XVIII. *Acoustical Notes.*—VIII.

Partial Tones of Stretched Strings of Variable Density—Maintenance of Vibrations by Impact of Water Drops—Discrimination between Sounds from directly in front and directly behind the Observer—The Acousticon—Pitch of Sibilants—Telephones.

By Lord Rayleigh, *O.M., Pres.R.S.**

Partial Tones of Stretched Strings of Variable Density.

If ρ_0 be the longitudinal density of a string of length l stretched with tension T_1, the periodic time (τ_r) of the rth component vibration is given by

$$\tau_r^2 = \frac{4l^2\rho_0}{r^2 T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (0)$$

In 'Theory of Sound,' 2nd ed. § 140, an approximate theory is given applicable when the density, in place of being strictly uniform, has the value $\rho_0 + \delta\rho$ where $\delta\rho$ is relatively small. We have

$$\tau_r^2 = \frac{4l^2\rho_0}{r^2 T_1}\left\{1 + \frac{2}{l}\int_0^l \frac{\delta\rho}{\rho_0}\sin^2\frac{r\pi x}{l}\,dx\right\} \quad . \quad . \quad . \quad (1)\dagger$$

"If the irregularity take the form of a small load of mass m at the point $x=b$, the formula may be written

$$\tau_r^2 = \frac{4l^2\rho_0}{r^2 T_1}\left\{1 + \frac{2m}{l\rho_0}\sin^2\frac{r\pi b}{l}\right\}. \quad . \quad . \quad . \quad (2)$$

These values of τ_r are correct so far as the first power of the small quantities $\delta\rho$ and m, and give the means of calculating a correction for such slight departures from uniformity as must always occur in practice.

As might be expected, the effect of a small load vanishes at nodes, and rises to a maximum at the points midway between consecutive nodes. When it is desired merely to make a rough estimate of the effective density of a nearly uniform string, the formula indicates that attention is to be

* Communicated by the Author.

† The r^2 in the denominator was inadvertently omitted. It may be worthy of note that the method is not limited to the case of density originally uniform. In general, if Y be the normal function corresponding to a variable density ρ, τ_r^2 is altered in the ratio

$$1 + \frac{\int \delta\rho\, Y^2\,dx}{\int \rho\, Y^2\,dx}.$$

FIG. 1.

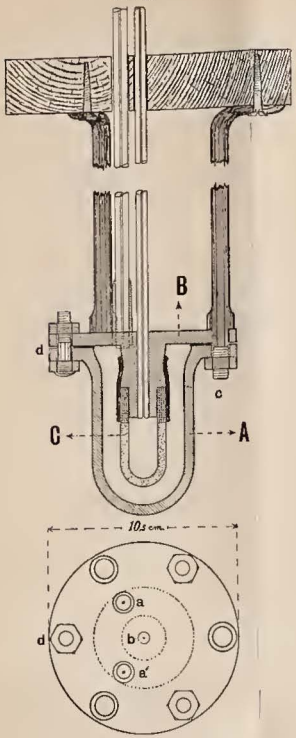
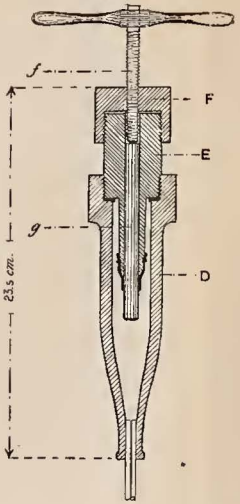


FIG. 2.



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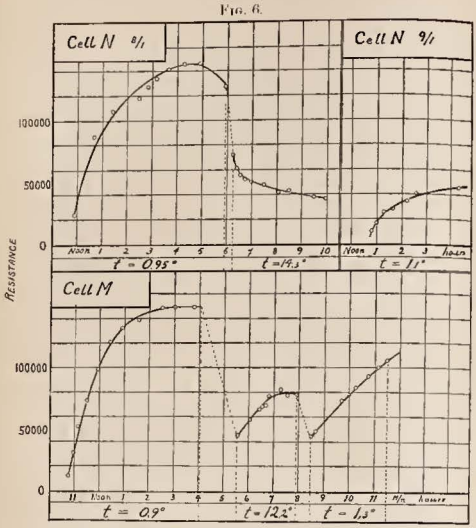
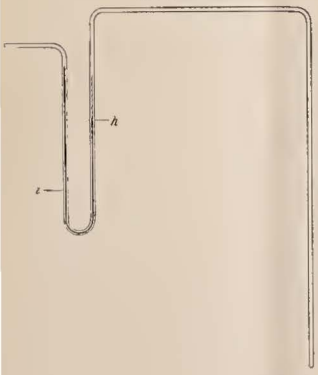


FIG. 3.



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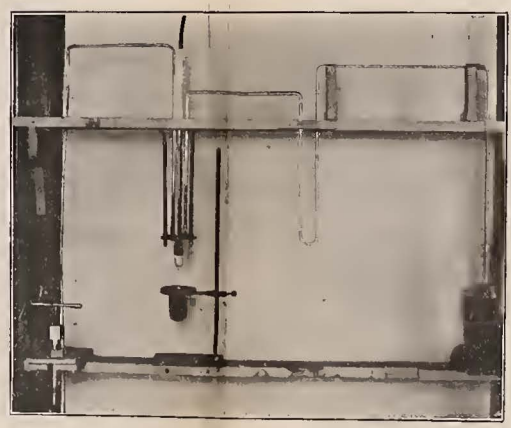
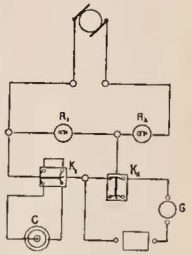


FIG. 5.



given to the neighbourhood of loops rather than to that of nodes.

The effect of a small variation of density upon the period is the same whether it occur at a distance x from one end of the string, or at an equal distance from the other end. The *mean* variation at points equidistant from the centre is all that we need regard, and thus no generality will be lost if we suppose that the density remains symmetrically distributed with respect to the centre. Thus we may write

$$\tau_r^2 = \frac{4l^2\rho_0}{r^2 T_1}(1+\alpha_r), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$\alpha_r = \frac{2}{l}\int_0^{\frac{1}{2}l} \frac{\delta\rho}{\rho_0}\left(1-\cos\frac{2\pi r x}{l}\right) dx. \quad . \quad . \quad . \quad (4)$$

In this equation $\delta\rho$ may be expanded from 0 to $\frac{1}{2}l$ in the series

$$\frac{\delta\rho}{\rho_0} = A_0 + A_1\cos\frac{2\pi x}{l} + \ldots + A_r\cos\frac{2\pi r x}{l} + \ldots, \quad (5)$$

where

$$A_0 = \frac{2}{l}\int_0^{\frac{1}{2}l}\frac{\delta\rho}{\rho_0}dx, \qquad A_r = \frac{4}{l}\int_0^{\frac{1}{2}l}\frac{\delta\rho}{\rho_0}\cos\frac{2\pi r x}{l}dx. \quad . \quad (6, 7)$$

Accordingly,

$$\alpha_r = A_0 - \tfrac{1}{2}A_r. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation, as it stands, gives the changes in period in terms of the changes of density supposed to be known. And it shows conversely that a variation of density may always be found which will give prescribed arbitrary displacements to all the periods. This is a point of some interest.

In order to secure a reasonable continuity in the density, it is necessary to suppose that $\alpha_1, \alpha_2, \ldots$ are so prescribed that α_r assumes ultimately a constant value when r is increased indefinitely. If this condition be satisfied, we may take $A_0 = \alpha_\infty$, and then A_r tends to zero as r increases.

As a simple example, suppose that it be required so to vary the density of a string that, while the pitch of the fundamental tone is displaced, all other tones shall remain unaltered. The conditions give

$$\alpha_2 = \alpha_3 = \alpha_4 = \ldots \alpha_\infty = 0.$$

Accordingly,

$$A_0 = A_2 = A_3 = \ldots = 0, \qquad A_1 = -2\alpha.$$

Thus by (5)

$$\delta\rho/\rho_0 = -2\alpha_1\cos(2\pi x/l). \quad . \quad . \quad . \quad . \quad (9)''$$

I have recently made a few observations on the vibrations of loaded wires in illustration of these formulæ. A simple case is afforded by (2). A lump of wax is attached at the centre of a stretched wire, so that $\sin^2(r\pi b/l)=0$ when r is even, and when r is odd takes the value unity, independently of the particular value of r. It is easily verified on trial that the partial tones of even order are undisturbed, and that those of odd order, though displaced, still constitute a *harmonic series*. The observation is best made with the aid of resonators by comparison with the partial tones of a note on the harmonium. The load may conveniently be chosen so as to depress the pitch by one or more complete semitones. If also the tension (due to a weight) be suitably adjusted, both series of partial tones may be brought into tune with the harmonium, either exactly, or (what is better in practice) approximately.

A more elaborate experiment was next attempted in illustration of the result expressed in (9). But some modification is necessary, inasmuch as in practice we are limited to *positive* loads. But a uniform loading has little significance from our present point of view, lowering all the partial tones by the same musical interval and being capable of compensation by varying the tension (T_1). It suffices therefore to take $\delta\rho/\rho_0$ proportional to $1-\cos(2\pi x/l)$; and the experiment consists in verifying that the first partial is depressed relatively to all the other tones, and that these latter retain their harmonic relations.

The length of the wire was adjusted to 720 mm., and loads proportional to $1-\cos(2\pi x/l)$ were attached at intervals of 30 mm., beginning at $x=0$ and corresponding to angles 0°, 15°, 30°, &c. The unit load (about $\frac{1}{4}$ gm. of wax) was at the points distant from the ends by one quarter of the length (90°, 270°), and the maximum load (2) was at the centre (180°). At the ends (0°, 360°) the loads were zero. By varying the tension the whole was tuned conveniently to suit the harmonium.

The results agreed fairly well with theoretical anticipations. The second and third partials retained very accurately their harmonic relation (fifth). The fourth and fifth partials were too sharp relatively to the second and third by nearly half a semitone. The depression of the first partial relatively to the second and third was 3 semitones or a minor third.

These estimations of pitch were made separately by Mr. Enock and myself, and in the case of the higher partials demanded some care. Useful indications are afforded by but partially depressing the key of the harmonium note, whereby

the pitch is lowered relatively to the normal. On the other hand it appeared, rather to my surprise, that a lowering of the bellows pressure (sometimes convenient in order to diminish the intensity of sound) slightly *raises* the pitch.

When the loads of wax were all removed, the first partial tone rose nearly an octave, indicating that the loading had been very severe.

In a second experiment the loads were reduced to less than half, the unit being taken equal to 0·1 gm., the distribution of the loads and the length of the wire being as before. Comparisons, either directly with the harmonium or with forks as intermediaries, gave the following results. The second partial of the wire was 5 beats per second flat on (the octave overtone of) harmonium B. The third partial was 4 beats per second flat on the corresponding partial of B. The fourth partial was 2 beats flat on B, and the fifth partial was 2 beats sharp on B. If we regard the two last partials as in tune with B, the second partial is out by 5 vibrations in about 256, or one in 55, viz. between a third and a quarter of a semitone. The first partial of the loaded wire was almost exactly two semitones below harmonium B.

I had supposed that the small deviations from harmonic relations exhibited by the second and higher partials might be due to the discontinuity of the loading or to the fact that the loads were not sufficiently concentrated upon the line of the wire. But subsequent observations upon an unloaded wire showed very similar deviations. The length was the same but the tension was diminished so as to bring the pitch back to B. The first partial of the wire was now one per second flat, the second $1\frac{1}{2}$ per second flat, the third 2 per second flat, the fourth 1 per second *sharp*, the fifth $3\frac{1}{2}$ per second sharp, and the sixth about 8 per second sharp, all referred to the corresponding partials of harmonium B. Since the partials of the harmonium note are necessarily in tune, harmonic relations among the partials of the wire would require beats always upon the same side and numerically as the natural numbers 1, 2, 3, 4, 5, 6. It may be noted that the wooden bridges, by which the vibrating portion of the wire was limited, were provided above and below with sharp edges, and were free to turn. In this way the vibrating portion was well defined, and the tension (due to weights hung over a pulley) was freely transmitted.

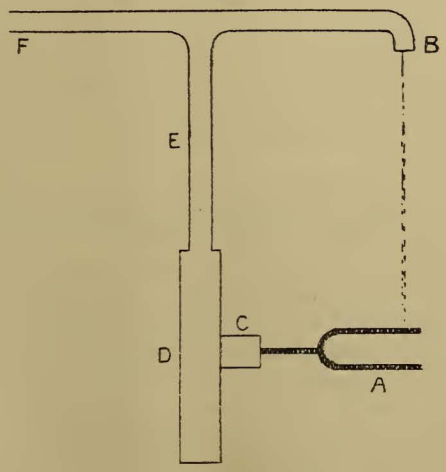
Maintenance of Vibrations by Impact of Water Drops.

It is known that jets of liquid tend to resolve themselves under capillary force into more or less uniform processions

of drops, the distance between the drops, or the number of drops passing a given point in unit time, depending upon the diameter and velocity of the jet. When a jet, otherwise undisturbed, is under the influence of a regular vibrator of suitable pitch, the resolution becomes absolutely uniform and takes its frequency from the vibrator *. If the procession of drops thus regularized is caused to strike a second vibrator of like pitch, the latter will usually be excited. The arrangement is simplified and any question of insufficient agreement of pitch is eliminated, if both parts be assigned to the *same* vibrator which at once regularises the resolution of the jet and is itself maintained in vibration by the impact of the drops.

The experiment has been tried with tuning-forks of pitch 256 and 320. The reaction between the fork A and the jet issuing at B (fig. 1) is effected through a branch tube E

Fig. 1.



terminating in a metal box D. The box is provided with a wooden socket cemented on a flexible face, to which the fork is screwed. The vibrations of the fork, transmitted through its stalk, cause the flow at B to be slightly variable, and if the adjustments are suitable determine a stream of drops of the same frequency. The orifice at B, perforated in a thin metal plate, is of about 2 mm. diameter. The tubes are of lead, allowing slight adjustments by bending; the supply

* The theory of these effects, on lines roughly sketched by Savart and Plateau, is given in Proc. Roy. Soc. xxix. p. 71, 1879; Scientific Papers, i. p. 377; or 'Theory of Sound,' 2nd edition, ii. p. 362.

of water may be either directly from a tap or preferably from an aspirator bottle. The head of water, about 30 cm., must be adjusted; and it is to be remarked that the question is not merely one of accommodating the natural pitch of the jet to that of the fork. There is also the phase to be considered, for the impact of the drops may check, as easily as encourage, an existing vibration. A slight alteration in the distance between A and B may here be useful. In practice attention should be given to the place of resolution of the jet, easily discerned in a suitable light. When this is brought up as near to the orifice as possible, it will be known that the vibration is vigorous and that the phase relation is suitable.

The experiment was quite successful. Both the forks referred to spoke well and steadily when suitable resonators were held near their prongs. But the arrangement is hardly to be recommended for general purposes. The use of water is messy, and unless care be taken is likely to end in rusting the forks. Moreover, the vibrations are not especially vigorous—in comparison, for example, with those which may be obtained electromagnetically. Another objection is to be found in the circumstance that drops of water remaining attached to the fork must render the precise pitch uncertain.

Discrimination between Sounds from directly in front and directly behind the Observer.

As already* mentioned, I am now unable to make the discrimination myself, even in the case of the voice used naturally; so that all that I can report relates to the observations of others. It would seem, however, that even youthful listeners are not always able to pronounce with certainty. In experiments made with some young people in a long corridor, they were able to discriminate among themselves whether a voice came from in front or behind, but when I spoke they made mistakes. The speaker facing towards the listener gave, for example, the numerals one—two—three &c.; and there were enough assistants moving backwards and forwards to eliminate information which might otherwise be given by footsteps. Why my voice afforded less foundation for a judgment was not clear—possibly in consequence of its graver pitch, or because its quality was less familiar. The corridor was so long that the observations were not appreciably disturbed by echos.

As mentioned before, Mr. Enock is able in many cases to

* Phil. Mag. vol. xiii. p. 231 (1907).

discriminate front and back when the voice is used naturally. But I find that both indoors and outside he could be deceived. Thus when standing on the lawn only a short distance in front of him, but facing *from* him, I gave the numerals, he judged that I was behind him, and this erroneous judgment was not disturbed even when I conversed freely with him. It would appear that there is not much to go upon, and that when an erroneous impression has once been made it is not easily disturbed by the slight indications available. Probably the turning away of the speaker softens the sibilants and other high elements in the sound, somewhat in the same way as is done by the external ears of the listener when he faces away from the sound. It must be understood that in these experiments the ears were used in a natural manner, without the aid or hindrance of special reflectors.

The repetition and extension of these observations would be of interest ; it would be best carried out under the supervision of a physicist young enough to be able himself to form judgments as to the front or back situation of the easier sounds, *e.g.* of the voice. The precautions necessary are indicated in former papers.

The Acousticon.

This instrument, intended to aid the hearing of the partially deaf, is composed of a simple battery, microphone, and telephone circuit. There is nothing special about the battery (one or two dry cells) or the telephone. But the microphone is unusually efficient. The disk which receives the sound is of carbon, about $\frac{1}{2}$ mm. thick, and is clamped at the circumference. Bearing against it are six groups of small ($\frac{1}{2}$ mm.)

Fig. 2.

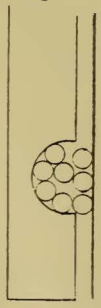
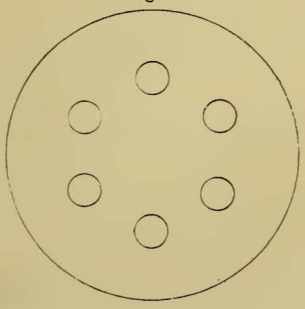


Fig. 3.



carbon spheres, having the appearance of shot, held in hemispherical cups cut out in a thick plate of carbon (figs. 2, 3). In use the microphone may be worn like a medal upon the

breast, or it may stand upon the table, *e. g.* at dinner. In a large model there are two microphones, as described, connected in parallel. The instruments upon which I experimented were kindly lent me by a friend who had found them efficient, though fatiguing to the ear.

For my purpose the microphone and telephone were placed in separate rooms, so that nothing could be heard except through the instrument. The reproduction of speech, given at about one foot away from the microphone, was better than anything I had ever heard before. The first impression was that all the consonantal sounds were completely rendered, but this turned out to be an illusion. In listening to the numerals, given in order, the observer would feel confident that he heard the *f* in *five* and the *s* in *six*. But if the initial sound was prolonged—*fffive*, *ssix*, the observer could not tell until he heard the sequel which it was going to be. Further, if the sounds were given as *ssive*, *ffix*, they were heard normally as five and six. It was plain that there was no difference in the rendering of *f* and *s*. I am informed that this is a well-known difficulty in ordinary telephoning, and that in spelling a name containing *f* or *s* it is usual to say "*f* for Friday" or "*s* for Saturday." But the articulation of the acousticon is so superior that it was surprising to find the failure complete. The characterisation of *sh* was not much better, though after a little practice I could distinguish it from *s* or *f*, but probably only by a greater loudness.

These failures might have been ascribed to my rather defective hearing, but other observers with normal hearing did no better. When, however, the iron plate of the telephone was replaced by one three times as thick, a difference between *f* and *s* could be detected, though both were rather weak. The above observations were made with the small model and usually with but one battery cell. In the case of the large model microphone and with two cells in action, it was just possible to hear some difference between *f* and *s*, the usual iron plate of the telephone being employed.

The question arises as to *how* the acousticon aids defective hearing. A failure to distinguish *f* and *s* seems a bad beginning. I could not find that the general loudness of speech was increased by the instrument. When the speaker stood some 20 feet away from the microphone, I could hear better directly from the situation of the microphone than when in the further room I listened through the instrument. Possibly a clue may be found in the behaviour of the acousticon in presence of low notes. These are very feebly transmitted. Pure tones from forks making 128, 256, 384 vibrations per second, held in front of Helmholtz resonators and close to the

microphone, were poorly heard, and of the sound received the greater part seemed to be overtones. Of course low notes are not necessary for speech, otherwise women would be at a disadvantage, contrary to all tradition. But can we suppose that low notes are actually deleterious?

At this point one recalls the observations of A. M. Mayer * upon the obliteration of higher sounds by graver ones. These observations have not attracted the attention they deserve. The author himself sufficiently emphasises their importance but he does not appear to have followed them up, as he announced the intention of doing. It was proved that while higher and feebler sounds could be entirely obliterated by louder and graver ones, on the other hand a feeble graver sound remained audible in the presence of a more powerful acuter one. "Indeed in this case as in all others where one sound remains unaffected by intense higher notes, the observer feels as though he had a special sense for the perception of a graver sound—an organ entirely distinct from that which receives the impress of the higher tones.

"That one sonorous sensation cannot interfere with another which is lower in pitch is a remarkable physiological discovery . . . ."

If we suppose, as I think we may, that one type at any rate of deafness involves obtuseness to the higher elements of sound upon which the intelligibility of speech largely depends, while the hearing of graver sounds is unimpaired, Mayer's principle suggests that advantage may ensue from an instrumental suppression of the graver components. A scientific friend has informed me that a relative of his was insensitive to grave sounds and that when addressed in a railway carriage expostulated against being shouted to, as if his hearing was less interfered with by the noises of the train than that of normal persons. There is said indeed to be one type of deafness which finds advantage in such noises, but perhaps only because people speak louder. It would be interesting to experiment upon such a case in detail. For my own part I can hear scarcely anything of what is said to me in the train.

Another indication of the importance of the higher elements in speech is afforded by the advantage experienced by many deaf people from placing the hands behind the ears, palms forward and curved. The tick of a clock, for example, is much enhanced. Perhaps the most striking effect is upon the sound of the wind whistling through trees. Artificial reflectors may of course replace the hands, but for the best results they need to be rather nicely shaped.

* Phil. Mag. ii. p. 500 (1876).

Pitch of Sibilants.

In connexion with the audibility of sibilants it is desirable to have some idea of their character in respect of pitch. Doubtless this may vary over a considerable range. In my experiments the method was that of nodes and loops *, executed with a sensitive flame and sliding reflector. A hiss given by Mr. Enock, which to me seemed very high and not over audible, gave a wave-length (λ) equal to 25 mm. with good agreement on repetition. A hiss which I gave was graver and less definite, corresponding to $\lambda = 32$ mm. The frequency of vibration would be of the order 10,000 per second, more than 5 octaves above middle *c* †.

Telephones.

Some miscellaneous observations upon telephones may here be recorded. From the fact that an improved articulation of the sibilants and some other consonants accompanies a thickening of the telephone plate, although at considerable cost in other respects, it was thought that a similar advantage might be attained by the introduction of a condenser into the electrical circuit. This entails no further complication if the transmitter, as well as the receiver, be a Bell instrument; but if a microphone be employed as transmitter, a small transformer must also be introduced, whose ratio of transformation may, if desired, be 1 : 1. The line, receiver, and condenser then constitute electrically a secondary circuit, and by choice of a suitable capacity the proper tone of this circuit may be tuned to any desired pitch. The earliest consideration of resonance in an electrical circuit in response to a periodic force was probably by Maxwell ‡ in connexion with Grove's "Experiment in Magneto-Electric Induction." If L, R be respectively the self-induction and resistance of a circuit, C the capacity of the interposed condenser, the current (dx/dt), elicited by the imposed electromotive force Ee^{int}, is given by

$$\left(-Ln^2 + iRn + \frac{1}{C}\right)x = E\,;$$

* Phil. Mag. vii. p. 149 (1879); Scientific Papers, i. p. 406.

† Compare Wiersch, *Drude Ann.* xvii. p. 1001 (1905). "Sieht man von ihren tiefsten Partialtönen ab, welche lediglich infolge Resonanz der Mundhöhle beigemischt sind, so entstehen die eigentlichen Reibelaute durch eine Schwingungszahl, welche minimal derjenigen des Grundtones der Luftsäule einer einseitig gedeckten Pfeife von ca. 14 mm. Länge entspricht, maximal aber im Bereiche der Unhörbarkeit liegt." This paper contains some interesting observations upon the influence of the *proper tone* of a telephone plate upon the articulation.

‡ Phil. Mag. May 1868; Maxwell's Scientific Papers, ii. p. 121.

so that the maximum effect occurs when

$$CLn^2 = 1,$$

i. e., when the natural frequency of the circuit coincides with that of the imposed force. *For this pitch* the self-induction is compensated. The effect of short-circuiting the condenser may be represented by taking $C = \infty$. If, apart from phase, the current is unaltered by short-circuiting the condenser, the capacity in action must be equal to one-half of the most favourable capacity, or else must be so great as not to be distinguishable in experiment from infinity. It is to be noted that the accurate compensation of self-induction can be effected for only one pitch, and that in practice the advantage will be limited to a range of pitch not exceeding an octave.

Such experiments as I have been able to make did not exhibit any improvement in articulation as the result of including a condenser in the circuit. It is possible that the simple theory above stated is too much interfered with by complications such as eddy-currents in the iron or hysteresis in the action of the condenser. A distinct resonance could be attained in the region of the higher notes of my harmonium (about 2000 vibrations per second) when an *additional* self-induction was included in the circuit. Such resonance would be practically limited to a region including 3 or 4 semitones, and when at the best pitch the condenser was put out of action by a short-circuiting key there was a very marked falling off of intensity. It is probable that this subject has already received attention from those engaged in the endeavour to improve the telephone as a practical instrument; if not, I think it would be worthy of investigation. There should be no great difficulty in securing several electrical resonances at pitches suitably distributed, in addition to the mechanical resonance of the plate.

If my memory serves me, it was observed in the early days of telephony that the sounds remained audible even though the usual ferrotype plate were replaced by one of other material. The experiment is easily tried. When a telephone provided with a copper or aluminium plate was included in a circuit with a battery (of one or two cells) and a simple carbon microphone (after Hughes), the ticking of a watch placed near the microphone was easily audible, and (it happened) with different quality in the two cases. On the other hand a plate of mica was silent under similar conditions, although when the excitation is very violent, as with a make-and-break arrangement, sounds may be heard without a plate

at all. The telephone employed was bipolar and of modern manufacture.

In another experiment the permanent steel magnets were removed from the telephone and replaced by a soft-iron U which could be magnetized at will by an independent electric current, the coils and pole-pieces of the telephone remaining undisturbed. Without a magnetizing current but little of the ticks of the watch could be heard from the copper plate, but when the soft iron was magnetized the sounds became distinct as before.

I had supposed at first that this experiment might discriminate between the two possible explanations of the sound, the one depending upon traces of iron as an impurity in the copper or aluminium, the other invoking the action of induced currents circulating in the metallic plates. But it appears that in either case the efficiency would be promoted by a high constant magnetization of the pole-pieces.

The copper disk, weighing about one gram, was of a kind unlikely to contain appreciable iron and its action was not affected by washing with hydrochloric acid. When a mica disk, which of itself gave no sound, was dusted over with 1 mg. of fine iron filings attached with varnish, only a very faint sound could be made out. A similar mica disk coated with $\frac{3}{4}$ gm. of copper filings from the same material as that of the disk, yielded no sound. A similar telephone with copper disk was included in the circuit for the sake of rapid comparison throughout the experiments.

From these results it appeared unlikely that the effects were to be attributed to traces of iron in the other metals, and this conclusion was confirmed in a varied form of the experiment tried later. Mr. Enock prepared two flat coils of fine covered copper wire weighing together about 2 gms. These were mounted separately on pieces of mica afterwards cemented with wax to the main disk so as to encircle the magnetic poles. When the ends of the flat coils were disconnected, nothing was heard ; but on completing the circuit of one or both coils the effects were very distinct, both with the carbon microphone and watch as before used or with another arrangement giving a more powerful action. We may conclude, I think, that the sounds of a copper disk are due to alternating currents induced in it and reacting upon the nearly constant magnetism of the pole-pieces.

Terling Place, Witham,
Jan. 1908.

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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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"Meditationis est perscrutari occulta; contemplationis est admirari perspicua .... Admiratio generat quæstionem, quæstio investigationem, investigatio inventionem."—*Hugo de S. Victore.*

——"Cur spirent venti, cur terra dehiscat,
Cur mare turgescat, pelago cur tantus amaror,
Cur caput obscura Phœbus ferrugine condat,
Quid toties diros cogat flagrare cometas,
Quid pariat nubes, veniant cur fulmina cœlo,
Quo micet igne Iris, superos quis conciat orbes
Tam vario motu."

J. B. Pinelli ad Mazonium.



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ERRATA.

Page 519, line 20, *for* few cubic centimetres *read* few cubic millimetres.

Page 520, line 4 from bottom, *for* glass *read* gas.

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