3

71417
com
10
Lib 2

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

CONDUCTED BY

LORD KELVIN, G.C.V.O. D.C.L. LL.D. F.R.S. &c.

GEORGE FRANCIS FITZGERALD, M.A. Sc.D. F.R.S.

AND

WILLIAM FRANCIS, Ph.D. F.L.S. F.R.A.S. F.C.S.

"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." Just. Lips. *Polit.* lib. i. cap. 1. Not.

VOL. XLVIII.—FIFTH SERIES.
JULY—DECEMBER 1899.

LONDON:

TAYLOR AND FRANCIS, RED LION COURT, FLEET STREET.

SOLD BY SIMPKIN, MARSHALL, HAMILTON, KENT, AND CO., LD.; WHITTAKER AND CO.; AND BY ADAM AND CHARLES BLACK;—T. AND T. CLARK, EDINBURGH; SMITH AND SON, GLASGOW;—HODGES, FIGGIS, AND CO., DUBLIN;—PUTNAM, NEW YORK,—VEUVE J. BOYVEAU, PARIS;—AND ASHER AND CO., BERLIN.

74655

COMM

CONTENTS OF VOL. XLVIII.

(FIFTH SERIES).

NUMBER CCXC.—JULY 1899.

Page

Messrs. Charles W. Waidner, Jr., and Francis Mallory on a Comparison of Rowland's Mercury Thermometers with a Callendar-Griffiths' Platinum Thermometer; a Comparison of the Platinum Thermometer with a Tonnelot Thermometer standardized at the Bureau International; and a Reduction of Rowland's Value of the Mechanical Equivalent of Heat to the Paris Nitrogen Scale 1

Mr. S. R. Milner and Prof. A. P. Chattock on the Thermal Conductivity of Water 46

Prof. C. G. Knott on Reflexion and Refraction of Elastic Waves, with Seismological Applications 64

Prof. J. C. Beattie on the Leakage of Electricity from Charged Bodies at Moderate Temperatures 97

Mr. E. F. J. Love on the Joule-Thomson Thermal Effect: its Connexion with the Characteristic Equation, and some of its Thermodynamical Consequences 106

Mr. G. V. MacLean on the Velocity of Electric Waves in Air. (Plate I.) 115

Dr. H. Starke's Reply to the Investigation of Mr. A. A. Campbell Swinton "on the Reflexion of Cathode Rays" .. 132

Rev. O. Fisher on the Residual Effect of a Former Glacial Epoch upon Underground Temperature 134

Dr. E. H. Barton and Prof. W. B. Morton on the Criterion for the Oscillatory Discharge of a Condenser 143

Dr. E. H. Barton and Prof. W. B. Morton: Supplementary Note to Paper "On the Criterion for the Oscillatory Discharge of a Condenser" 148

Lord Rayleigh on the Theory of Anomalous Dispersion 151

Dr. John Henderson on Cadmium Standard Cells 152

Dr. C. Chree on Denudation and Deposition 156

Page

Notices respecting New Books:—
J. P. Wrapson and W. W. Haldane Gee's Mathematical and Physical Tables 158
Proceedings of the Geological Society:—
Mr. S. S. Buckman on Gravel at Moreton-in-the-Marsh (Gloucestershire) 159
Mr. A. E. Salter on the Occurrence of Pebbles of Schorl-rock from the South-west of England in the Drift-deposits of Southern and Eastern England 159
Prof. T. G. Bonney and Miss C. A. Raisin on Varieties of Serpentine and Associated Rocks in Anglesey 160
Mr. H. H. Arnold-Bemrose on a Sill and Faulted Inlier in Tideswell Dale (Derbyshire) 160
Prof. T. G. Bonney and Rev. Edwin Hill on the Relations of the Chalk and Drift in Möen and Rügen 161
Prof. Grenville A. J. Cole on a Critical Junction in the County of Tyrone 161
Mr. C. T. Clough and Dr. W. Pollard on Spinel and Forsterite from the Glenelg Limestone 162
Prof. J. E. Marr on Limestone-Knolls in the Craven District of Yorkshire and elsewhere 163

NUMBER CCXCI.—AUGUST.

Mr. C. S. Whitehead on the Effect of a Solid Conducting Sphere in a Variable Magnetic Field on the Magnetic Induction at a Point outside 165
Messrs. J. P. Kuenen and W. G. Robson on the Mutual Solubility of Liquids.—Vapour-Pressure and Critical Points 180
Mr. Benjamin Davies on a new form of Amperemeter and Voltmeter with a Long Scale 204
Mr. J. Rose-Innes and Prof. Sydney Young on the Thermal Properties of Normal Pentane.—Part II. (Plate II.) 213
Dr. R. A. Lehfeldt on the Vapour-Pressure of Solutions of Volatile Substances 215
Mr. R. W. Wood on the Photography of Sound-Waves by the "Schlieren-Methode." (Plates III. & IV.) 218
Lord Kelvin on the Application of Force within a Limited Space, required to produce Spherical Solitary Waves, or Trains of Periodic Waves, of both Species, Equivoluminal and Irrotational, in an Elastic Solid 227
Lord Kelvin on Magnetism and Molecular Rotation..... 236
Notices respecting New Books:—
H. Weber's Lehrbuch der Algebra, Vol. II. 240

NUMBER CCXCII.—SEPTEMBER.

Page
Signor Quirino Majorana on the Contact Theory. (Parts I. and II.) 241
Signor Quirino Majorana on the Contact Theory (Attraction of Unlike Metals).—Part III. 255
Mr. C. G. Lamb on the Distribution of Magnetic Induction in a Long Iron Bar 262
Prof. J. A. Fleming, and Messrs. A. W. Ashton and H. J. Tomlinson on the Magnetic Hysteresis of Cobalt. (Plates VII. & VIII.) 271
Messrs. John Trowbridge, T. C. and I. C. Howe on the Explosive Effect of Electrical Discharges 279
Mr. J. Rose-Innes on the Ratio of the Specific Heats of Air . 286
K. Tsuruta's Thermodynamic Notes (No. 1).—On the Alleged Sign of "Specific Heat of Saturated Ether Vapour" 288
Mr. A. G. M. Michell on the Elastic Stability of Long Beams under Transverse Forces 298
Notices respecting New Books:—
Mr. O. Heaviside's Electromagnetic Theory, Vol. II. .. 309
M. H. Poincaré's La Théorie de Maxwell et les Oscillations Hertziennes 313
Proceedings of the Geological Society:—
Mr. J. R. Dakyns on the Limestone-Knolls below Thorpe Fell, between Skipton and Grassington in Craven 313
Mr. A. Vaughan Jennings on the Geology of the Davos District 314
Mr. F. R. Cowper Reed on the Lower Palæozoic Bedded Rocks of the Waterford Coast 315
Intelligence and Miscellaneous Articles:—
Notes on the Zeeman Effect, by H. M. Reese 317
A possible Mechanism of Magnetization, by J. J. Taudin Chabot 319

NUMBER CCXCIII.—OCTOBER.

Lord Rayleigh's Investigations in Capillarity 321
Dr. J. H. Vincent on the Photography of Ripples.—Fourth Paper. (Plates V. & VI.) 338
Prof. D. B. Brace on Achromatic Polarization and Differential Double Refraction 345
Prof. R. B. Owens on Thorium Radiation 360
Lord Kelvin on the Application of Force within a Limited Space, required to produce Spherical Solitary Waves, or Trains of Periodic Waves, of both Species, Equivoluminal and Irrotational, in an Elastic Solid 388

Notices respecting New Books :— Page
Prof. G. Lippmann's Unités Électriques absolues 393
Prof. Alex. McAulay's Octonions, a Development of Clifford's Bi-Quaternions 394
H. Arthur Korn's 'Abhandlung über Dynamik, von D'Alembert' 395
Mr. W. R. P. Hobbs's The Arithmetic of Electrical Measurements 396
Proceedings of the Geological Society :—
Messrs. James Park and Frank Rutley on the Rhyolites of the Hauraki Goldfields (New Zealand) 396
Mr. J. B. Hill on the Progressive Metamorphism of some Dalradian Sediments in the Region of Loch Awe. 397
Mr. F. A. Bather on a Quartzite Pebble from Bowdon, Cheshire 398
Mr. C. A. Matley on the Geology of Northern Anglesey. 399
Mr. John Parkinson on an Intrusion of Granite into Diabase at Sorel Point (Northern Jersey) 399
Vallauri Prize 400

NUMBER CCXCIV.—NOVEMBER.

Prof. A. P. Chattock on the Velocity and Mass of the Ions in the Electric Wind in Air 401
Prof. D. B. Brace on a New Spectrophotometer and an Optical Method of Calibration 420
Dr. R. A. Lehfeldt on the Theory of the Electrolytic Solution-Pressure 430
Mr. R. S. Willows on the Variation of the Resistance of certain Amalgams with Temperature 433
Dr. G. Johnstone Stoney: Survey of that part of the range of Nature's Operations which Man is competent to Study 457
Mr. W. C. D. Whetham on the Coagulative Power of Electrolytes 474
Prof. F. J. Jervis-Smith on the Effect of Heat on the Discharge of an Induction-Coil, in which the Primary Circuit is interrupted by an Electrolytic Break 477
Hon. R. J. Strutt on the Dispersion of the Cathode Rays by Magnetic Force 478
Mr. A. Hutchinson on Stokesite—a New Mineral from Cornwall 480
Dr. Walter Stewart on the Disintegration of Platinum and Palladium Wires at High Temperatures 481
Notices respecting New Books :—
Prof. A. Reychler's Outlines of Physical Chemistry 484

NUMBER CCXCV.—DECEMBER.

Page
Mr. Bruce V. Hill on Accidental Double Refraction in Liquids. 485
Mr. Arthur W. Warrington on Hydrometers of Total Immersion 498
Mr. G. A. Schott on the Determination of the Point of Maximum Density of a Liquid by the Hydrometer of Total Immersion 506
Prof. H. L. Callendar on a Practical Thermometric Standard. 519
Prof. J. J. Thomson on the Masses of the Ions in Gases at Low Pressures 547
Prof. C. G. Knott and Prof. T. Gray on Reflexion and Refraction of Elastic Waves 567
Notices respecting New Books:—
Prof. H. Crew's The Elements of Physics 569
Proceedings of the Geological Society:—
Mr. W. Gibson and Dr. Wheelton Hind on a Series of Agglomerates, Ashes, and Tuffs in the Carboniferous Limestone Series of Congleton Edge 570
Mr. E. J. Garwood on the Glacial Phenomena of Spitsbergen 571
Dr. C. Davison on the Cornish Earthquakes of March 29th to April 2nd, 1898 571
Prof. T. T. Groom on the Geological Structure of Portions of the Malvern and Abberley Hills 572

Index 573

XXI. *Photography of Sound-Waves by the "Schlieren-Methode."* By R. W. Wood*.

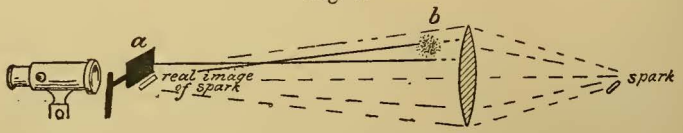
[Plates III. & IV.]

I HAVE always felt that the very beautiful optical method devised in 1867 by Toepler†, for the study of "Schlieren," or striæ, is not as well known outside of Germany as it deserves to be, and trust that the photographs illustrating this paper are sufficient excuse for bringing it before the readers of the 'Philosophical Magazine.' Sound-waves in air were observed by Toepler, but they have never to my knowledge been photographed. When seen subjectively, the wave-fronts, if at all complicated, cannot be very carefully studied, as they are only illuminated for an instant, and appear in rapid succession in different parts of the field of the viewing-telescope.

It occurred to me that if these waves could only be photographed, a most valuable set of lantern-slides could be prepared for illustrating to students the changes in the form of the wave-front, after undergoing reflexion, refraction, &c.

In teaching the subject of optics we are compelled to resort to diagrams when dealing with the wave-front, and the student is apt to get the idea that it is merely a sort of a conception, and that the ray is the real thing. I have found these pictures of sound-waves very valuable in checking this notion, showing as they do the bending of the wave-front by refraction through a prism, the converging wave from a concave reflector, the secondary wavelets formed by transparent and opaque gratings, the diffraction produced by the edge of an obstacle, and other phenomena connected with wave-motion. The apparatus used for photographing the waves is essentially the same as that which Toepler used for viewing them, except that the illuminator has been improved so as to give a much stronger light than the one originally used, which was far too feeble to affect the photographic plate.

Fig. 1.



The general arrangement of the "Schlieren" apparatus is shown in fig. 1. A good-sized achromatic lens of the finest

* Communicated by the Author.

† Wied. *Ann.* cxxxi. p. 33.

quality obtainable, and of rather long focus, is the most important part of the device. I have been using the object-glass of a small telescope figured by the late Alvan Clarke. Its diameter is five inches, and the focal length about six feet. I have no doubt but that a smaller lens could be used for viewing the waves, but one of at least this size is desirable for photographing them.

This lens is mounted in front of a suitable source of light (in the present case an electric spark), which should be at such a distance that its image on the other side of the lens is at a distance of about fifteen feet.

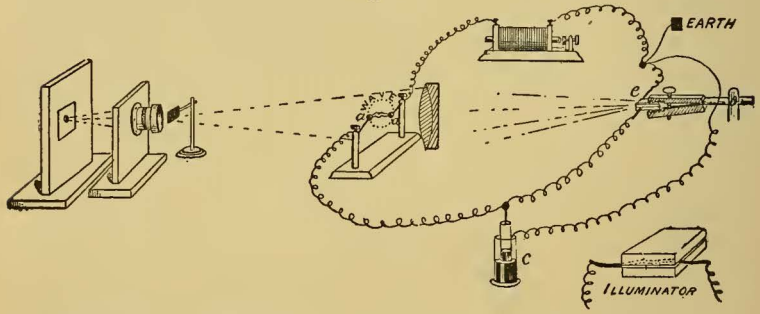
The image of the spark, which we will suppose to be straight, horizontal, and very narrow, is about two-thirds covered with a horizontal diaphragm (*a*), and immediately behind this is placed the viewing-telescope. On looking into the telescope we see the field of the lens uniformly illuminated by the light that passes under the diaphragm, since every part of the image of the spark receives light from the whole lens. If the diaphragm be lowered the field will darken, if it be raised the illumination will be increased. In general it is best to have the diaphragm so adjusted that the lens is quite feebly illuminated, though this is not true for photographic work. Let us now suppose that there is a globular mass of air in front of the lens of slightly greater optical density than the surrounding air (*b*). The rays of light going through the upper portion of this denser mass will be bent down, and will form an image of the spark below the diaphragm, allowing more light to enter the telescope from this particular part of the field ; consequently, on looking into the instrument, we shall see the upper portion of the globular mass of air brighter than the rest of the field. The rays which traverse the under part of "*b*," however, will be bent up on the contrary, forming an image of the spark higher up, and wholly covered by the diaphragm, consequently this part of the field will appear black. It will readily be understood, that with the long path between the lens and the image a very slight change in the optical density of any portion of the medium in front of the lens will be sufficient to raise or depress the image above or below the edge of the diaphragm, and will consequently make itself manifest in the telescope.

The importance of using a lens of first class quality is quite apparent, since variations in the density of the glass of the lens will act in the same way as variations in the density of the medium before it, and produce unequal illumination of the field. It is impossible to find a lens which will give an

absolute even feeble illumination, but a good achromatic telescope-objective is perfect enough for every purpose. A more complete discussion of the operation of the apparatus will be found in Toepler's original paper in the *Annalen*. The sound-waves, which are regions of condensation, and consequent greater optical density, make themselves apparent in the same way as the globular mass of air already referred to. They must be illuminated by a flash of exceedingly short duration, which must occur while the wave is in the field of view.

Toepler showed that this could be done by starting the sound-wave with an electric spark, and illuminating it with the flash of a second spark occurring a moment later, while the wave was still in the field. A diagram of the apparatus used is shown in fig. 2. In front of the lens are two brass balls (*a*, *a*), between which the spark of an induction-coil passes,

Fig. 2.



immediately charging the leyden-jar *c*, which discharges across the gap at *e* an instant later. The capacity of the jar is so regulated that the interval between the two sparks is about one ten-thousandth of a second. The field of the lens is thus illuminated by the flash of the second spark before the sound-wave started by the first spark has gone beyond the edge of the lens.

To secure the proper time-interval between the two sparks it is necessary that the capacity of the jar be quite small. This limits the length and brilliancy of the illuminating-spark, and with the device employed by Toepler I was unable to get enough light to secure photographs of the waves. After some experimenting I found that if the spark of the jar was passed between two thin pieces of magnesium ribbon pressed between two pieces of thick plate-glass, a very marked improvement resulted. With this form of illuminator I found

that five or six times as much light could be obtained as by the old method of passing the spark between two brass balls.

The spark is flattened out into a band, and is kept always in the same plane, the light issuing in a thin sheet from between the plates. By this arrangement we secure a light source of considerable length, great intensity, and bounded by straight edges, the three essentials for securing good results. The glass plates, with the ribbon terminals between them, must be clamped in some sort of a holder and directed so that the thin sheet of light strikes the lens: this can be accomplished by darkening the room, fastening a sheet of paper in front of the lens, and then adjusting the plates so that the paper is illuminated as much as possible. The image formed by the lens will be found to have very sharp straight edges, on one of which the edge of the diaphragm can be set in such a manner as to allow but very little light to pass when the intervening medium is homogeneous; a very slight change, however, in any portion may be sufficient to cause the entire amount of light passing through that portion to pass below the diaphragm and enter the telescope.

This arrangement of the spark-terminals between plates of glass is also convenient for spectroscopic work, as the spark is concentrated into a very narrow bright line, which is fixed in position instead of wandering about, as it is apt to do when passed between uncovered terminals.

For photographing the waves the telescope was removed, and a Zeiss photographic objective put in its place. A vertical board was firmly clamped behind this in such a position that the image of the balls, between which the sound-spark passed, would be in focus on a plate held against it. This arrangement was used instead of a camera, because it was necessary to move the plate rapidly during the exposure, to prevent the image of more than one wave being formed on the same place. It was found that simply holding the plate in the hand against the vertical board and advancing it slowly from left to right, at the same time giving it a rapid up-and-down motion, answered every purpose. It must be borne in mind that the image of the 5-inch lens on the plate was only about 13 mm. in diameter, and that the field surrounding it was dark; consequently a large number of images could be secured on a single plate.

These images showed the waves in different stages of development, for the time-interval between the two sparks varied between rather wide limits. This was really an advantage, for on a single plate it was possible to pick out a series showing the successive changes in the form of the

wave-front produced by reflexion, refraction, &c. Fig. 1 is a contact-print of a small portion of one of the plates, and shows the images exactly as they were obtained on the sensitive film. The other figures are all enlarged, and in general a series of four is given in each case to show the successive positions of the wave-front. Each picture shows the circular field of the telescope-lens with the two brass rods crossing it and supporting in the centre the two balls between which the sound-spark passes. The hot air rising from the spark appears in most of the pictures like a puff of steam above the ball.

In fig. 2 we have a series illustrating the reflexion of a spherical wave from a flat plate, which theory demands shall be also a sphere with its centre of curvature below the plate. The sound-wave appears as a circle of light and shade surrounding the image of the central balls. In No. 3 of the series it will be noticed that the wave-front is gaining a little where it is moving in the warm air above the spark, the front bulging out a little in this place.

Fig. 3.—Here the wave starts in the focus of a concave reflector, and the transformation of the spherical wave into a plane wave is beautifully shown, the reflected portion, or echo, appearing as an approximately straight line moving up out of the reflector. This series is very useful in illustrating the action of a concave mirror on light emanating from its focus.

A very beautiful series of eight pictures, illustrating the action of a concave reflector on a wave starting at some distance outside of its focus, is shown in fig. 4. The diverging spherical wave-front after reflexion is seen to be converging, the wave shrinking to a point in the fourth one of the series, the point being the conjugate focus of the reflector. From now on it diverges, and passes out of the reflector, as is shown in the succeeding pictures.

It is interesting to follow the changes which the wave-front undergoes on entering and leaving the mirror. In No. 1 of the series we have the wave entering the mirror, the portions which have already encountered the steep sides of the reflector and undergone reflexion trailing along behind. In No. 2 the whole wave has undergone reflexion, the central portion is transformed into a converging shell, with which a portion of the two waves reflected from the steep sides appears to unite. As the converging shell shrinks up towards the focus the remnants of the two lateral wave-fronts run in under it and come together just as the main disturbance comes to focus (Nos. 3 and 4).

The wave now begins to diverge again, and moves up out of the reflector, a portion encountering the walls and suffering reflexion. These reflected portions appear to be converging shells, for they are always "concave up" until they get to the top of the mirror, when they are "concave down." In one or two pictures they seem to have come to a focus, although the point is not as sharply defined as the principal focus. It will be seen that light and shade of the wave are interchanged after the passage of the disturbance through the focus. This I am wholly unable to explain. It must be remembered that the light and shade of the wave do not correspond to rarefaction and condensation, but depend merely on the position of the diaphragm which cuts off a portion of the image of the spark. It will be seen that the waves are always light above and dark below, or *vice versa*, according as the diaphragm is above or below the image. If the edge of the diaphragm covers the upper edge of the spark's image, any portion of the medium in front of the lens which has an optical density greater than the surrounding medium (its density, however, shading off gradually to that of the rest of the medium) will appear bright above and dark below, since the rays which traverse the upper portion are bent down and get under the diaphragm's edge, increasing the illumination, while the rays traversing the under portion are bent up, and are consequently intercepted by the diaphragm. A region of less optical density would act in the opposite manner, and appear dark above and light below.

Some of the photographs were taken with the diaphragm above the spark, and some with it below, consequently both arrangements of light and shadow will be found in the pictures.

The reflexion of a wave does not affect the distribution of the light and shade, but passage through a focus always seems to, the appearance being what we should expect if a condensation-wave swept into the focus and a rarefaction-wave swept out, a circumstance that is scarcely possible.

The refraction of sound in a medium denser than air is shown in fig. 5, where we have a rectangular tank with sides made of plane-parallel glass, and covered with a collodion film of soap-bubble thickness made by the method described by Toepler. Ordinary collodion is diluted with about ten parts of ether, poured on a small piece of plate-glass and immediately drained off. As soon as it is quite dry a rectangle is cut with a sharp knife on the film. Toepler's method of removing the film was to place a drop of water on one of the cuts, and allow it to run in by capillarity; but I have had

better success by proceeding in the following manner :— One end of the plate is lowered into a shallow dish of water, and the plate inclined until the water comes up to one of the cuts. By looking at the reflexion of a window in the water it is possible to see whether the film commences to detach itself from the glass. If all goes well it will float off on the surface of the water along the line of the knife-cut, and it should be slowly lowered (one end resting on the bottom of the dish) until the rectangular piece detaches itself and floats freely on the surface. The edges of the tank are well greased and then lowered carefully upon the film, to which they will adhere. The whole must then be lifted from the water in an oblique direction, when the film will be found covering the tank and exhibiting the most beautiful interference-colours. The tank was filled with carbonic acid and placed under the origin of the sound-wave. On striking the collodion-film the wave is partly reflected and partly transmitted, and it will be seen that the reflected component in air has moved farther than the transmitted component in the carbonic acid. The spherical wave-front is transformed into an hyperboloid on entering the denser medium. This is well shown in No. 3 of the series, where the wave in air, moving at higher velocity, has passed out of the field entirely, and there remains only the slower-moving disturbance in the denser gas. In No. 4 the wave is seen returning up through the tank after having struck the bottom.

After several failures I succeeded in constructing a prism with its two refracting faces of this exceedingly thin collodion, which, when filled with carbonic acid, showed the bending of the wave-front, exactly as we figure it in diagrams for light. It was necessary to have the collodion thinner than before, since if we are to photograph the wave after twice traversing the film, we must lose as little energy as possible by reflexion. Fig. 6 shows the refraction in a carbonic-acid prism, the bending being particularly noticeable in No. 4, on which I have, with a pair of dividers, traced out the position which the wave-front would have occupied had it not traversed the prism.

Fig. 7 shows the refraction of the wave when the tank was filled with hydrogen, in which the sound travels faster than in air. The bulging down of the wave-front is very noticeable, though it is not as great as we should expect. This may be due to the fact that the hydrogen is not pure, owing to diffusion, but as a steady stream was flowing from a generator into the tank throughout the experiment, and as no "Schlieren" are to be seen, I do not think that this can fully explain the discrepancy.

In fig. 8 we have the refraction of the wave through a hydrogen prism, the front being bent forward instead of backward, corresponding to the passage of light through a prism of air in glass.

The formation of a sound-shadow and the diffraction of sound is beautifully shown in fig. 9, a small strip of glass having being mounted just below the balls. In No. 1 the wave has just struck the plate, and the reflected portion is starting off. In No. 3 the wave has quite passed the obstacle, and a distinct shadow, or region of no disturbance, is seen close under the plate, while in No. 4 the wave has, by diffraction, curled round the edge and joined together again, obliterating all trace of the shadow.

The principle of Huygens, that any point on a wave-front may be considered as the centre of a new disturbance, is illustrated in fig. 10, two slits having been mounted below the source of the wave. The slits were made by fastening rather broad strips of glass together, with a gap of 3 mm. width between them. (They do not show distinctly as slits in the photographs on account of two narrow strips used to fasten the broader pieces together.) In No. 1 the wave has struck the upper slit, and the small notch which marks the aperture is seen to be the centre of a new disturbance, which is nearly a complete hemisphere. This secondary wave then encounters the second slit, which in its turn becomes the centre of a new disturbance, while that portion of the wave which does not pass through the aperture bounces back and forth between the two plates, as is seen in the last two pictures of the series.

The union of a number of secondary waves from several parallel slits into a new wave, a rough example of a diffraction-grating, is shown in fig. 11, where we have a number of narrow strips of glass mounted so as to form a hemi-cylinder, at the centre of which the wave starts. There are also secondary wavelets reflected from the strips, and these unite into a converging wave which returns to the origin.

Fig. 12 illustrates the reflexion of a wave from a grooved surface, the grooves being rectangular in form. No. 1 of the series is interesting, as the time-interval between the two sparks was exceptionally short, and the wave has scarcely emerged from behind the balls. In the last one of the series two wave-fronts are seen, one arising from the union of the secondary wavelets reflected from the tops of the ridges, the other formed by the disturbances coming from the bottom of the grooves. Reflexion from a corrugated surface is shown

in fig. 13, the secondary wavelets in No. 3 of the series being especially beautiful.

Figs. 12 and 13 are interesting in their connexion with reflexion-gratings.

The formation of a musical note by the reflexion of a single pulse from a flight of steps is shown photographed in fig. 14. This phenomenon is often noticed on a still night when walking on a stone pavement alongside a picket-fence, the sound of each footstep being reflected from the palings as a metallic squeak, which Young has pointed out to be analogous to the power of a diffraction-grating to construct light of a definite wave-length.

In No. 1 of the series the wave has just reached the top step of the flight. In No. 2 it is half-way down, and we begin to see traces of the secondary wavelets, while in No. 3 it has reached the bottom, and we see a beautiful train of little reflected waves, constituting a musical note of very high pitch. The wave-length of the reflected disturbance is of course twice the height of the steps, or very nearly. In No. 4 the original wave is out of the field, and there remains only the train.

The passage of one sound-wave through another is shown in fig. 15. Here we have two spark-gaps arranged in series before the lens, so that two waves are started simultaneously. The wave-fronts are seen to have passed through each other without suffering any disturbance whatever.

The condensation in these waves must be very considerable, since they show on the photographic plate even when the apparatus is not made very sensitive, that is, when a considerable part of the light passes below the diaphragm. Under these circumstances it is impossible to see them with the telescope, or only with the greatest difficulty.

In conclusion a few words regarding the apparatus may be helpful to those wishing to repeat the very beautiful experiments of Toepler. An induction-coil capable of giving a three or four inch spark is about right, while a good-sized test-tube partly filled with mercury, and standing in a cylinder of mercury, will be found most convenient for a leyden-jar. The balls between which the sound-spark passes should be adjusted so as to obtain almost the maximum spark possible, which will in general be rather less than half what the coil will give between its terminals. I find that the best results are obtained when the sparks give off the same crackle found desirable in experiments with Hertz waves. Fresh plates of glass should be put in the illuminator every little while.

It is not at all difficult to get the apparatus to work properly, and I have no doubt but that it could be made to work on quite a small scale with a good photographic objective of rather long focus. The objective of a good-sized spy-glass would also give good results. Toepler was, I believe, of the opinion that he got more uniform results with an influence-machine than with the coil. He certainly found the time-interval between the two sparks to be more constant. This, however, is no object in photographic work, for the wide variation is the very thing that makes the pictures interesting.

Physical Laboratory of the University of Wisconsin,
Madison, May 20th.

XXII. *On the Application of Force within a Limited Space, required to produce Spherical Solitary Waves, or Trains of Periodic Waves, of both Species, Equivoluminal and Irrotational, in an Elastic Solid.* *By* Lord Kelvin, *G.C.V.O., P.R.S.E.*

[Concluded from the May Number.]

§ 21. THE strictly equivoluminal motion thus represented in equations (58) consists of outward-travelling waves, having direction of vibration in meridional planes, and very approximately * perpendicular to the radial direction, and amplitude of vibration equal to $\frac{3}{2}h\frac{q}{r}\sin\theta$, where θ denotes the angle between r and the axis. The gradual change from the simple motion $\xi = h\sin\omega t$ at the surface of the rigid globe, through the elastic solid at distances moderate in comparison with q, out to the greater distances where the motion is very approximately the pure wave-motion represented by (58), is a very interesting subject for detailed investigation and illustration. The formulas expressing it are found by putting $v=\infty$ in (45), and using this equation to determine $F_2(t)$ in terms of $F_1(t)$; then using (47) to determine $F_1(t)$ in terms of $\mathfrak{F}(t)$ given by (51); and then using (55) and (56), for which $v=\infty$ makes $t_2=t$, to determine ξ, η, ζ. They are as follows:—

$$\left.\begin{aligned}\xi = \mathrm{B}(r,t)x^2 - \left(\frac{q}{r}\right)^3\frac{h}{2}\left[\left(1-\frac{3u^2}{q^2\omega^2}\right)\sin\omega t + \frac{3u^2}{q^2\omega^2}\sin\omega t_1 - \frac{3u}{q\omega}\cos\omega t\right] \\ -\left(\frac{q}{r}\right)^2\frac{3h}{2}\frac{u}{q\omega}\cos\omega t_1 + \frac{q}{r}\frac{3h}{2}\sin\omega t_1 \\ \eta = \mathrm{B}(r,t)xy\,; \qquad \zeta = \mathrm{B}(r,t)xz\end{aligned}\right\} \quad (59)$$

* Rigorously so, if the wave-length and q are each infinitely small in comparison with r.