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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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"Meditationis est perscrutari occulta; contemplationis est admirari perspicua . . . . . Admiratio generat quæstionem, quæstio investigationem, investigatio inventionem."—*Hugo de S. Victore.*

—"Cur spirent venti, cur terra dehiscat,
Cur mare turgescat, pelago cur tantus amaror,
Cur caput obscura Phœbus ferrugine condat,
Quid toties diros cogat flagrare cometas;
Quid pariat nubes, veniant cur fulmina cœlo,
Quo micet igne Iris, superos quis conciat orbes
Tam vario motu."

J. B. Pinelli ad Mazonium.



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SUPPLEMENT TO VOL. XVI. FOURTH SERIES.

LV. *On the Theory of Sound.* By J. J. WATERSTON, *Esq.**

IN a paper on the Integral of Gravitation that appeared in the Philosophical Magazine of May last, I endeavoured to draw attention to the principle of physical causation supplied to us by the mechanical theory of heat,—how it leads us to study natural phænomena in their dynamical sequence, and suggests the arranging of problems of molecular statics so that they may present the aspect of motor transition to the mathematical inquirer. It may perhaps be of advantage, as testing the general argument, to discuss the question, *Can the theory of the propagation of sound developed by Newton and Laplace be viewed, consistent with the modern idea of heat, as a correct exposition of the actual mode in which pulses are transmitted through air*? It appears from recent memoirs, that this opinion is still generally entertained by physicists; and the theory of Laplace has in some instances been considered so perfect as to afford the means of deducing the specific heat of air from the velocity of sound, and this by zealous professors of the mechanical theory of heat.

The ideas upon which Laplace's theory was constructed are to be found in the second chapter of the second book of the *Mécanique Céleste*. The following extracts show how very remote they are from those now universally prevalent:—

Page 105, line 8. "Doubtless it is necessary to admit that between the molecules of air there is a repulsive force which is only sensible at imperceptible distances: the difficulty consists in deducing from it the laws of elastic fluids. This may be done by the following considerations."

P. 105, line 4 from bottom. "I suppose then that these molecules retain the heat by their attraction, and that their mutual repulsion is due to the repulsion of the molecules of the heat which I assume to extend to an insensible sphere of activity."

P. 111, line 9. "But what ought we to understand by the temperature u, and what is its measure? It appears natural to

* Communicated by the Author.

take for this measure only the density of caloric produced in a space by the radiation of neighbouring bodies."

P. 111, line 19. "A supposition that it appears very natural to admit is, that the action of the caloric of a molecule of gas on the caloric of another molecule, &c."

P. 113, line 6. "We have supposed, in that which precedes, that the caloric of a molecule was retained on it by the attraction of the molecule, which experienced no sensible action except by the repulsive force exerted on this caloric by the caloric of the surrounding molecules."

P. 117, line 16. "It is then extremely probable that the attractive force of the caloric of one molecule by another molecule is insensible in the state of gas."

Professor Thomson, who has written largely on the dynamical theory of heat (Philosophical Magazine; Edinb. Phil. Trans.; London Phil. Trans.), has the following in a memoir "On the Thermal Effects of Fluids in Motion" (Phil. Trans. 1854, p. 361):—

"In the notes (Prof. Thomson's notes) to Mr. Joule's paper on the Air-engine (Phil. Trans. 1852, p. 82), it was shown that if Mayer's hypothesis be true, we must have approximately—

Specific heat of air with constant pressure	·2374
Specific heat of air with constant volume	·1684

because observations on the velocity of sound, with *Laplace's theory*, demonstrate that $k = 1{\cdot}410$ within $\frac{1}{700}$th of its own value. Now the experiments at present communicated to the Royal Society prove a very remarkable approximation to the truth in that hypothesis*; and we may therefore use these values as very close approximations to the specific heats of air."

The ratio k was found by MM. Gay-Lussac and Welter to be 1·37, and by MM. Clement and Desormes 1·35. The same experiments repeated by the author of the article 'Hygrometry' in the *Encyc. Brit.*, impressed him with the conviction that the *initial* ratio was exactly $\frac{4}{3}$.

Here, therefore, we have an instance of a zealous adherent and expounder of the dynamical theory of heat showing such confi-

* At p. 341, under the heading "Theoretical deductions from these Experiments," Prof. Thomson arrives at the conclusion that air and carbonic acid "evolve more heat than the amount mechanically equivalent to the work of compression," thus representing the experiments as proving an *exception* to Mayer's hypothesis. That such cooling effects must take place in consequence of the *deviation* from Mariotte's law discovered by Regnault, if Mayer's hypothesis and the dynamic theory of heat hold good, seems obvious enough (see Phil. Mag. vol. xiv. p. 279). So that they are actually a delicate test of the accuracy of that hypothesis; and taken as supplementary to Mr. Joule's previous experimental inquiries, seem completely to establish it as a theory proved by induction.

dence in Laplace's views, such conviction of their truth to nature, that he considers them entitled to supersede the result of direct observation; and no attempt is made to reconcile the statical theory of caloric with the *vis viva* theory of heat; nor is a necessity for doing so apparently even felt as a preliminary step to the adopting of one of the deductions of the former as an ally to the deductions of the latter.

As Mr. Joule's name is conjoined with that of Professor Thomson in this memoir, it is perhaps necessary to assume that he also approves of this manner of deducing the ratio of specific heats.

Another instance of the commingling of statical and dynamical theories is to be found in a memoir by M. Masson, "On the Correlation of the Physical Properties of Bodies," which appeared in a recent number of the *Annales de Chimie*. M. Masson also adopts the mechanical theory of heat (see § III. chap. 2). Yet in the beginning of the same chapter he writes, "Laplace has discovered the true mathematical expression of the velocity of sound in gas." Further, M. Masson states that he has found, by experimenting according to the method employed by MM. Gay-Lussac and Welter, that the value of k is actually what is required to make Laplace's formula agree exactly with the best observations on the velocity of sound. No details are given, although the amount of compression in such experiments is of some importance, as the ratio increases with this amount; and the formula has only to do with the initial ratio, or when the change of density is infinitely small. M. Masson has also computed the velocity of sound in a considerable number of composite gases and vapours from the pitch of the sound given by an organ pipe while immersed in them; and thence, employing Laplace's theorem, obtains the respective values of k, which are thus found to range from 1·26 to 1·42 in the composite gases, and from 1·06 to 1·27 in the vapours.

Thus all authorities, both of the statical and dynamical school, seem to agree that Newton and Laplace's theory of sound is a perfect representation of nature, and that its success is as complete as the theory of gravitation.

Newton, in several parts of the *Principia*, calls upon us to keep in mind that his principles are mathematical, not philosophical. At the beginning of Book 3 he thus expresses himself:—"In the preceding books I have laid down the principles of philosophy; *principles not philosophical, but mathematical, such to wit as we may build our reasonings upon in philosophical inquiries.*" Again, after proving in Prop. 23, Book 2, that "particles flying each other with forces that are reciprocally proportional to the distances of their centres, compose an elastic

fluid whose density is as the compression," he adds, in the scholium, "*But whether elastic fluids do really consist of particles so repelling each other, is a physical question. We have here demonstrated mathematically the property of fluids consisting of particles of this kind, that hence philosophers may take occasion to discuss that question.*"

In Prop. 47 of Book 2, Newton shows that "if pulses are propagated through a fluid, the several particles of the fluid going and returning with the shortest reciprocal motion are always accelerated or retarded according to the law of the oscillating pendulum." It is assumed that the elastic force is proportional to the density; and in the direction of the pulse the fluid is supposed to be divided into *physical lineolæ, which are expansible and contractile,* and exhibit a force that resists compression inversely as their breadth. The mathematical reasoning defines the law by which the breadth of these lineolæ must vary while they go and return,—and hence the law of the difference in the breadth of two adjacent lineolæ, and consequently the law of the accelerative force operating on each corpuscle, which is thus found to be the same as a body moving in a cycloid is subject to under the influence of gravity.

Newton's fundamental hypothesis is, that the particles of air in the direction of the pulse are *successively* agitated with like motions; that both the dynamic condition and the static force of repulsion, which is determined by the length of the line that separates two adjacent particles (called a lineola), is transferred onwards in the direction of the pulse from one particle to the next adjacent in regular succession.

The demonstration takes account of three orders of magnitudes :—1, the breadth of a pulse (L); 2, the breadth of an oscillation of a particle ($2l$); 3, the length of a lineola (λ), each considered as infinitesimal with respect to the preceding.

If the motion of a particle forward and backward in the line $2l$ corresponds to that of a cycloidal pendulum, *i. e.* if the relation between the accelerative force (acting in the line of motion), the acquired velocity, and the time is the same in the line $2l$ as in the complete cycloid, the force in this line must vary simply as the distance (y) from its middle point. The value thus assigned to the force implies that δ, the difference between the lengths of two adjacent lineolæ, should vary also in this proportion. If a semicircle is described on the diameter $2l$, y is the cosine of an arc, of which x being the sine, we have $\delta \div y \div dx$; so that the differential of the lineola ought to be equal to the differential of the sine, and hence the absolute magnitude of the deviation of the length of a lineola from its mean length ought to be proportional to the sine.

Thus if the motion of a particle is oscillatory, like a complete cycloidal pendulum, the required sequence of force demands the above specific sequence of change in the distance of the particles. Again, if the motion of each particle is oscillatory, the required sequence in its velocity (viz. that it should vary as x the sine) demands also a specific sequence of change in the distance of the particles; and this sequence is precisely the same as what is required by the sequence of force.

To obtain a clear idea of the proof of this (which is a problem of pure mathematics), we may suppose with the same radius l another semicircle to be described, placed also in the line of the pulse, and removed to the distance λ from the preceding semicircle. Let a third also be drawn, removed the same distance λ from the second. We have further to suppose these semicircles divided into as many parts (aa_1, a_1a_2, a_2a_3, &c.; bb_1, b_1b_2, b_2b_3, &c.; cc_1, c_1c_2, c_2c_3, &c.), beginning at where the line of pulse intersects them as λ enters into L (or $\frac{L}{\lambda}$ number of parts). The length of each of these parts or *steps* is thus $\frac{\lambda}{L}2l\pi=s$ (being infinitesimal with regard to λ).

Having made this construction, we have next to consider that the motion of each particle to be oscillatory must be such that, at the instant when particle A has traversed the versed sine of aa_n, the particle B (next in *advance* of A) being one *step* behind in its motion, has traversed only the versed sine of bb_{n-1}, and particle C the versed sine of cc_{n-2}. If B had traversed as many steps as A, the distance λ that separates them would not alter; but since it is a step behind, AB is at this point less than λ by the difference between vers aa_n and vers bb_{n-1}, or vers aa_n — vers aa_{n-1}, which equals $s \cdot \sin aa_{n-\frac{1}{2}}$. In the same way, C being a step behind B, their distance is less than λ by $s \cdot \sin aa_{n-1\frac{1}{2}}$. Thus we have $BC-BA=s(\sin aa_n-\sin aa_{n-1})=s\cos aa_n\frac{s}{l}$. [Here s, being an absolute magnitude, has to be divided by the absolute radius l to represent the differential of arc.] At the beginning of the vibration $n=1$, and $\cos aa_n=$ radius; hence with B at the initial point b, C at c_{-1} (a step back on the returning half of the previous oscillation), and A at a_1 (the points on the circle being supposed projected on the diameter), the difference

$$BC-BA=\frac{s^2}{l}=4l\left(\frac{\pi\lambda}{L}\right)^2.$$

This initial amount determines the accelerative force acting at the beginning of the motion of each particle, which is obtained by comparing it with the reciprocal of λ, which represents the

whole static force of repulsion between two particles at the distance λ. This force having to support the weight of $\frac{H}{\lambda}$ particles (H being the height of a uniform atmosphere), $\frac{1}{\lambda}$ represents the force $\frac{H}{\lambda}g$, viz. a force that in one second is capable of communicating a velocity of $\frac{H}{\lambda}g$ feet per second.

To obtain the value of the initial force acting on particle B when it is at b, we have the following proportion:

$$\frac{1}{\lambda} : \frac{1}{\lambda} - \frac{1}{\lambda - \frac{s^2}{l}} :: \frac{H}{\lambda}g : \frac{s^2}{l\lambda}\frac{H}{\lambda}g = Hg4l\left(\frac{\pi}{L}\right)^2.$$

The time (τ) taken by a particle to traverse $2l$, with this force diminishing as the distance from the centre of the semicircle, is the same as the time required for one oscillation of a pendulum whose length is l, if subject to an influence of gravity equal to this force, and is the same as the time taken by the pulse to travel through $\frac{1}{2}$L. By the law of the pendulum, τ is equal to π multiplied by the square root of quotient of length of pendulum by force of gravity, hence

$$\tau = \pi\sqrt{\frac{l}{\pi^2}\frac{L^2}{Hg4l}} = \tfrac{1}{2}L\sqrt{\frac{1}{Hg}};$$

and the velocity of the pulse per second is $\sqrt{Hg}$.

This supposes Mariotte's law maintained; but the experiments made at the suggestion of Laplace, proved that for rapid compressions the elastic force increases in a higher ratio than the density; and for small increments of density, the correspondent increments of elastic force are very nearly $\frac{4}{3}$ those computed on the hypothesis that Mariotte's law is maintained; that is, as if the repulsive force of a particle supported the weight of $\frac{4}{3}\frac{H}{\lambda}$ particles: hence Newton's theorem for the velocity of sound in elastic fluids is strictly represented by $\sqrt{\frac{4}{3}Hg}$.

The numerical results from this formula, taking Regnault's data, compared with observations made at low temperatures (so that the influence of aqueous vapour should be avoided), show a difference at most of about $\frac{1}{356}$, or as if $\frac{4}{3}$ should be augmented $\frac{1}{118}$th part. Thus no objection to Newton's theory can well be made on the ground of its not according with observation, as no theory can be expected to embrace all the circumstances that may affect the result; *e. g.* the repulsive force may not emanate from the centre of the particles, so that the size of the particles may

influence the result. Also the repulsive action is necessarily assumed to be limited to adjacent particles, not extending through the interstices of these to the particles beyond (for such is the extraordinary and improbable hypothesis required to deduce Mariotte's law from a static repulsive force). This may be supposed subject to modification during vibratory action.

But the hypothesis upon which the mathematical demonstration rests is open to three grounds of objection:—1. It does not take account of the condition of the front of a pulse when the particles from a condition of rest enter into the cycle of motion defined by the theory. 2. The force of repulsion between two adjacent particles required by the theory is extravagantly large. 3. The other physical properties of gases are not deducible from the hypothesis.

To these may be added, that the dynamical theory of heat has suggested another hypothesis which is free from these objections, and which therefore claims a preference according to Newton's first "rule of reasoning in philosophy," viz. "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. To this purpose the philosophers say that Nature does nothing in vain, and *more is in vain when less will serve*; for Nature is pleased with simplicity, and affects not the pomp of superfluous causes."

1. The theory does not take account of the condition of the front of the pulse, or rather of the front of the first of the series of pulses of which a sound consists. This is apparent if we consider that a particle is represented by the theory as at rest at each extremity of its oscillation, and at those points the accelerative force is at its maximum, and is derived from the difference between the lengths of the lineolæ that issue from the particle in front and in rear. The front lineola cannot differ from the mean length so long as the front particle is at rest unaffected by the advancing pulse. The rear lineola is less than the mean length by a certain small amount α. If the front particle were in action in a pulse cycle, the length of the front lineola would be greater than the mean length of a lineola by the same amount α, so that the accelerative force at each extremity of the oscillation of a particle is represented by 2α; and unless it were so, the condition required to sustain the beautiful relation of velocity and propelling force would be wanting. But at the front of the first pulse the lineola does not differ from the mean length, so that the accelerative force is represented by α, and this is only one-half the amount required by the theory to begin the oscillation. In truth, the demonstration only applies to a pulse having similar pulses operating on both sides.

2. The force of repulsion between two adjacent particles re-

quired by the theory is extravagantly large. The recent advances in the theory of heat have, in a measure, compelled us to realize the dynamic value of natural forces. To compute the absolute value of the repulsive force acting between two adjacent molecules of air, we have to consider that it has to support the gravity of the number of molecules in the height of a uniform atmosphere $\left[\frac{H}{\lambda}\right]$; it must therefore exceed the force of gravity of one molecule in this ratio. Now the force of gravity in one second can communicate a velocity of 32 feet per second, so that the force of repulsion between two adjacent molecules of air must be capable in one second of communicating a velocity of $32\,\frac{H}{\lambda}$ feet. The absolute value of λ, the distance between two adjacent molecules of air, we can now with great probability deduce from the phænomena of capillarity (Phil. Mag. vol. xv. p. 1). At the boiling-point of water the number of molecules of steam in a cubic inch is the same as the number of molecules of air in the same volume. At 86° the number of layers of aqueous molecules in a cubic inch is 215 millions (Phil. Mag. vol. xv. p. 11). Hence at ordinary temperatures the distance between two adjacent molecules of air must be about $\frac{1}{17}$th of a millionth of an inch, and the value of $\left(32\,\frac{H}{\lambda}\right)$, the velocity communicable in a second, is 160 thousand times the velocity of light. Can we for a moment believe that such a force has any real existence, that it is other than a mathematical fiction?

3. The other physical properties are not deducible from the hypothesis of a static force of repulsion. The deductive power of Newton's theory is confessedly limited to Mariotte's law and the velocity of sound. Laplace, by the invention of calorific atmospheres, is allowed to have added to these Dalton and Gay-Lussac's theory of expansion; but it is a question whether the reciprocal action between heat-atmospheres and molecules, which he expresses by mathematical symbols, can be realized by the mind. In judging of this, we must not forget the chapter of the *Mécanique Céleste*, in which the author speculates upon what the laws of motion would have been if force had been as a function of the velocity, instead of as the simple velocity*. What is to be expected from a superstructure resting upon such a foundation as this reveals? Nevertheless, granting that

* The author of the article 'Virtual Velocities' in the Penny Cyclopædia has the following remark upon this chapter of the *Mécanique Celeste*:—"We have never met with any one who could give us an intelligible account of the meaning of this investigation."

Mariotte's law, Dalton and Gay-Lussac's law, and the velocity of sound are represented by the statical hypothesis, we have still Dalton and Graham's law of diffusion and diffusive velocity; Gay-Lussac's law of volumes; Dulong and Petit's law of specific heat, extended to the more simple gaseous bodies by Haycraft and the French physicists; the law of latent heat partially discovered by Gay-Lussac and Welter's experiments; also the diminution of temperature in ascending the atmosphere,—all as yet undeduced from any statical theory of elastic fluids. It may be that additions to the mathematical hypotheses of Laplace will be attempted with the view of extending their capacity, as indeed there seems to be no limit to this artificial and barren system of procedure, which is as far removed from the simplicity of nature as the hideous epicycles of Ptolemy.

There is another mode by which pulses may be conceived to be transmitted, which admits of being set forth in a popular way.

Suppose we range a number of ivory balls in a straight line upon a billiard table, and strike the first of the row upon the second, the initial velocity will be carried forward from one ball to the next adjacent, and so will make its appearance in the last —supposing perfect elasticity and no resistance in rolling— undiminished as if the motion of the first ball had continued, and the impulse had been carried by it alone, and not transferred by impact through others. These balls, confined to one line, are supposed to be in motion among themselves, so that those adjacent alternately strike against each other in opposite directions; the end ones being reflected from the cushions, and then back again after striking the next adjacent ball, the *vis viva* in one direction being at all times equal to the *vis viva* in the opposite.

If we now suppose one cushion to move forward with comparatively slow velocity, each time the adjacent ball strikes it it will be reflected with a velocity greater than that with which it impinged. This increment of velocity it transfers to the next ball, and so on; and the velocity with which the impulse is transferred along the line is equal to the common velocity with which the balls move. We may suppose the line extended indefinitely, and the motion of the cushion to be alternately forwards and backwards. While the adjacent ball impinges many times during each advance and retreat—during the former carrying forward a succession of small increments of *vis viva*, during the latter a succession of small decrements of *vis viva*—a pulse is formed, the intensity and duration of which depends on the motion of the cushion, but the velocity of propagation upon the motions of the balls, upon their common velocity.

Now suppose these balls reduced excessively in their dimensions, and to be *perfectly* elastic as well as the cushion, and we shall have obtained an idea of how pulses may be conveyed in a manner quite different from that depending upon a statical repulsive force between adjacent particles. Instead of such stupendous force, we have to substitute molecules simply moving with the velocity of a cannon-ball, and assume the atmosphere to maintain its elasticity by its particles striking against each other with such velocity, which, viewed cosmically, by no means exceeds what is moderate, and even highly probable if heat is molecular motion.

Such a theory of elastic fluids was started by Mr. Herapath so far back as 1821 in the pages of the Annals of Philosophy, and has more recently suggested itself to M. Krönig, as we find noticed by M. Clausius in his memoir "On the Nature of the Motion we call Heat" (Phil. Mag. vol. xiv. p. 108).

The following is an extract from Mr. Herapath's memoir, p. 278, vol. i. Annals of Philosophy, April 1821:—"..... if gases, instead of having their particles endued with repulsive forces, subject to so curious a limitation as Newton proposed, were made up of particles or atoms mutually impinging on one another, and the sides of the vessel containing them, such a constitution of aëriform bodies would not only be more simple than repulsive powers, but, as far as I could perceive, would be consistent with phænomena in other respects, and would admit of an easy application of the theory of heat by intestine motion. Such bodies, I easily saw, possessed several of the properties of gases: for instance, they would expand, and if the particles be vastly small, contract almost indefinitely; their elastic force would increase by an increase of motion or temperature, and diminish by a diminution; they would conceive heat rapidly and conduct it slowly; would generate heat by sudden compression, and destroy it by sudden rarefaction; and any two having ever so small a communication, would quickly and equally intermix."

At p. 341, in paragraph beginning "These impulses," &c., and in those which succeed, ending "have from one another," we have a very clear announcement of the mode by which a static force of pressure is counterbalanced by a dynamic force of elastic impact.

At p. 345, Prop. 8. "The same things remaining, the elasticity of a gas under a variable temperature and compression is proportional to its numeratom (number of atoms in constant volume) and the *square of its temperature* conjointly; or the elasticity varies as the *square of the temperature* directly, and the simple of the space inversely.

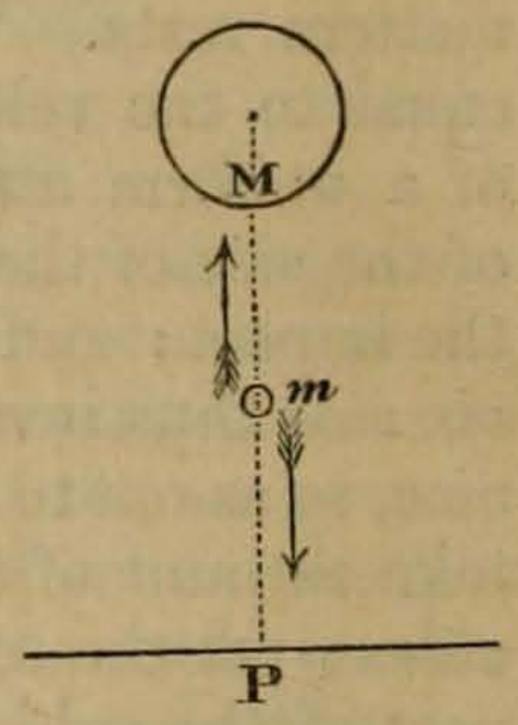
Mr. Herapath unfortunately assumed heat or temperature to be represented by the simple ratio of the velocity instead of the square of the velocity—being in this apparently led astray by the definition of motion generally received—and thus was baffled in his attempts to reconcile his theory with observation. If we make this change in Mr. Herapath's definition of heat or temperature, viz. that it is proportional to the *vis viva* or square velocity of the moving particle, not to the momentum or simple ratio of the velocity, we can without much difficulty deduce, not only the primary laws of elastic fluids, but also the other physical properties of gases enumerated above in the third objection to Newton's hypothesis. In the Archives of the Royal Society for 1845–46, there is a paper "On the Physics of Media that consists of perfectly Elastic Molecules in a state of Motion," which contains the synthetical reasoning upon which the demonstration of these matters rests. The velocity of sound is therein deduced to be equal to the velocity acquired in falling through three-fourths of a uniform atmosphere. This theory does not take account of the size of the molecules. It assumes that no time is lost at the impact; and that if the impacts produce rotatory motion, the *vis viva* thus invested bears a constant ratio to the rectilineal *vis viva*, so as not to require separate consideration. It also does not take account of the probable internal motion of composite molecules; yet the results so closely accord with observation in every part of the subject, as to leave no doubt that Mr. Herapath's idea of the physical constitution of gases approximates closely to the truth. M. Krönig appears to have entered upon the subject in an independent manner, and arrived at the same result; M. Clausius, too, as we learn from his paper "On the Nature of the Motion we call Heat" (Phil. Mag. vol. xiv. p. 108).

The physics of such media is a study that must be ungenial, perhaps repulsive, to mathematicians brought up in the statical school. The fundamental hypothesis does not permit us at once to transfer the subject to the domain of pure mathematics, as Newton's hypothesis converted physical astronomy into a purely mathematical study. The *mode* of action by which certain phænomena make their appearance must be realized at each step in conformity with the conservation of force: the causal relation must be ever present to the mind, or no true progress can be made. On the other hand, the mathematics required is simple, and almost every one of the applications of the theory admits of popular illustration. An attempt of this kind I have given above in reference to the conveyance of sonorous impulses, introducing a dynamic theory of sound. The strict demonstration of the velocity is given in the memoir above referred to. A few further illustrations may here be added to show in brief the capabilities

of the theory; but it is difficult to limit these to one branch, all the physical properties of elastic fluids being so interwoven with each other; and it is an admirable instance of the simplicity of nature, that the cluster of elegant quantitative relations which the physics of gases present, should flow from the constitution assigned, which indeed is the simplest that it is possible to imagine.

The velocity of sound is not affected by the height of the barometer, but it is sensibly influenced by a change of temperature. This latter is to be looked for; since the velocity of the particles of air increases with the temperature, the velocity with which they convey pulses must increase in the same proportion: but it is not so obvious that the height of the barometer or weight of the atmosphere should have no effect either to accelerate or retard.

Let m be an elastic ball traversing the vertical P M backwards and forwards from the sphere, M, to the plane, P, the surfaces of m, M, and P being perfectly elastic. The condition of permanence in the mean distance of M from P requires that the impacts of m upon M should have the effect of changing the velocity of M downwards into the same velocity upwards. Gravity affects M in the interval of time that elapses while m descends from M to P and ascends from P to M; during half this time gravity is employed in destroying the upward motion of M, and during the second half in producing the velocity downwards with which it encounters m on its return,—m and M thus meeting each other, and separating after impact with the same velocity, but with directions reversed.



The relation between the distance MP($=\lambda$), the velocity of $m(=v)$, the weight of $\mathrm{M}\left(=\frac{\mathrm{H}}{\lambda}m\right)$, and of m, is very simple, and enables us to compute the absolute value of v.

The time taken by m to traverse MP is $\frac{\lambda}{v}$ part of a second; and in this time gravity communicates to M the velocity $\frac{\lambda}{v}g$.

From the law of elastic collision, two bodies impinging and reflected back in the direction they came with unaltered velocities, must have their velocities inversely proportioned to their masses, so that

$$\mathrm{M} : m :: v : \frac{vm}{\mathrm{M}} = \frac{\lambda}{\mathrm{H}}v = \frac{\lambda g}{v} \text{ as above.}$$

Thus we have $v^2 = Hg$, or the velocity is that produced by gravity in a body falling through $\frac{1}{2}H$. [Strictly, the square velocity of air-molecules must be six times this, because the above calculus only takes account of the action in one of the six rectangular directions of space.]

To trace the influence of the barometer or weight of the uniform atmosphere, we may suppose the weight of M doubled and λ reduced one-half: this leaves H and v unaltered; so that if the density of air increases as the weight of the uniform atmosphere, the velocity of sound is unaffected by the barometer. If with M constant the density represented by $\frac{1}{\lambda}$ diminishes, H must increase in the same ratio, and thence $v^2 \doteqdot \lambda$, or the volume under constant pressure as the square molecular velocity,—which conforms to Dalton and Gay-Lussac's law, if $v^2 \doteqdot$ temperature from zero of gaseous tension.

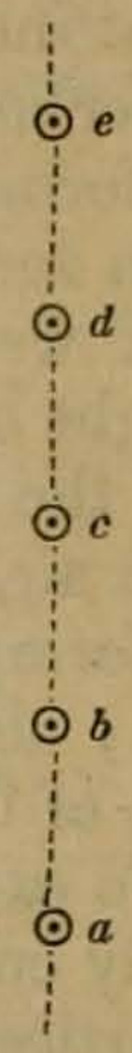
If we view these relations in another elastic fluid, where the weight of the molecule is twice that of air, M being unaltered, and the number of molecules in a unit volume also the same as with air, we have H inversely as m, or one-half the height of a uniform atmosphere of air, and v^2 reduced in the same proportion; also the velocity of sound reduced inversely as the square root of the molecular weight or specific gravity of the gas.

To explain the increase of temperature that arises from suddenly condensing air, we may imagine an elastic ball traversing a vertical between two horizontal plates and striking alternately against them. Those plates being also considered as perfectly elastic, the velocity of the ball will continue uniform without its motion being impaired. If we now suppose the distance between the plates to be gradually diminished by one of them assuming a velocity incomparably less than that of the ball, the ball will, each time it strikes this advancing plane, receive an increment to its velocity, and thus to its *vis viva*.

Let v represent the velocity of the ball, δ the distance between the planes, $\frac{v}{n}$ the velocity of the plane. The number of impacts upon the advancing plate in a unit of time is $\frac{v}{2\delta}$. The velocity after one impact has increased from v to $v + \frac{v}{n}$, and the square velocity from v^2 to $v^2 + \frac{2v^2}{n}$, the increment being $\frac{2}{n}$ of the square velocity; at the same time the decrement of space is $\frac{v}{n}$ (the space moved over by the plate in a unit of time) divided by $\frac{v}{2\delta}$; this

gives $\frac{2\delta}{n}$; and the ratio of this to δ is also $\frac{2}{n}$: thus the increment of *vis viva* is equal to the decrement of distance, irrespective of the velocities and distance, and is equal to the mechanical force exerted by the plate. In an elastic medium, the increment of absolute temperature is equal to one-third the decrement of volume, and the increment of temperature is the equivalent of the force expended in the act of compression. We thus gain a knowledge of the mechanical equivalent of heat, and further deduce the amount of *vis viva* in a gas to be equal to the work performed by its pressure acting through three times its volume.

The diminution of temperature found when ascending the atmosphere may be illustrated by supposing a series of elastic balls, *a*, *b*, *c*, *d*, &c., to be arranged in a vertical, and moving in the vertical so that those adjacent shall alternately strike against each other at the extremity of their up-and-down motion without any transference of *vis viva*, which requires that they should encounter each other with equal velocities, e. g. *b* in its up-motion striking *c*, and in its down-motion striking *a*. Now we have to mark, that, between the upper and lower impact, *b* receives an accession of *vis viva* from the force of gravity which is proportional to the vertical distance traversed; so that comparing the *vis viva* of *b* with any other of the series, such as *z*, we shall find that the higher ball *z* has less *vis viva* than *b*, and the difference is equal to gravity acting through *bz*. Thus in the atmosphere we might expect the decrease of temperature to be uniform if its constitution agrees with this hypothesis.



The strict demonstration represents this to be the case, and that the gradient of temperature is 1° in 319 feet; also that the absolute height of the atmosphere is six times the height of a uniform atmosphere, the density in a stratum as the fifth power of the depth of that stratum below the summit, and the elastic force or height of barometer as the sixth power of that depth.

Here we find that the diminishing temperature, in ascending the atmosphere, is represented as the natural condition of vertical equilibrium; and the question occurs, may not the increasing temperature found in descending through the earth's crust be also its natural condition of vertical equilibrium? This may be cited as one of many instances of the suggestive power of the *vis viva* theory, marking it as specially the natural introduction to the dynamic theory of heat, and as likely to promote a beneficial change in the application of mathematics to molecular physics generally. Upon this account it seems to merit the attention of the educational authorities in the higher depart-

ments of scientific instruction. The subject, however, is so remote from practical application, that there is little hope of any impression being made in such quarters for many years. In the mean time it seems the duty of those who have profited by it, to do what lies in their power to proclaim its merits and acknowledge the value of the idea first struck out by Mr. Herapath, and perhaps saved from oblivion by the Philosophical Magazine of that period.

Edinburgh, Nov. 6, 1858.

LVI. *On Solar Spots, and the Method of determining their Depth. By* Father Angelo Secchi, *Professor of Astronomy, and Director of the Observatory of the Roman College**.

It seems that the attention of astronomers and natural philosophers has been now turned again in a special manner to the study of the chief luminary of our system,—while some time ago it appeared to be abandoned, as though they feared to be overcome by the force of such a mass of light.

From the unwearied attention of men of science much has been already discovered in the sun, and there is a hope of our yet knowing much more concerning it; it is therefore with pleasure that we see this subject followed up with general ardour, particularly after the result of the recent experiments on the solar temperature.

The most interesting question is that of the nature of the solar spots, with respect to which, although a great number of natural philosophers are agreed in admitting them to be gaps in the solar atmosphere, yet from time to time the old hypotheses respecting them are revived, which suppose them to be clouds, or something else indefinable; and it is for that reason that I deem it to be fitting not to recede from what was established by some of my recent observations, which tend to subvert for ever all those ancient hypotheses that took their origin only from the inferior quality of the instruments with which the observations were made, or from want of experience on the part of the observers themselves.

The luminous coating of the sun, to which the name photosphere has been given, is without doubt in a fluid state, as is shown by its prodigious instability; and the spots are nothing else than gaps in this coating, that allow of the sensibly dark body of the sun being seen. In another memoir of mine contained in the Acts of the *Academia di Nuovi Licei*, I have dwelt strongly upon this point, the fundamental idea of which is not

* From the *Astronomische Nachrichten*, No. 1148; communicated by W. G. Lettsom, Esq.