of the violin; and secondly, I pressed my stethoscope with its concave membrane on the bottom-piece of the violin, precisely under the sounding-post, and attached the india-rubber tube to the flame-apparatus. The results in the latter case were as follows.

On the G-string *g* showed the figure of the octave in weak wave-formed flames, which, as far as *b*, rose to sharply defined clearly cut flames. With $\bar{c}$ the latter fell quite suddenly into one single broad, short, and faint flame, in which I could perceive only the smallest trace of the octave when played forcibly. Already the D-string only showed simple flame series, which for $\bar{d}$ $\bar{e}$ $\bar{f}$ $\bar{g}$ were rounded, wave-like, and weak, but on playing $\bar{a}$ became again stronger. The $\bar{a}$ on the A-string gave very high and deeply cut flames, $\bar{b}$ still stronger ones, which fell, however, at $\overline{\overline{c}}$ and became quite weak. Up to $\overline{\overline{g}}$ and $\overline{\overline{a}}$ on the E-string every trace was lost of the small flame-points which had appeared at the last overtones.

On the connexion of the interior air with the apparatus, the insensibly graduated picture of the octave from *g* changed into a single sharply defined flame at *b*; this attained such an extraordinary height at $\bar{c}$, as though it had been produced by the vibrations of an organ-pipe provided with a capsule at the node.

The note $\bar{d}$ also showed a series of high and sharply defined flames, which, however, quite disappeared at $\bar{e}$ to give place to the weak rounded-off waving lines as far as $\bar{a}$.

This sudden appearance of very high flames in the region of $\bar{c}$ is explained by the circumstance that the lowest proper note of the interior air of the violin is precisely $\bar{c}$. For the upper notes I obtained the same result as with the stethoscope; that is to say, the notes $\bar{a}$ and $\bar{b}$ again gave much stronger vibrations than $\bar{e}\ \bar{f}\ \bar{g}$, and than the upper $\bar{\bar{c}}\ \bar{\bar{d}}\ \bar{\bar{e}}$ &c.; so that the second peculiar note of the interior air, or rather of the whole system formed by the violin, seems to be in the region of $\bar{a}$ and $\bar{b}$.

With regard to sound, we have in this case certainly been able only to make evident the transition from the figure of the octave to that of the simple note. The siren shows much better the gradual disappearance of the higher upper note from the musical sounds when their fundamental tone is raised. To this end I intercept the impulse above the open perforated plate by means of an arched aperture which expands into a small tube, and is placed immediately above a part of the apertures so as to permit them to affect the flame, while I cause the rotation of the plate to increase from its lowest to its highest swiftness by increased pressure of the air. The mirror then shows at the lowest notes very large and dense flame-groups; these change towards the middle of the great octave into more clearly defined and deep-slit waves, with at first five, then towards *c* and *d* with four flame-points. At *g* and *a* the number of the points falls to three, at $\bar{c}$ and $\bar{d}$ to two; and at $\bar{a}$ the last trace of the octave disappears from the sound; after this all the still higher notes only show single flame-pictures.

But the result of this experiment is essentially different if when a sounding-chest is fastened over the perforated plate. It first intensifies the upper harmonics of the sound, then the lower, and lastly the fundamental itself: this causes the flame-groups no longer to become simpler gradually and in accordance with the height of the notes, but to show rather sudden changes alternately rising and disappearing. Thus the sound of a siren, over the perforated plate of which a resonance-box giving the note $\bar{\bar{c}}$ was placed, after showing a few complicated and faint pictures when the plate was slowly rotated, produced on reaching the pitch *c* clearly a large flame in agreement with the fundamental tone: this flame had four summits, derived from overtone 4, which coincided with the proper note of the sounding-chest. On turning the plate more rapidly, the flame-picture became simpler, until at *f* it became one single flame, so that the overtone 3 must be quite wanting in this sound of the siren. The ascending scale had scarcely passed *f* when there appeared between each two large flames a small but sharply defined flame, which quickly increased in size, and towards $\bar{c}$ reached nearly the height of the chief flames, where the effect of the resonance-box confirmed the fact of its being the overtone 2 of the sound of the siren. Above $\bar{c}$ the smaller flame leant always more towards the larger one, until at $\bar{a}$ it completely disappeared in it: after this again only single flames appeared (Pl. I. fig. 7).

In order to make the sound in these experiments act strongly on the capsule, I provided the resonance-box with a tube, and placed its interior in direct connexion with the flame-apparatus. These experiments, in which the air-impulses of the siren are prevented from passing immediately into the atmosphere, being compelled to pass through a resonator which remains unchanged for all the fundamental tones of sound, give a visible picture of the process of the formation of vocal sounds; for it is known that the air contained in the cavity of the mouth, when speaking or singing the same vowel in different tones, is always tuned to the same note, so that the mouth must act on the air-waves produced in the larynx in the same way as the sounding-chest on the air-impulses of the siren. Nevertheless the series of flame-pictures of the same vowel, sung in the tones of two octaves, does not show such sudden changes as might have been expected without closer research.

In order to produce the pictures of the vowels, I sing them into a small funnel-shaped mouthpiece which is connected with the cavity before the membrane by a short india-rubber tube; thus they reach the capsule with great intensity (fig. 6).

I had already in 1867 sketched and had painted the pictures of the vowels *u, o, a, e, i* sung to the notes of the two octaves from C to $\bar{c}$. I proceeded in the following manner. In order to be sure that I had not changed the character of the vowel in the transition from one tone to another, I first verified the proper note of the mouth with the tuning-fork; then, while I sang into the apparatus, an artist drew the picture which he saw in the mirror. I also drew the same picture independently: and if both our drawings were identical they were looked upon as correct; if, however, there were discrepancies, I repeated the experiment until the error was discovered.

The five finished drawings (Pl. II. fig. 8) were unfortunately too late for the Exhibition; but I was able to exhibit them at the Meeting of the Association of Natural Philosophers at Dresden in 1868. I delayed their publication until now because I wished to revise them with precision, but was always prevented by the delicate state of my throat, which did not permit me such fatiguing experiments. But now, since I can no longer hope to recover, I have used my best endeavours to make the pictures correct, and give them forth, not indeed as absolutely perfect, but as nearly so as it was possible for me to make them.

The drawings themselves are much more difficult than might be supposed, particularly of the large flame-groups of the deeper notes, not only on account of the evanescence of the pictures, but also because the flame-summits do not always follow each other, but are partly situated beneath one another, so that it appears as though different flame-groups were intermingled, or rather pushed partly one before another. But these flames, whose background, so to say, is formed by other flames, easily escape observation, particularly if the back ones are not so high, nor the front ones so low, that the bright summits of the latter stand out upon the blue lower parts of the former. We can indeed, by a rapid rotation of the mirror, separate all the summits from each other; but then the whole group becomes difficult to observe, on account of its great length and the great bending of the flames.

However imperfect the drawings may be on account of the absence of some details, yet they give true pictures of the general outline as portrayed in the mirror. If, for example, the vowel A be sung on the note C, the picture shows a group from which a tall bright flame rises near a smaller very blue one; after these come a whole mountain of regularly toothed flames. Now it is quite possible that this ridge has really 9 summits, whilst I have only drawn 8; for it has sometimes appeared to me that there were more than that number on days when I produced this very low note stronger and purer than usual; but this does not change the character of the whole group, which could never be mistaken for that of U, O, E, or I sung in the same note. In any case, therefore, these pictures appear to me sufficient for the representation of the great difference in the appearance of the sound of the five vowels, sung on the same note, as well as to show the manner of the change of the flame-pictures of the same vowel from one note to another. But this is the chief point, and indeed all that can be attained with certainty by the apparatus; for just on account of its great sensitiveness we must not hope for absolutely correct pictures. The details in the group change most remarkably, not only when the same vowel is sung in the same note by different voices, but also when the same voice gives vowel and note with different intensity. A very slight change in the condition of the voice is sufficient to effect great changes in the flame-pictures. For instance, when my throat is weary, instead of obtaining the picture as drawn of U sung on *e*, I get only a small flame and two tall broad ones, the last in place of two and two in the picture; and similar simplifications are made in all the flame-groups.

In order to see first what influence may be expected from the fixed notes of the mouth-cavity on the flame-pictures, I will give a general view by drawing for each vowel, sung in each note of the two octaves from C to $\bar{c}$, the harmonic overtone to which the characteristic note approximates, and the number of vibrations by which they differ.

For O, A, and E, I take the characteristic notes (given by Helmholtz) $\bar{b}$, $\bar{\bar{b}}$, and $\bar{\bar{\bar{b}}}$; but, differing from the former opinion of Donders and Helmholtz, I have found for U and I the notes b and $\bar{\bar{\bar{\bar{b}}}}$; so that the five chief vowels are all an octave distant from each other, and the characteristic note of the lowest vowel, viz. U, unites with the lowest note which it is possible for the mouth to intensify by resonance.

In the definition of these notes there is no question of an absolutely exact number of vibrations. If, for example, I find the most powerful resonance of the mouthpiece for U giving between 220 and 230 vibrations, I may take equally 224 or 225 vibrations as the characteristic note of U. I make this remark here particularly because, in a short address to the Paris Academy (April 25, 1870) on the before-mentioned definitions of the characteristic notes of U and I, I gave as the average simple vibrations for U, O, A, E, I, 225, 450, 900, 1800, and 3600—but after a subsequent revision, the equally correct numbers 224, 448, 896, 1792, and 3584. The former are indeed easier to retain, but they refer to no note in use; whereas the latter numbers show the vibrations of the seventh overtone of $\underline{C_1}$, C, c, $\bar{c}$, and $\bar{\bar{c}}$ ($\bar{c}$=256 vibrations.).

In the following Table the first column contains the vowel, the second the note sung, and the third and fourth the two overtones of the sound of that note between which the characteristic sound of the vowel falls, together with the number of vibrations by which one of these notes is lower and the other higher than the proper note of the mouth-cavity.

U 448	C	3(*g*)	− 64	+ 64	4($\overline{c}$)	O 896	C				
	D	3(*a*)	− 16	+128	4($\overline{d}$)		D	6($\overline{a}$)	− 32	+112	7($\overline{d}$)
	E	2(*e*)	−128	+ 32	3($\overline{b}$)		E	5($\overline{gis}$)	− 96	+ 64	6($\overline{b}$)
	F	2(*f*)	−107	+ 64	3($\overline{c}$)		F	5($\overline{a}$)	− 42·6	+128	6($\overline{\overline{c}}$)
	G	2(*g*)	− 64	+128	3($\overline{d}$)		G	4($\overline{g}$)	−128	+ 64	5($\overline{b}$)
	A	2(*a*)	− 22	+192	3($\overline{e}$)		A	4($\overline{a}$)	− 42·6	+170·6	5($\overline{\overline{cis}}$)
	B	1(B)	−208	+ 32	2($\overline{b}$)		B	3($\overline{fis}$)	−176	+ 64	4($\overline{b}$)
	c	1(*c*)	−192	+ 64	2($\overline{c}$)		*c*	3($\overline{g}$)	−128	+128	4($\overline{\overline{c}}$)
	d	1(*d*)	−160	+128	2($\overline{d}$)		*d*	3($\overline{a}$)	− 32	+256	4($\overline{\overline{d}}$)
	e	1(*e*)	−128	+192	2($\overline{e}$)		*e*	2($\overline{e}$)	−256	+ 64	3($\overline{b}$)
	f	1(*f*)	−106·7	+234·6	2($\overline{f}$)		*f*	2($\overline{f}$)	−213·4	+128	3($\overline{\overline{c}}$)
	g	1(*g*)	− 64	+320	2($\overline{g}$)		*g*	2($\overline{g}$)	−128	+256	3($\overline{\overline{d}}$)
	a	1(*a*)	− 21·4	+405·2	2($\overline{a}$)		*a*	2($\overline{a}$)	− 42·6	+384	3($\overline{\overline{e}}$)
	b			+ 32	1($\overline{b}$)		*b*	1($\overline{b}$)	−416	+ 64	2($\overline{b}$)
	$\overline{c}$			+ 64	1($\overline{c}$)		$\overline{c}$	1($\overline{c}$)	−384	+128	2($\overline{\overline{c}}$)

A 1792	C		14		
	D	12($\overline{\overline{a}}$)	− 64	+ 80	13($\overline{\overline{\overline{c}}}$)
	E	11	− 32	+128	12($\overline{\overline{b}}$)
	F	10($\overline{\overline{a}}$)	− 86	+ 85	11
	G	9($\overline{\overline{a}}$)	− 64	+128	10($\overline{\overline{b}}$)
	A	8($\overline{\overline{a}}$)	− 86	+128	9($\overline{\overline{b}}$)
	H	7	−112	+128	8($\overline{\overline{b}}$)
	c		7		
	d	6($\overline{\overline{a}}$)	− 64	+224	7
	e	5($\overline{\overline{gis}}$)	−192	+128	6($\overline{\overline{b}}$)
	f	5($\overline{\overline{a}}$)	− 85	+256	6($\overline{\overline{\overline{c}}}$)
	g	4($\overline{\overline{g}}$)	−256	+128	5($\overline{\overline{b}}$)
	a	4($\overline{\overline{a}}$)	− 86	+341	5($\overline{\overline{\overline{cis}}}$)
	h	3($\overline{\overline{fis}}$)	−352	+128	4($\overline{\overline{b}}$)
	$\overline{c}$	3($\overline{\overline{g}}$)	−256	+256	4($\overline{\overline{\overline{c}}}$)

E 3584	C		28
	c		14
	$\overline{c}$		7
I 7168	C		56
	c		28
	$\overline{c}$		14

The characteristic tone of U approaches therefore to the third overtone of D and E, to the second of A and B, and to the fundamental *a* and *b*; and in point of fact in the flame-groups of D and E we can perceive a forking into three, in those of A and B into two chief divisions; while the flame-pictures of *a* and *b* show a great preponderance in intensity of the fundamental over the accessory tones.

The characteristic tone of O does not approach any of the overtones of the sounds that are sung (except C) nearer than about half a tone; therefore it has but little effect on the flame-pictures. At *a*, where it approaches the second, and at *d*, where it approaches the third overtone, we perceive clearly the forking into two and three parts respectively; but the more complicated groups of A, F, and D do not show any particular prominence of the tones 4, 5, and 6. This was to be expected, as the air in the mouth cannot strongly vibrate if its pitch, as in this case, differs half a tone from the note already weak when sounded.

The characteristic tone of A approaches no overtone nearer than 16 vibrations, except at C and *c*, where it coincides with the 14th and 7th overtones. Nevertheless the pictures of C and *c* do not show the existence of the 14th and 7th overtones, probably because these notes are so high and weak in the larynx that they cannot vibrate the atmosphere in the mouth sufficiently to act on the flame.

The characteristic tones of the vowels E and I are too high to have any effect on the flame; and thus E sung on $\bar{c}$ shows but such a picture as does a fundamental distantly accompanied by its octave, instead of a group of seven summits.

I, sung on the same note, shows only a series of simple flames which seem to indicate a simple tone. This simplicity of the flame-picture here, however, is only apparent, as in all the pictures of I.

The wide, large, and almost forkless flames shown by the different groups are really mostly whole tufts of flame, which appear somewhat confused when the note is weakly given; but when on the contrary, it is blown with force, and particularly at the mouthpiece, numerous bright points may be clearly seen, which indicate the presence of very high accessory notes. It is very fatiguing to sing I, and when pitched low is so difficult that I was compelled to omit the sounds from C to F in the drawings.

I made an experiment to see whether the flame-picture would take any different form if I placed the tube, instead of before the mouth, at the back part of it, and then sang A on *f*; but with the exception of increased intensity, the result in both cases was the same.

The whispered vowels had but a slight effect on the flame

The bands of light in the mirror appeared under their influence like an alternately darker and lighter ribbon with irregular small teeth; and the whole was so cloudy and undefined that I could even discover no difference between the different vowels. The semivowels *m* and *n* gave such similar pictures that I could not distinguish between them. I have sketched them for the notes *e̅, g, e, c* (fig. 9, Pl. I.); deeper notes showed longer, but still misty and undefined periods. Of course in these experiments I was obliged to put my nose instead of my mouth to the instrument. The quivering R, silently pronounced, shows a series of flame-summits of different elevations pretty regularly forked or toothed. In the small rotating mirror, with a plate 15 centims. wide, of which I generally make use, these summits appeared to me to follow each other irregularly. But when I employed a larger one, 40 centims. wide, I perceived the regular periodicity of the whole group, which was repeated four or five times in the width of the mirror. The teeth, which are spread over the whole of the flame-summits, are simply caused by the air-current. Of this we can easily convince ourselves by placing the tongue at a little distance from the gums instead of permitting it to vibrate on them, and then expelling the air violently through the narrow aperture. The band of flame then appears serrated, without any individual flame-summits rising above it. But if we intone the R, the picture of the note unites with that of the letter, and there ensues such a confused series of single flames and whole groups of dissimilar height and form, that in the evanescence of the picture it is impossible to decipher them. I have sought to give the character of the voiceless R in fig. 10, Pl. I.

The different characteristics of the voiceless explosives P, T, and K are easily recognized. At P the flame suddenly rises straight up high above the average line, then shows two or three similar *élancements,* which are followed by a few rapidly decreasing ones. Both the high and low chief movements show as at R the indentations caused by the air-current.

The rise of the flame is less sudden at T, neither is it so high; and the deep incisions are wanting, which at P in the commencement show two or three rapid *élancements*. At K, which is articulated further back in the mouth, there is still less a sudden rise of the flame; but the picture begins with a regular rising and falling wave, followed by a few rapidly diminishing ones of the same form. The indentation of the whole picture is the same as in P and T.

If we utter one of these consonants many times in succession while continually turning the mirror, we rarely see the picture well; it is therefore better to place the mirror so that the flame shows in one corner, and with a slight turn must pass over its

XIV. *On Manometric Flames.* *By* Dr. Rudolph König (*of Paris*).

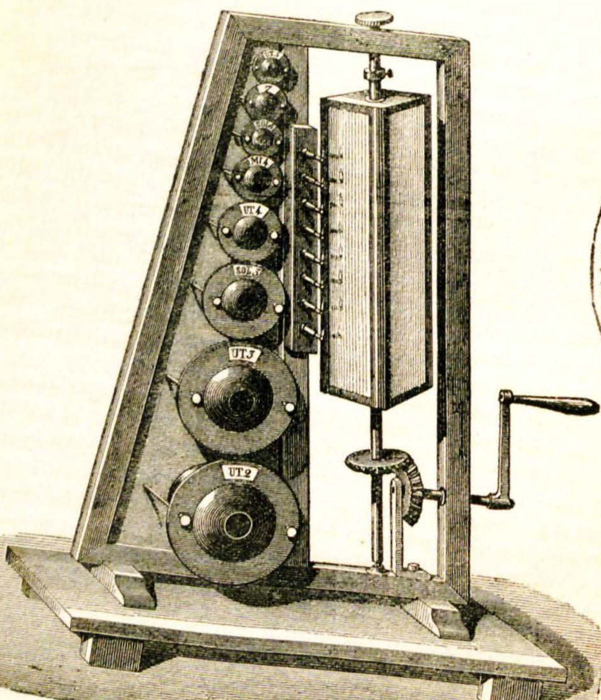
[Concluded from p. 18.]

[With a Plate.]

Decomposition of Sounds into their simple Tones.

THE same resonators (Helmholtz's) which serve for analysis of sounds by means of the ear, are also of use for the visible dissection of sounds by the flames. To this end I construct an apparatus with eight resonators tuned to the harmonic notes of *c*, each of which is connected with a manometric flame. These eight flames are placed in a slanting line one above the other, and show, in the rotating mirror fixed in the same direction, eight parallel bands of light when in repose, and when in vibration eight waved lines (fig. 11). Of course in this case each

Fig. 11.



flame must be perfectly independent of the other, and each flame vibrate only when its particular resonator is put in action by a note in unison; the notes *not* contained in the series of resonators must have no effect whatever on any of the flames. In order to show how far the apparatus fulfils these conditions, I usually employ a series of tuning-forks on sounding-chests, which, particularly a few moments after being sounded, give almost simple notes.

I first take forks which are in tune with the resonators, and sound them singly, and show that only the bands of light which correspond to their notes dissolve into vibrations, so that several simple notes must be sounded to cause the appearance of several serrated bands of light. By means of a tuning-fork not in tune with the resonators, I can then show that its note, even when sounded with considerable force, has no effect on the flames. A note sounded with very great intensity may indeed have some effect on all the flames, through the resonators; but this case will not give rise to error, as all the flame-series appear equal, whereas, when resonance takes place, the number of the single flame-waves in the series increases upwards in the proportion of 1 : 2 : 3 &c., and their width, of course, decreases in the inverse ratio.

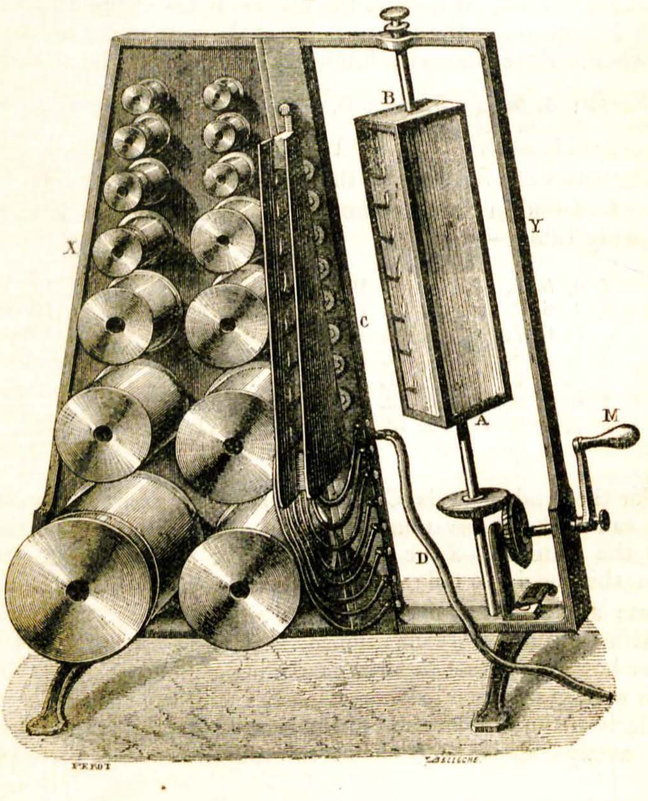
After demonstrating in this way the nature of the apparatus, I produce before it a sound whose fundamental is c; and the serrated bands of light then show by what harmonic notes the fundamental is accompanied, as well as the relative intensities of these notes. If before the apparatus we play the g of the violin, for which the apparatus has no resonator, the octave $\overline{g}$ vibrates strongly, and the $\overline{c}$ of the same instrument resolves, together with the flame of the fundamental, that of the octave $\overline{\overline{c}}$. An open organ-pipe, of small diameter, tuned to c, when forcibly blown, set the first five flames in vibration, but the third vibrated more strongly than the octave. A closed organ-pipe with the same fundamental caused the twelfth to appear very strong, and the fifth very weak. A protruding tongue without a sounding-cup resolved the first six harmonic notes with pretty regularly decreasing intensity.

On singing the vowel U, the octave as well as the fundamental shows rather strong vibrations, and only sometimes a trifling effect may be observed on the third note. D, on the contrary, excites the flames of the third and fourth notes very strongly, while the vibrations of the octave are weaker than with U. The fifth flame-band is serrated, but slightly, with O. With O A the region of greatest intensity becomes higher; it is the fourth and fifth notes which show the deepest indentations in the band of light, while the lower notes are weaker. With A all the flames are resolved up to the seventh, and the fourth, fifth, and sixth vibrate with great force. When E is sung, we see the fundamental accompanied by the octave weakly, and very strongly by the twelfth. The double octave and its third show vibrations of moderate intensity; and the seventh flame shows traces of the existence of the seventh tone. The letter I sung on *c* gives a strong movement to the flames of the octave and the fundamental only, while all the other flames are in repose.

The resonators 7 and 8 ($\overline{\overline{\overline{c}}}$) of the apparatus cause their flames to vibrate with difficulty, and the notes must be very strong. We have now reached the limits within which the flames can be usefully employed.

As this apparatus does not permit us to choose the fundamental tone of the vowel or of any other sound which is to be analyzed, it is adapted rather to demonstration than to further investigation. However, to make it more useful for the latter purpose, I have constructed a second model (fig. 12), in which the

Fig. 12.



eight spherical resonators are replaced by fourteen universal resonators. These resonators consist each of a cylinder, its length about equal to its diameter, which is formed by two pipes placed one within the other. The outer of these pipes terminates at one end in a hemisphere, from which the tube for the ear is carried, as in the spherical resonators. The opposite end of the inner pipe is closed by a plate, in the middle of which there is an opening for the passage of the enclosed air to the exterior atmosphere. This arrangement permits us by drawing out the pipe to increase the mass of air in the resonator, and to lower its tone by a third. On the inner tube lines are drawn which indicate how far the outer one must be drawn out for the different notes. The deeper resonators of the series are so constructed that the highest note of the larger shall always reach to the lowest of the next smaller one. In the higher-toned resonators this would not be sufficient, because the sixth, seventh, and eighth accessory notes approach each other so nearly that the necessity might occur of forming two of them with the same resonator. Since, therefore, the highest notes of the deeper are a whole note above the lowest notes of the next upper resonator, the whole series contains the following notes:—1, G–B; 2, B–*dis*; 3, *dis–fis*; 4, *fis–a*; 5, a–$\overline{c}$; 6, $\overline{c}$–$\overline{e}$; 7, $\overline{e}$–$\overline{gis}$; 8, $\overline{gis}$–$\overline{\overline{c}}$; 9, $\overline{\overline{c}}$–$\overline{\overline{e}}$; 10, $\overline{\overline{d}}$–$\overline{\overline{f}}$; 11, $\overline{\overline{e}}$–$\overline{\overline{gis}}$; 12, $\overline{\overline{f}}$–$\overline{\overline{a}}$; 13, $\overline{\overline{gis}}$–$\overline{\overline{\overline{e}}}$; 14, $\overline{\overline{\overline{c}}}$–$\overline{\overline{\overline{d}}}$.

The series of overtones for the notes of both octaves from C–$\overline{c}$ are to be found in the resonators placed opposite to each in the following Table:—

C :	2, 4, 5, 6, 7, 8, 9, 10.	*c* :	2, 5, 7, 8, 9, 11, 13, 14.
D :	2, 4, 6, 7, 8, 9, 10, 11.	*d* :	2, 6, 8, 9, 10, 12, 13, 14.
E :	3, 5, 6, 7, 8, 9, 10, 11.	*e* :	3, 6, 8, 9, 11, 13, 14.
F :	3, 5, 7, 8, 9, 10, 11, 13.	*f*	3, 7, 8, 11, 12, 13.
G :	1, 4, 6, 7, 8, 9, 10, 11,	*g* :	4, 7, 9, 11, 13.
A :	1, 4, 6, 8, 9, 10, 11, 12,	*a* :	5, 8, 9, 12, 14.
B :	1, 5, 7, 8, 9, 11, 12. 13,	*b* :	5, 8, 11, 12.
		$\overline{c}$:	5, 8, 11, 13.

For the fundamentals C–F the resonators are wanting, but one can make observations up to the ninth note of the sound. For the sounds G–*d* the resonators serve to the eighth note; then they begin to fail; at *e* we can employ only six flames, at *f* five; and at last at $\overline{c}$ but three for the overtones.

Although, as before mentioned, it is indicated on each resonator how far it must be drawn out for the different notes, yet it is as well, in order to have exact results with the apparatus, particularly if the fundamental of the sound to be investigated does not exactly coincide with one of the indicated notes, to employ the following mode of giving the desired pitch to the resonators in question.

Tune a string of the sonometer to the fundamental tone of the sound, and produce on it the harmonic notes one after another. Then place the proper resonator in communication with the ear instead of with the manometric capsule; and while the india-rubber tube is in the ear, it becomes very easy to determine their arrangement and the position for the strongest resonance.

After having tuned eight of these resonators to *c* and its over-tones, I repeated the same experiments with this apparatus as I tried on the spherical-resonator apparatus, and obtained exactly the same results. There was not the least sign of any weakened sensitiveness in the flames; so that this apparatus appears to me exactly fitted for more exact and searching experiments on sounds in general, and particularly those of the human voice, at least those composed of notes which do extend beyond $\overset{\equiv}{c}$. It is to be remarked that direct employment of the resonators with the ear does not succeed far beyond this limit.

Unfortunately I am now convinced that the state of my voice does not permit me to investigate any further in this direction, as I had intended; so I must be content to show the capabilities of the apparatus, as I shall again, when describing the method of experimenting on the vowel-sounds, and others also, by the elimination of single accessory notes, or whole series of them.

Interference-phenomena.

In my description of the results obtained by the combination of the notes of two organ-pipes I have not mentioned *unison.* The combination of two notes in unison has a special interest, on account of the communication of the vibrations and the interference-phenomena which may be observed therein. I therefore preferred deferring their description until now, when I could explain them in connexion with other similar experiments.

If we place two organ-pipes tuned in unison in communication with two flames and sound only one of them, the flame of the other shows that its air-column also vibrates in sympathy through communication; and this passing on of the vibrations takes place even if the organ-pipes are not in exact unison with each other, and therefore when sounded together cause *beats* to be heard. But it is to be remarked that in this case the sympathetic pipe does not form its own vibrations, but only vibrations which are exactly in unison with those of the one acting on it, so that beats are neither heard nor their effects seen in the flame. If, however, we blow the second organ-pipe and thus cause its own vibrations, they unite with the resonance-vibrations, and the flame shows clearly, by its violent flickering, the existence of beats, which are also heard distinctly.

I draw particular attention to this isolated occurrence of the resonance-vibrations in the air-column, because it is not exhibited by the influence-phenomena of two strings stretched above the same sounding-board; but the proper vibrations combined with resonance-vibrations appear in the string influenced, without its being struck or bowed.

It is known that the beats of two such mutually sympathetic strings accommodate themselves to each other in such a manner that the one reaches the maximum amplitude of its vibrations when the other is at the minimum. Now the flames of the two sympathetic organ-pipes exhibit the same phenomenon, for as the one rises the other falls; both, however, must be blown at the same time, whilst it is only necessary to play on one of the strings.

When the pipes are in perfect unison, and their single vibrations mutually adapt themselves in the same way as the beats did, *i. e.* that in the node of the one there is a condensation of the atmosphere when in the other a rarefaction takes place, then the whole process can be clearly observed in the two flames if we place them one beneath the other in a vertical line. Both flames show their vibrations unweakened; yet their individual pictures in the rotating mirror are not beneath each other in the two lines, but alternate.

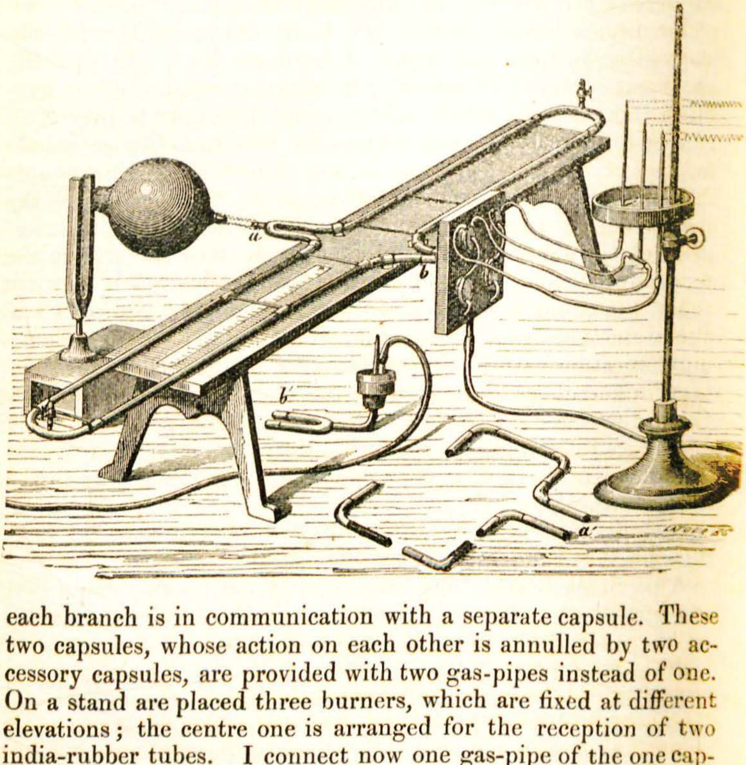
If both notes act together on the same flame, they, of course, at the beats show more violent flickerings than did the two flames; for the latter were produced by direct and by sympathetic and therefore unequally strong vibrations in the same air-column, whilst the present ones are formed by direct and therefore nearly equally strong notes in two similar air-columns. If the two notes are approximated gradually to unison, we observe that the oscillations cannot be made slower at will, as with tuning-forks, but at a certain limit they disappear suddenly, and both air-columns vibrate as one system, *i. e.* as two somewhat differently tuned bodies that are so closely united and therefore act so strongly on one another that neither can give its proper note in its integrity, and the consequence is that only a single intermediate note is produced. This note is more powerful than that of a single organ-pipe; and the flame shows in the centre of its interior a brilliant waist, which rises above a non-brilliant blue broad hollow space. As it approaches perfect unison more and more, the height of this dark space increases, the brilliant waist vanishes; and when unison is attained the flame appears in complete repose. At the same moment the strong fundamental has almost disappeared, and we hear the first overtone clearly produced; for it is known that, there being a difference of half a vibration-period between two equal sounds in unison, while the fundamental and the odd overtones are destroyed, all the even overtones in both sounds vibrate without difference of phase and strengthen one another. The flame also makes the octave recognizable in the rotating mirror, since we see a series of low wide flame-pictures, of which each single one is forked. It is well in this experiment to employ a rather stronger air-pressure, in order to increase the intensity of the octave in the sound of the pipes.

As this prominence of the octave at the interference of the fundamentals of two sounds is demonstrated particularly well by means of the double siren of Helmholtz, I represented the phenomenon in this case also by the flames. To this end I provided each of the two sounding-chests over the turning plates with a tube, which permitted its interior space to be placed in direct communication with the tube leading to the capsule. This tube was of india-rubber, thus retaining the power of movement within certain limits for the upper wind-chest of the siren, so as to be able by its different positions to produce the interference or to withdraw it. Invariably, if we approach the siren-chest to the interference-place, we see the great vibrations of the fundamental gradually disappear, and the short forked flame take their place as the picture of the octave.

A particular apparatus, which I construct for the observation of interference-phenomena of the most various kinds, is founded on the method first employed by Herschel, and after him by many natural philosophers. This is to produce interference by permitting the waves from the same source to traverse two courses differing in length by half a wave, and then to reunite them. It consists of a tube that between its ends branches into two arms, one of which can by drawing out be lengthened at will (fig. 13). If we wish a complete interference, we must introduce a simple note into the tube, which is joined to a resonator, before which we sound the proper tuning-fork. If we now lengthen the one arm until the difference of length of the two is equal to half the wave-length of the note of the tuning-fork, the waves coming from the two arms are mutually destroyed at the other end of the tube; and if we fix this into a small cavity, over which a manometric capsule is placed, we see, on drawing out one of the arms of the tube, how the at first deeply serrated flame-series in the rotating mirror gradually transforms itself into a simple band of light, until the difference of a half wave-length is attained.

But the interference can be shown still more beautifully by another arrangement. Instead of causing the arms united to a single tube to act on a capsule, I place a small apparatus to both exits of the two tube-branches; this is so arranged that now

Fig. 13.



each branch is in communication with a separate capsule. These two capsules, whose action on each other is annulled by two accessory capsules, are provided with two gas-pipes instead of one. On a stand are placed three burners, which are fixed at different elevations; the centre one is arranged for the reception of two india-rubber tubes. I connect now one gas-pipe of the one capsule with the highest burner, one pipe of the other capsule with the lowest, and by means of the remaining two exit-pipes I place both capsules in communication with the centre burner. If I now strike the tuning-forks while the lengths of the tube-branches are equal, the three flames in the rotating mirror show three equally deeply serrated flame-series one above another, of which the centre alone changes into a simple band of light on lengthening one of the branches a half wave-length of the note, while both the other flames continue to vibrate with unchanged intensity. Thus we have here at the same time a view of the action of the sound-waves when they approach through the one arm alone, when they have passed through the second only, and also when they arrive united at the flame after passing through both.

If in these experiments we employ instead of a tuning-fork with a resonator an open organ-pipe of not too great diameter, during the interference of the waves of the fundamental the vibrations of the octave become again prominent. By interference we can remove not only the fundamental, but any overtone we please from a sound, as may be clearly demonstrated with the above-described covered pipe. I conduct the sound into the apparatus, while I connect with it, after the removal of the gas-burner, the capsule at the end by means of an india-rubber tube. If I then draw out the one tube so far that interference ensues for note 3, the centre flame in the mirror shows the simple flame-series of the fundamental, while the two others form the picture before described (fig. 5, Pl. I.), resulting from the combination of notes 1 and 3. In the same way we can banish from vowel-sounds various overtones, or rather whole series of them, which offers a new and fruitful method for the investigation. In these experiments the arrangement with three flames is particularly useful, because the upper and lower flames remaining always unchanged permits the slightest alteration in the middle one to be observed. Thus, for example, the vowel U sung on $\bar{c}$ into the apparatus shows the fundamental only weakly, accompanied by the octave. If we place the apparatus so that the waves of $\bar{\bar{c}}$ interfere, every trace of this octave is lost, whereas on the interference of the fundamental two narrow flames of almost equal height take the place of each wide flame; these narrow flames represent the octave, now almost alone. With O sung on the same note (when the fundamental is accompanied much more strongly by the octave than with U) we can make the same experiments; only here at the interference of the octave the note 3 becomes prominent, whilst the wide flame of the fundamental spreads out into three diminishing summits. A sung on *c*, at the interference of the third note brings forward strongly the octave with the fundamental. If the waves of the octave interfere, there appears a group of five flame-summits, which appear to indicate the notes 1, 3, and 5. If we suppress the fundamental and with it the notes 3, 5, &c., there appears a simple flame-series, which is formed by the octave alone.

These phenomena are nevertheless not always of so simple a nature as in these examples, when it is a question of more composite flame-groups of the deeper sounds; and therefore I will now call attention to the fact that, on lengthening one of the tubes of the apparatus, we often see suddenly very great changes in the flame-picture when the former is between the interference-points of two successive overtones of the sound. This is then the interference-point of the lower octave, or twelfth of a higher overtone of the sound, which is in this way removed.

In the place of the forked tube into which in all the foregoing experiments the note or sound was introduced, we can put two separate tubes of exactly equal length and form, each consisting of three separate pieces inserted one in another and capable of being turned round so that we can move the two openings at their ends in any direction we please without alteration of the length of the tube or of the form of its turnings. This arrangement permits then the entrance of the note of two different points of a vibrating body into the apparatus—for example, of two vibrating bridges of a plate with contrary signs, or of the same place on its opposite surfaces: in both these cases the interference takes place when the two paths are equal, and the tone first becomes audible when the interference is destroyed by lengthening one of the compound tubes.

In order to adapt the apparatus to the demonstration of the wave-lengths of a note in different gases, and for the experiments of Zoch, I have provided the pipes with two cocks, which serve to fill and empty them. Of course, if we experiment with any other gas than atmospheric air, the resonator cannot remain in direct communication with the interior of the pipe; and therefore we must in that case place between them a small cavity, which is divided in the centre by a thin membrane into two halves—the one to be united with the pipe, the other with the resonator. Besides we must then have india-rubber rings to draw over the ends of the tubes which are only placed within each other, so that the gas cannot escape at these places.

It is, of course, understood that this apparatus permits the direct observation of different interference-phenomena by the ear, and consequently the repetition of the experiments of Mach, Quincke, and others. For this purpose we have but to place one of the forked tubes before the apparatus and connect the former with the ear by an india-rubber tube.

XV. *On the best Arrangement of* Wheatstone's *Bridge for measuring a given resistance with a given Galvanometer and Battery.*

By Oliver Heaviside, *Great Northern Telegraph Company, Newcastle-on-Tyne*.*

In the figure, *a, b, c,* and *d* are the four sides of the electrical arrangement known as Wheatstone's bridge or balance, *e* the galvanometer, and *f* the battery branch. Throughout this paper *d* is supposed to be the resistance to be measured, and *e* and *f* both known. The problem is to find what resistances should be given to the sides *a, b,* and *c* (which we are able to

* Communicated by the Author.