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MAX PLANCK INSTITUTE
FOR THE HISTORY OF SCIENCE

MATHEMATICAL
DISCOURSES
CONCERNING
Two New Sciences
RELATING TO
Mechanicks and Local Motion,
IN
FOUR DIALOGUES.

I. Of the Reſiſtance of Solids againſt Fraction.

II. Of the Cauſe of their Coherence.

III. Of Local Motion, *viz.* Equable, and naturally Accelerate.

IV. Of Violent Motion, or of PROJECTS.

By GALILEO GALILEI,
Chief Philoſopher *and* Mathematician *to the Grand Duke of* TUSCANY.

With an APPENDIX concerning the Center of Gravity of SOLID BODIES.

Done into *Engliſh* from the *Italian*,
By THO. WESTON, *late Maſter, and now publiſh'd by* JOHN WESTON, *preſent Maſter, of the Academy at* Greenwich.

LONDON
Printed for J. HOOKE, at the *Flower-de-Luce*, over-againſt St. *Dunſtan's* Church in *Fleet-ſtreet*. M. DCC. XXX.

To the HONOURABLE

Col. *JOHN ARMSTRONG*,

Surveyor-General

OF

HIS MAJESTY's ORDNANCE.

SIR,

IT was the laudable Cuſtom of Authors, in ancient Times, to inſcribe their Works to truly Illuſtrious and Worthy Men; ſometimes to Men eminent for their univerſal Benevolence and publick Merit to Mankind, and ſometimes to ſuch as were eminent for their Knowledge in the Author's Subject. The frequent Services you have been, and are conſtantly call'd to by your PRINCE for your Country's Good,

denote you of the *former* Claſs; and your Experimental, as well as Contemplative Knowledge in GALILEO's Doctrine, ranks you with the *latter*.

On theſe Accounts, *Sir*, together with a View of fulfilling my late Brother's Intentions, I preſume (as, I know, he deſign'd) to publiſh this following Tranſlation under your Name. Thus am I faithful to the Deceaſed, in rendring you all juſt Acknowledgments for your Approbation of his Undertaking; and am favour'd with an Opportunity of profeſſing my being, with Sincerity and all due Reſpect,

SIR,

Your moſt Oblig'd,

and moſt Obedient

Humble Servant,

J. Weſton.

THE

TRANSLATOR's PREFACE.

THESE Select Pieces *of our Excellent Author were, many Years ago, publiſh'd in an* Engliſh *Dreſs by Mr.* Salusbury, *in the ſecond Tome of his Mathematical Collections and Tranſlations; which is now become very ſcarce: Whereupon a new Tranſlation of them being much deſired and promoted; the Countenance I (however unequal to the Attempt) have receiv'd from the Judicious, has been too prevalent for me to decline it.*

In Conſequence whereof I now make publick a Second Tranſlation (done from the Elzevir *Edition of the* Italian*); and, as it is meerly ſuch, I conceive there need nothing more be added on my Part: But whether it be (as I preſume my Friends expect it ſhould be) neater, and more agreeable to the preſent Taſte, and leſs obſcure than the former, I willingly ſubmit to the Deciſion of the candid Reader.*

As

As to our Author, his univerſally eſtabliſh'd Fame almoſt wholly anticipates my ſaying any thing of him, or his Works. " *It is confeſs'd by all,* [ſays the ingenious " Dr. *Pemberton*] *That* GALILEO *greatly improved* " *Philoſophy, by ſhewing, that the Power in Bodies,* " *which we call* Gravity, *occaſions them to move down-* " *wards with a Velocity equably accelerated; and that* " *when any Body is thrown forwards, the ſame Power* " *obliges it to deſcribe in its Motion that Line, which is* " *called by Geometers a* Parabola. [This is the grand " Fundamental in the Art of Gunnery.] *When a Body* " *falls perpendicularly, it is known how long Time it* " *takes in deſcending from any Height whatever: And* " *if it be thrown forwards, we know the real Path,* " *which it deſcribes; we can determine in what Dire-* " *ction, and with what Degree of Swiftneſs, it muſt be* " *projected, in order to its ſtriking againſt any Object* " *deſired; and we can alſo aſcertain the very Force,* " *wherewith it will ſtrike. Sir* ISAAC NEWTON " [continues the Doctor] *has farther taught, That this* " *Power of Gravitation extends up to the Moon, and* " *cauſes that Planet to gravitate towards the Earth:* " *He has prov'd likewiſe, that all the Planets gravitate* " *towards the Sun, and towards one another; and that* " *their reſpective Motions follow from this Gravitation.* " *And as* GALILEO *has ſhewn more fully than was* " *known before, what Effects were produced in the Mo-* " *tion of Bodies by their Gravitation towards the Earth;* " *ſo Sir* ISAAC NEWTON, *by this his Invention, has* " *much advanced our Knowledge in the Celeſtial Mo-* " *tions.*

For

For an Account of our Author's Diſcoveries in the Heavens, and Improvements in Aſtronomy, by Help of the Teleſcope, who was the firſt that fitted and applied that Inſtrument to Celeſtial Obſervations, I refer the Reader to the following Engliſh *Verſion of the Preface to the Original in* Italian; *and ſhall here only remind him of the known hard Fate this great Man, from his ſuperior Knowledge in Aſtronomical Matters, met with, in that he was, by a Conclave of bigotted Cardinals, condemned for Hereſy, for maintaining the Doctrine of the Motion of the Earth, and the Stability of the Sun; it being repugnant (as they ignorantly and maliciouſly adjudged) to the Authorities of ſacred Scripture. A very remarkable Effect of Popiſh Infallibility!*

The only Objection to our Author, as I have heard of, is, that he is too prolix: Nil rectum putat niſi quod ipſe facit, *may be juſtly applied to ſuch Objector: All know, that the Advantage, which accrues from the Dialogue Way of Writing, makes more than ſufficient Amends for the Prolixity inevitably attending it. Moreover, our Author's Digreſſions and Reflections are neither trifling nor abſurd, but ſuch as are curious, pleaſant and uſeful; as will evidently appear upon a fair Peruſal.*

T. W.

THE

PREFACE

OF THE

EDITOR of the Original in *Italian.*

SINCE Civil Society is kept up by the mutual Aſſiſtances Men receive from one another; and ſince Arts and Sciences do largely contribute to this End, it has been always cuſtomary to reſpect the Inventors of them, which indeed our wiſe Fore-fathers did to a very great Degree; whoſe Veneration for them did ſo riſe in Proportion to the Excellency and Uſefulneſs of the Invention, that they did not ſcruple to make them equal with the Gods; and with one Conſent by this the greateſt Mark of their Eſteem for ſuch Perſons, whom they look'd upon as the Authors of their Well-being, to immortalize their Name: And certainly ſuch as by their great Penetration have diſcover'd and corrected the Fallacies of many Propoſitions laid down by eminent Perſons, and for many Ages receiv'd for Truth, are no leſs worthy of Praiſe

 than

than the former; nay, this they deſervedly claim, tho' they only detect the Error, and do not diſcover the Truth; which is a Task ſo difficult, that it forced the Prince of Orators upon this Wiſh, *Utinam tam facile poſſem vera reperire, quam falſa convincere!* And I muſt needs ſay, that whatever Praiſe has been merited this Way, is juſtly due to theſe latter Ages, who, by continually ſifting the Works of the ſagacious Ancients, have brought the Arts and Sciences, found out by them, to a greater Degree of Perfection; which Improvement goes on daily, eſpecially in Mathematical Sciences, wherein (not to mention others that have ſucceeded well) I make no Doubt, but that all will readily give the Preference to our *Galileus*, who has not only, with clear Demonſtrations, detected the fallacious Reaſonings, and unjuſt Concluſions, that others had paſs'd current (as abundantly appears by his Writings) but alſo as he was the firſt that diſcover'd and made us acquainted with *Jupiter*'s four Satellites, by Help of the Teleſcope (which Inſtrument, tho' invented in theſe Parts, yet was improv'd by him): To him we owe alſo the true Diſcovery of the Milky Way, of the Spots in the Sun, the Inequality of the Moon's Surface, the Emulous Phaſes of *Venus*, the Ring about *Saturn*, and of the Quality and Diſpoſition of Comets: All which the ancient Aſtronomers and Philoſophers knew nothing of; ſo that it may be truly ſaid, that Aſtronomy has deriv'd a new Luſtre from him: And as the Power and Wiſdom of the great Creator is no where ſo conſpicuous as in the Firmament and heavenly Bodies, ſo it is no ſmall Addition to this great Man's Praiſe, to have been the firſt that has led us into the Knowledge of

of them; and that by Ocular Teſtimony, notwithſtanding their immenſe Diſtance, ſince, according to that known Saying, *One View of the Object is of more Service to the Learner, than Rules a thouſand times repeated*, becauſe Intuitive Knowledge (as one expreſſes it) goes hand in hand with Definition. But his great Parts and Induſtry diſcover themſelves no where ſo much as in this Treatiſe, from which it appears that he was the Inventor of two New Sciences, which he has demonſtrated Geometrically. And what is ſtill more remarkable in this Treatiſe is, that one of theſe Sciences is employ'd about an Eternal Subject, the Principal in Nature, about which all the great Philoſophers have from time to time buſied themſelves, I mean *Local Motion*, which is attended with many ſurprizing Accidents, whereof none has been hitherto in the leaſt minded, much leſs demonſtrated. The other Science, which is alſo demonſtrated from its proper Principles, is employ'd about that Reſiſtance of Solids, whereby they are preſerv'd from being cruſh'd to Pieces by outward Force: Which Knowledge is of very great Uſe, eſpecially in Mechanical Arts and Sciences, as being productive of many Accidents and Propoſitions hitherto taken no Notice of. And indeed in this Treatiſe, and in this only, a firſt rational Entrance is made into theſe two Sciences, thence juſtly call'd New, and a regular Proceſs carried on by a great Number of Propoſitions actually demonſtrated, and a Paſſage laid open to infinite others, as the intelligent Reader will eaſily grant.

A TABLE

A

TABLE

OF THE

CHIEF MATTERS treated on in this Book.

DIALOGUE I.

1. *Of the Reſiſtance of Solids againſt Fraction,* Pag. 1

DIALOGUE II.

2. *Of the Cauſe of the Coherence in Solids,* 159

DIALOGUE III.

3. *Of Local Motion;* 227
   i. e. ——— *Equable,* 228
   ——— *Naturally Accelerate,* 237

DIALOGUE IV.

4. *Of Violent Motion, or of* PROJECTS, 371

5. *An* APPENDIX, *concerning the Center of Gravity of Solids,* 447

6. *A* TABLE *of the moſt Notable Things,* 489

GALILÆUS

HIS

DIALOGUES.

DIALOGUE I.

Concerning the RESISTANCE *of* SOLIDS, *againſt* FRACTION.

SALVIATI, SAGREDO, *and* SIMPLICIUS.

SALV. THE conſtant Employments (Gentlemen) in your famous *Arſenal* of *Venice*, and eſpecially thoſe relating to what we call MECHANICS, ſeem to me to afford, to a ſpeculative *Genius*, a large Field to Philoſophize in, there being a great Number of Artificers conſtantly employ'd in forming Engines and Machines of all Sorts; amongſt whom, as well on account of the Obſervations made by their Predeceſſors, as of thoſe which their own Care and Induſtry helps them to, there muſt certainly be ſome very skilful, and ſuch as are able to give the juſt Reaſons of Things.

SAGR. You are partly in the right, Sir: Being naturally curious, I frequently divert myſelf by viſiting that Place, and talking with the OVERSEERS OR FOREMEN: By converſing with theſe, I own I have often found great Helps in my Enquiries concerning the Reaſon of Effects, which were wonderfully abſtruſe, and, in a manner, incredible; but yet ſometimes I have continued at a great Loſs, and, indeed, have deſpaired of ever underſtanding how that could follow, which, though I could not ſee why it ſhould be ſo, my very Senſes proved to me was true. As to what that old Man, not long ſince, told us, tho' it be, I own, the common Opinion, I can't but think it altogether falſe, as are many other Concluſions among the Unskilful, who often pretend to account for Things they know little or nothing of.

SALV. You are ſpeaking, I believe, of his laſt Aſſertion, when we asked him, Why that *larger* Veſſel which was to be launch'd, was arm'd with ſo much greater *Apparatus* of Stays, Supporters, and other Proviſions, than the ſmaller Sort were? When his Anſwer was, That this was by Way of Defence, leſt ſtriking againſt the Ground, it ſhould, by the Weight of its own vaſt Bulk, break its Back; An Inconvenience (continued he) *leſſer* Veſſels are not ſubject to.

SAGR. I was ſo, and chiefly of this laſt Concluſion of his, which I always took for a vulgar Error, *viz.* That in theſe, and other *Machines*, we muſt not argue from leſſer Things to greater, ſince many Inventions ſucceed in leſſer

leſſer Machines, which, upon Trial afterwards, will by no Means do in larger. But now all Mechanical Reaſonings being founded on *Geometry*, wherein I don't ſee how the Greatneſs or Smallneſs of Circles, Triangles, Cylinders, Cones, or of any other Planes or Solids, can alter the Properties or Paſſions of thoſe Planes or Solids: I cannot ſee, if the greater Machine be made in all its Parts ſimilar to, and in the ſame Proportion with thoſe of the leſſer, which is uſeful and fit for the Purpoſes it was deſigned; I cannot ſee, I ſay, why this larger Machine ſhould not be equally exempt from unlucky and deſtructive Accidents.

SALV. The vulgar Opinion, as to this Affair, is, I own, abſolutely falſe, and its Contrary, *viz. Many Machines may be made more perfectly in large, than in ſmall*, may be advanced with more Truth by half. Thus, for Example, A Clock that ſhews and ſtrikes the Hours, may be more accurately made of ſuch, or ſuch a Size, than of a leſs. With much better Reaſon do others more skilful conclude, who refer the Cauſe of the Event or Effect of ſuch large Machines, not coming up to that which is collected from the pure and abſtracted Demonſtrations of *Geometry*, to the Imperfection of *Matter* which is ſubject to many Changes and Defects. But here I don't know whether I may, without Suſpicion of Arrogance, ſay, That that Recourſe to the Imperfection of Matter which may blemiſh the moſt pure Mathematical Demonſtrations, is not ſufficient to excuſe that Deviation of Machines in the *Concrete*, from what was conceived of them when conſider'd abſtractedly, and in *Idea* only.

Nay, I affirm, That tho' we abſtract all Imperfection from *Matter*, and ſuppoſe it perfect, invariable, and free from all accidental Alteration; yet, only becauſe it is material, it comes to paſs, that the greater Machine, made of the ſame *Matter*, and in the ſame Proportion with the leſſer, ſhall anſwer the leſſer in a moſt exact Symmetry, in all Circumſtances, except in Strength, and Power of reſiſting violent Invaſions; for herein it ſhall not anſwer it, but the greater it is, ſo much, in Proportion, I affirm, it ſhall be the weaker. And now, becauſe I ſuppoſe *Matter* to be unalterable, or always the ſame, it is manifeſt, that concerning it, the moſt ſimple and pure Mathematical Demonſtrations neceſſarily offer themſelves. Therefore, *Sagredo*, fling away the Opinion to which you, and, may be, all others who have ſtudied *Mechanics*, have adhered to, *viz.* That Machines and Compound Engines, made of the ſame Matter, with the moſt accurate Obſervation, of the ſelf-ſame Proportions between their ſame Parts, ought to be equally, or, to expreſs myſelf better, proportionally diſpoſed to reſiſt, and to yield to external Aſſaults and Impulſes: For it may be geometrically demonſtrated, that the greater are always, in Proportion, leſs able to reſiſt than the leſſer; ſo that, in ſhort, there is a Term neceſſarily aſcribed, not only in all artificial Machines and Structures, but alſo in natural Ones, which neither Art nor Nature can paſs, provided the ſame Proportions be always obſerved, with the Identity of Matter.

SAGR. I am perfectly aſtoniſh'd!——However, from what you have ſaid, it ſeems to me neceſſarily to follow, That

That it is impoſſible to make two Machines of the ſame Matter, ſimilar and unequal, and which ſhall be able to reſiſt in the ſame Proportion with each other. And that 'tis alſo impoſſible to find two ſimilar Rods, or Staffs, of the ſame Wood, which ſhall be alike in Strength, but unequal in Size.

SALV. It is ſo, Sir; and that you may be further aſſured that you take my Meaning right, I affirm, That if we take a Staff of Wood, of ſuch Length, and Thickneſs, that being fix'd faſt in a Wall, at right Angles to it, *i. e.* parallel to the *Horizon*, it will juſt ſupport itſelf, *i. e.* ſo that if it was a Hair's Breadth longer, it would break by its own Weight; I ſay, there cannot poſſibly be another ſuch a one: I mean thus, Let, for Inſtance, the Length of the Staff be 100 times the Breadth; then, I ſay, no other Staff can be made of the ſame Matter, unequal to it, but ſimilar, *i. e.* whoſe Length is 100 times its Breadth, that can ſuſtain itſelf preciſely, as the former does, and no more; but all *bigger* than it will break, and thoſe *leſſer*, will be able to ſuſtain ſomething more than their own Weight: And what I ſay of the *State of ſuſtaining itſelf*, may be underſtood of every other State. And thus, if a Beam can ſuſtain a Weight of ten Beams, each equal to itſelf, another ſimilar Beam cannot in like manner ſuſtain a Weight of ten, each equal to itſelf. Now pray, Gentlemen, obſerve how Concluſions that are juſt, although they ſeem never ſo improbable at firſt View, yet, with a little Conſideration, fling off the Veil which obſcured them, and their Demonſtrations appear moſt pleaſant, ſimple, and eaſy. Who does not ſee, that

that a Horſe falling from the Height of 3 or 4 Yards, breaks his Bones, whilſt a Dog falling from the ſame Height, or a Cat from a Height 8 or 10 times as great, gets no Hurt; as neither does a Graſshopper from a Tower, nor an Ant from the Moon? Little Children take no Harm from their Falls, whilſt older People break their Limbs: And as leſſer Animals are, in Proportion, more robuſt and ſtronger than greater, ſo alſo leſſer Plants ſupport themſelves better than greater. I believe you both know, that an Oak 200 Feet high, cannot ſupport its Branches ſo ſpread, as thoſe of another of moderate Height. And that Nature cannot produce a Horſe as big as 20 Horſes, nor a Giant bigger than 10 Men, unleſs this be done by a Miracle, or the Proportion of the Members be much altered, and particularly of the Bones, by enlarging them greatly beyond the Meaſure of common Bones. In like Manner, if we ſuppoſe, that greater and leſſer artificial Machines can be with equal Eaſe made and preſerv'd, we run into a manifeſt Error. As thus, for Inſtance, ſmall Pyramids, Cylinders, and other ſmall Bodies, may be ſafely mov'd, laid along, and reared upright, without any Danger of breaking them, whilſt very large ones break of themſelves, or by every little Accident, and that for no other Reaſon, but on account of their own Weight. And here I can't help telling you of an Accident worth your Notice, as, indeed, are all thoſe Events that occur unexpectedly, eſpecially when the very Means uſed to prevent an Inconvenience, prove the main Cauſe of the future ill Event. There was a very great Pillar of Marble laid along, and two Rollers put under it near its Ends: After ſome Time, a certain Mechanic thought, the better to ſecure

cure this Pillar from breaking in the midſt by its own Weight, to put there a third Roller; this Opinion of his ſeemed generally very juſt, but the following Event prov'd the contrary; for, a few Months after, this Pillar crack'd and broke in two, juſt over the new Roller.

SIMP. A very wonderful and unexpected Accident indeed! eſpecially if it was cauſed by this Roller's being placed under its Middle.

SALV. Without doubt this was the Occaſion of it, and when once you know the Cauſe of this Effect, you'll ceaſe wondering. You muſt know then, upon taking the two Pieces of the Pillar off the Rollers, it was found, that one of theſe Rollers, on which one of the Ends of the Pillar had reſted, was, by Length of Time, grown rotten, and ſunk lower, and that in the Middle continuing ſound and ſtrong, occaſioned one Half of the Pillar to hang in the Air, without any Support at one End; ſo that its own exceſſive Weight occaſioned that which would not have happened if it had reſted only upon the two firſt Rollers; for then, if either End had given Way, the Middle would likewiſe have follow'd. And here now, ſurely, I may aſſert, That in a ſmall Pillar there had been no Danger of this, though it was of the ſame Stone, and of a Length anſwerable to its Thickneſs, in the very ſame Proportion as the Length and Thickneſs of the greater Pillar are to each other.

SAGR. I am now convinced of the *Truth* of the Fact; but don't ſee the *Reaſon* why, in the Augmentation of Matter,

Matter, the Reſiſtance and Strength ſhould not be increas'd at the ſame Rate: And this ſeems the more ſtrange to me, becauſe I ſee, on the contrary, in other Caſes, the Power of reſiſting Fraction increaſes much more than the Bulk of Matter does; Thus, for Example, let two Nails be drove into a Wall, the one twice as thick as the other, *that* ſhall bear a Weight, not only double, but triple, nay quadruple of *this*.

SALV. You may ſay octuple, and not err from the Truth; but this Effect is not contrary to the former, tho', at firſt Appearance, it ſeems ſo difficult.

SAGR. Therefore, *Salviatus*, remove theſe Doubts, and explain to us theſe Difficulties, if it ben't too much Trouble; for I can't but think, that this Matter of *Reſiſtance* will open to us a Field of noble and uſeful Contemplations. And if you pleaſe to let this be the Subject of To-Day's Diſcourſe, you'll much oblige us both.

SALV. With all my Heart; If I can but call to mind what I formerly learnt of our * *Academic*, who has made many Diſcoveries about this Matter; and theſe all built, according to his uſual Manner, on geometrical Demonſtrations, inſomuch that this of his may be very juſtly called a *New Science*. For tho' ſome of his Concluſions have been made by others, and firſt of all, indeed, by *Ariſtotle*, yet were they not deduced from their primary and undoubted Principles, by a fair and undeniable Method of Reaſoning. And now, becauſe, as I have told you, it is my Deſign to convince you by Demonſtration, and

* GALILÆUS.

and not content myſelf with perſuading you with Probabilities only; preſuppoſing that you have already, from others that have treated in a rational Way of *Mechanics*, learn'd enough for our preſent Purpoſe; Before I proceed, it is requiſite that we conſider what Effect that is which is produced in the Fraction, or Breaking of any Beam, or other Solid, whoſe Parts are firmly connected, becauſe this is the firſt Notion whereon the firſt and ſimple *Principle* conſiſts, which ſhould be, in our future Reaſonings, taken for granted.

The better to explain which, let us ſuppoſe a Cylinder or Priſm AB of Wood, or ſome other ſolid and coherent



Matter, to be firmly faſten'd above in A, and to hang perpendicularly; to whoſe other End B, let there be hung the Weight C. Now 'tis manifeſt, that how great ſoever the Tenacity and Coherence of the Parts of the Solid be, provided it be not infinite, it may be overcome by the drawing Weight C, whoſe Gravity, I ſuppoſe, may be

increaſed as much as we pleaſe, and that Solid will, at length, break as a Rope would: And as we know that the Reſiſtance of a Rope is occaſion'd by the Multitude of the Strings or Threads of which the Rope is compos'd; ſo in Wood we find its Fibres or Grain running length-ways, which enable it more powerfully to reſiſt Fraction, than any Rope of the ſame Thickneſs. But in a Cylinder of Stone or Metal, the Coherence of its Parts (which ſeems greater) depends on a Cement different from ſuch Strings, or from what in Wood we call the Grain; and yet by a ſtronger Force theſe alſo are broken.

SIMP. If it be as you ſay, I conceive well enough, that the Strings or Grain of the Wood, which is as long as the Wood itſelf, may make it ſtrong, and able to reſiſt almoſt any Force applied to break it. But how can a Rope compos'd of Hempen Threads, each not above two or three Feet long, be ſpun out to the Length of, for Inſtance, 100 Feet, and keep the ſame Strength? I would likewiſe know your Opinion concerning the Agglutination or Connection of the Parts of Metals, Stones, or any other Matter wanting thoſe Threads or Ligatures; which, nevertheleſs, if I am not miſtaken, are more tenacious than ſuch as have them.

SALV. To ſolve theſe Difficulties you ſtart, we muſt digreſs to new Speculations, and theſe but of little Uſe to our main Deſign.

SAGR. But if ſuch Digreſſions may lead us to the Knowledge of new Truths, where can the Diſadvantage be,

be, ſince we are not tied down to a ſtrict and conciſe Method, but talk to pleaſe ourſelves, if we do digreſs ſometimes, eſpecially when we are in Danger of loſing ſome Knowledge, which we have a preſent Opportunity of enquiring into, and which omitted, another, perhaps, may never offer itſelf? Beſides, who knows but by theſe Digreſſions we may gain ſuch Hints as are more valuable than the firſt-intended Concluſions? Therefore I intreat you, that you will ſatisfy *Simplicius*, and myſelf, no leſs deſirous than he, to underſtand what that Cement is which holds the Parts of theſe Solids ſo tenaciouſly join'd, and which can hardly be diſſolved. The Knowledge of which, muſt be a great Help to us to underſtand how the Parts of theſe Strings or Ligaments, of which ſome Solids are compoſed, cohere.

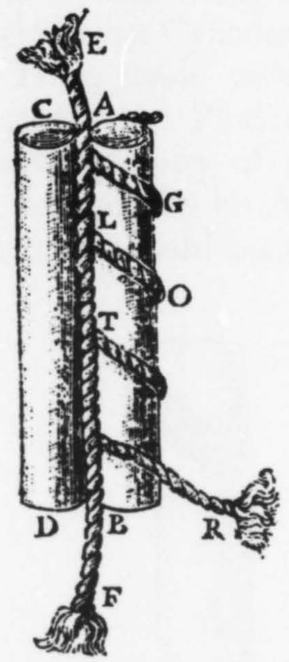
SALV. To pleaſe you, I'll do as you would have me. The firſt Difficulty then, is, How the Threads of a Rope, an hundred Feet long (none of which exceed two or three Feet in Length) can be ſo cloſely connected, as to require ſo great a Force to break them?

Now tell me, *Simplicius*, can you hold one End of one of theſe Hempen Threads between your Fingers ſo faſt, that I, pulling by the other End ſhould break it ſooner than get it from you? Surely you can: When, then, theſe Threads are not only at the Ends, but are all over graſped, or held very faſt, is it not manifeſtly more difficult to pluck the ſame from him that holds 'em, than to break them? Now the very ſame Thing is done in twiſting the Rope, *i. e.* the Threads, by theſe Means,

are ſo ſtrained, and bound ſo faſt one within another, that pulling the Rope with great Force, its Threads break aſunder, but are not parted from one another, or pull'd from their Hold, *i. e.* untwiſted, or drawn out to their firſt Length, or *that* they were of before they were twiſted together; as is plainly ſeen by viewing the Ends of the ſaid Threads in the broken Place, that are very ſhort, which, if the Rope had not been divided by breaking the Threads, but by ſeparating them from one another, they would not be. And now obſerve how one Truth draws on another: That Thread which, held faſt between the Finger and Thumb, did not give Way to, or follow him, who endeavour'd, with all his Might, to draw it from between them, reſiſteth, becauſe it was ſtaid by a double Compreſſion, ſince the Finger preſſed againſt the Thumb, as much as this againſt that: And, without doubt, could only one of theſe Preſſures be retained, the Thread would reſiſt the other Perſon's Endeavour to pull it away, half as much as it did when it was detained by both Preſſures conjointly.

But now, becauſe by lifting off the Thumb, for Inſtance, its Preſſure can't be taken away, but the Preſſure of the Finger muſt alſo be removed; let's ſee if there be no other Way to be found to preſerve one of theſe Preſſures, the other being moved, that is, to make that Thread, of its own accord, as it were, cling to the Finger, or other Solid, to which it is applied, and that ſo forcibly, that the more one draws the Thread to get it from the Finger, the faſter the Thread preſſes the Finger, and this is done if the Thread be wound round the Finger. For the

the better underſtanding whereof, ſee the following Figure. Let AB and CD be two Cylinders, and between them, let there be extended a Thread, which, for greater Plainneſs, we'll repreſent by the ſmall Cord EF. Now



there's no doubt but theſe two Cylinders being preſs'd hard againſt one another, the Cord EF, pulled by the End F, has a great Force to reſiſt, before it can be drawn from between the two Solids which compreſs it. But if one of theſe Cylinders be removed, though the Cord ſtill touches the other, yet ſhall it not, by ſuch Contact, be hinder'd from ſlipping away; but if at the Top of the Cylinder at A, the Cord be held but lightly, and then wound about the ſame in a ſpiral Manner, as AGLOTR, and if you pull it by the End R, 'tis manifeſt, that it will begin to

preſs

preſs the Cylinder, and if there were more Turnings or Windings about, the harder 'tis pull'd, the cloſer it will adhere to, or preſs the Cylinder. And if, by multiplying the Wreaths, the Contact be made longer, and conſequently more inſeparable, the more difficult it will be to withdraw the Cord, and make it give Way to the Force that pulls it. Now, I ſay, who does not ſee that the ſame Reſiſtance is in the Threads, which, with a thouſand and a thouſand ſuch Twinings, twiſt or ſpin the thick Rope? Nay, the Streſs of ſuch Twiſtings binds ſo tenaciouſly, that very ſtrong Bands are made of a few ſhort Ruſhes, tho' but a little twiſted together.

SAGR. Your Diſcourſe has made me ceaſe wondering at two Effects, the Reaſon whereof, I did not before well underſtand. One was, to ſee a Rope turned two or three times at moſt about the *Axis* of a Crane, adhere ſo firmly to it, as not to ſlip away, tho' it be drawn by the immenſe Force of a vaſt Weight. Moreover, by turning on the Crane, we ſee the ſaid *Axis*, by the ſole Contact of the Rope begirting it, to draw and raiſe up vaſt Stones, while one of little Strength, could hold and ſtay the other End of the ſaid Rope with his Hands. The other Effect I wonder'd at, was that of a ſimple, but clever, Inſtrument, invented by a young Kinſman of mine, by which, and by Help of a Cord, he could let himſelf down from a Window, without hurting or gawling his Hands, as he, not long before, had done to his great Pain. And that you may the better underſtand this, take the following Scheme.

Scheme. About a Cylinder, as AB, about two Inches thick, and eight or ten long, he cut a Notch ſpirally, of one turn and half, and no more, big enough to receive the Cord he deſign'd to uſe, and which he put into the Notch at the End A, and brought out at the other B: Afterwards he inclos'd that Cylinder and Cord in a Wooden, or rather a Tin Tube, made with Claſps or Hinges, to open or ſhut lengthways at Pleaſure. Then having made faſt the Rope above, graſping of the ſaid Tin Caſe with both his Hands, he hung by his Arms, whence, ſuch was the Compreſſion of the Cord between the ambient Tube



and the Cylinder, that, at Pleaſure, by clinching his Hands cloſer, he could ſtop himſelf, and by a little looſening his Hold, he could let himſelf down gently as he pleas'd.

SALV.

SALV. An ingenious Invention truly; and more fully to explain its Nature, methinks I have a Glimpſe of ſome Diſcoveries additional to it. But I will not now make any farther Digreſſion about this Matter, ſince you are ſo deſirous to have my Opinion concerning the Reſiſtance of other Bodies againſt Fraction, whoſe Texture is not of Strings and Threads, as is that of *Ropes*, and moſt Kinds of *Wood*, but the Connexion of whoſe Parts depends on other Cauſes; and which, in my Judgment, may be reduc'd to two Heads: The former of which, is Nature's much-talk'd-of Abhorrence of a *Vacuum*; for the other (the 'forementionʼd one being not ſufficient) we muſt introduce ſome glutinous Matter that may ſtrongly connect the Particles whereof theſe Bodies are compos'd.

And firſt of a *Vacuum*; where I will ſhew, by plain Experiments, what, and how great its Virtue is. And, firſt, we may ſee when we pleaſe, that if two flat Pieces of Marble, Metal, or Glaſs, exquiſitely plain'd and poliſh'd, be laid one upon another, the one may be eaſily mov'd horizontally off the other (a plain Proof there's no glutinous Matter holds them together) but if you endeavour to pull them vertically from one another, you'll find ſo great a Reſiſtance to ſuch Separation, that the upper one will lift the under one up, and carry it, tho' pretty thick and heavy, with it, and hold it up continually. This Experiment, I ſay, moſt evidently proves to us how much Nature abhors to admit a *Vacuum*, though for the ſhorteſt time imaginable, which would be between them before the Concourſe of the Particles of the circumambient Air could take Place, and fill it up. We farther ſee, that if

if theſe two Plates are not exactly poliſh'd, and conſequently, their Surfaces applied, don't touch one another exactly all over; that if you try gently to ſeparate them, there's nothing to hinder you but only the Weight; but with a ſudden Pull the under Plate will riſe with the upper one, and immediately fall down again, following the upper one only for that very ſmall Time, which ſerves for the Expanſion of that ſmall Quantity of Air which was between the two Plates, not every where touching one another, and for the Entrance of the other circumambient Air. Now no doubt but the like Reſiſtance which ſo ſenſibly diſcovers itſelf betwixt the two Plates, is in like manner found between the Parts of any Solid, and, as its concomitant Cauſe, conſtitutes their Connection, at leaſt in Part.

SAGR. Hold a little, pray, and let me offer a particular Conſideration, juſt come into my Mind; 'tis this: We ſee the lower Plate to follow the upper one, and, by a quick Motion, to be together raiſed with it; we thence are aſſured, that (contrary to the Opinion of many Philoſophers, and may be of *Ariſtotle* himſelf) Motion *in Vacuo* would not be inſtantaneous: For ſhould it be ſuch, the two fore-mention'd Plates, without the leaſt Reſiſtance, would be ſeparated, ſince the ſelf-ſame Inſtant of Time would be ſufficient for their Separation, as for the Concourſe of the ambient Air to fill up that *Vacuum*, which might remain between them. From the lower Plate's following the upper one, therefore, we may infer the Motion *in Vacuo* would not be inſtantaneous. And we may farther gather, that between thoſe two Plates there is ſome

Vacuity,

Vacuity, at leaſt for ſome very ſmall time, *i. e.* for ſo long as the Motion of the ambient Air laſts in its Concourſe to repleniſh or fill up the *Vacuum* : For was there no Vacuity there, there would be no Need of the Concourſe or Motion of the ambient Air. We muſt therefore conclude, that a *Vacuum* is admitted by Violence, or contrary to Nature, although, in my Opinion, nothing is contrary to Nature, but what is impoſſible, *i. e.* but what is *Not*.

But here now ariſes another Difficulty, and 'tis this ; Altho' Experience aſſures me of the Truth of the Concluſion, yet I do not underſtand the Cauſe to which ſuch an Effect may be aſcrib'd ; becauſe the Effect of this Separation of the two Planes is prior to the *Vacuum*, which by conſequence ſhould ſucceed it, and becauſe, in my Opinion, the Cauſe ought, if not in Time, yet in Nature, to precede its Effect, and a poſitive Effect ought to have a poſitive Cauſe : I can't conceive how the Cauſe of the Coherence or Coheſion of thoſe two Planes, and of their Repugnance to a Separation (which two Effects are already in act) ſhall be aſſign'd to a *Vacuum*, which, as yet, is not, but ought to follow ; for of Things not in Being, there can be no Operation, according to an infallible Maxim in Philoſophy.

SIMP. But ſince you grant *Ariſtotle* this Axiom, ſure you can't but grant another every whit as infallible, *viz.* That Nature does not attempt Impoſſibilities ; on which Axiom, in my Opinion, depends the Solution of our Doubt : Becauſe therefore a *Vacuum* is of itſelf impoſſible, thence Nature permits not that to be done, in conſequence of

of which a *Vacuum* muſt enſue : and ſuch an Action is the Separation of the Plates.

SAGR. Now (admitting what *Simplicius* alledges to be a ſufficient Solution of my Doubt) in purſuance to the Diſcourſe we begun with, this Repugnance to a *Vacuum* ſeems to me a ſufficient Cement to hold together the Parts of any Solid of Stone or Metal, or of any other Matter more denſe, and conſequently more able to reſiſt a Separation. For if-ſo-be a ſingle Effect hath but one only Cauſe, as I at preſent underſtand and believe, or if many be aſſign'd, they may be reduc'd to only one ; why ſhould not this Reaſon taken from a *Vacuum*, be ſufficient to account for all Reſiſtances ?

SALV. I will not at this time meddle with this Controverſy, *viz.* Whether a *Vacuum*, without the Help of any other Cement, be ſufficient firmly to hold together the ſcarcely ſeparable Parts of firm Bodies. But I affirm, that the Buſineſs of a *Vacuum*, which is of Force, and Concluſive as to the two Plates, is not of itſelf ſufficient firmly to connect the Parts of a ſolid Cylinder of Marble or Metal, which drawn lengthways with great Force and Strength, are at laſt ſeparated and divided. But now, if I have found a way of diſtinguiſhing this already-known Reſiſtance, depending on a *Vacuum*, from every other that may concur with it, to ſtrengthen the Connection, and can demonſtrate to you that it alone is not near ſufficient for ſuch an Effect ; will you not allow me to introduce ſome other ? Help him, *Simplicius*, ſince he is in Doubt what to anſwer.

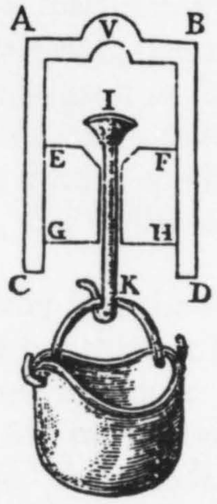
SIMP. *Sagredus's* not giving an Anſwer, without doubt proceeds from ſome other Reaſon, there being no manner of Room for Doubt, concerning ſo clear and neceſſary a Conſequence.

SAGR. You are right, *Simplicius*, for I was thinking if a Million of Gold from *Spain*, be not ſufficient to pay an Army, whether or no it would not be abſolutely neceſſary to provide ſome farther Means to make up the Deficiency. But go on, *Salviatus*; and ſuppoſing I admit your Conſequence, ſhew us how to ſeparate the Operation of a *Vacuum* from all others, and, by eſtimating it, ſhew us its Inſufficiency to produce the Effect we are talking of.

SALV. Well, I will ſhew you a Method of ſeparating the Power of a *Vacuum* from all other, and of meaſuring it: And to do this, we will take ſome continued Matter, whoſe Parts reſiſt a Separation on no other Account, but on that of a Repugnance to a *Vacuum*, ſuch as Water is, as has been at large demonſtrated in a certain Tract by our Academic; So that whenever a Cylinder of Water drawn, reſiſts a Separation of its Parts, it muſt be acknowledg'd, that this can be aſcrib'd to no other Reaſon, but that it may exclude a *Vacuum*. Now to make this Experiment, I have invented a Way, which, by the Help of the following Diagram, may be much better explain'd than it can be by bare Words.

Let

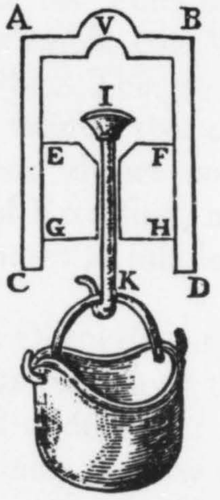
Let C A B D repreſent a Cylinder of Metal, or of Glaſs, made hollow and exactly turn'd: into whoſe Concave put a Cylinder of Wood, fitted moſt exactly to touch every where the other, which let be E G H F; which Cylinder may be mov'd upwards or downwards. This I would have bored in the Middle, for an Iron Rod to paſs thro', made



with a Hook at one End, as K, and let the other End, I, grow thicker, in Form of a Cone or Top; wherefore the upper Part of the Hole made for this Rod to paſs thro' muſt be hollow'd conically, ſo as moſt exactly to receive the conical End I, of the Rod I K, when drawn down by the End K. But now I would not have the Cylinder of Wood E H, when put into the concave Cylinder A D, touch the uppermoſt Superficies of the ſaid concave Cylinder, but to continue two or three Fingers Breadth from it; and the empty Space A F, I would have fill'd with Water, which may be put in by turning the Aperture C D

of

of the Veſſel upwards; then preſs down the moving Cylinder or Stopper EH, keeping the conical Part I of the Iron Rod a little Diſtance from the Hollow that was made in the Stopper to receive it, that the Air may have Room to go out, as it will do upon thruſting down the Stopper by the Hole the Iron Rod goes thro', provided



it be made a little wider than the Rod IK is. The Air being let out, and the Iron Rod pull'd to ſtop cloſe the Hollow of the Stopper by its conick Part I; then turn the Mouth of the Veſſel downwards again, and hang to the Hook K a Bucket, which fill with Sand, or with ſuch a Quantity of ſome other weighty Matter, 'till at length the upper Surface of the Stopper EF be ſeparated from the lower Surface of the Water, to which it was connected by no other Cauſe but the Repugnance to a *Vacuum*: And then if you weigh the Stopper with the Iron Rod, the Bucket, and all that was in it, you will have the Quantity

Quantity of the Force of the *Vacuum*. And if to a Cylinder of Marble, or Cryſtal, of the Thickneſs of the Cylinder of Water, you faſten ſuch a Weight, that, together with the Weight either of that Marble or Cryſtal itſelf, it may equal the Weight of all the fore-mention'd Things: If then the Cylinder breaks, we may moſt certainly affirm, that the ſole Cauſe of the Connection of the Parts of the Marble or Cryſtal, is that fetch'd from a *Vacuum*. But if this Cylinder does not break, but to break it, there's requir'd a Weight four times as great as the former, we may moſt certainly conclude, that the Reſiſtance of a *Vacuum* is one Part of 5; and that there is another Reſiſtance to Fraction 4 times as powerful, ſtill to be accounted for.

SIMP. I can't deny but the Invention is an ingenious one; but yet I can't but think it ſubject to many Difficulties, which make me in doubt about it; for who can aſſure us that the Air can't penetrate between the Glaſs Cylinder ABCD and the wooden one EFGH, tho' ſtopt as cloſe as you can with Tow, or ſome other plyant Matter? and I queſtion whether the Cone (I) will ſtop the Hollow made to receive it ſufficiently cloſe, altho' you wax it, or uſe any other Cement: And again, why may not the Particles of Water diſtend and rarefy themſelves? and why may not the Air, or Exhalations, or ſome other more ſubtile Matter, penetrate thro' the Pores of the Wood, or even Glaſs itſelf?

SALV. Theſe Difficulties *Simplicius* ſtarts, are not improper. And himſelf indeed in ſome meaſure helps us to a Remedy, as to what relates to the Penetration of the

Air

Air thro' the Wood, or between the Wood and the Glaſs. But I obſerve farther, that allowing the Difficulties urg'd to take Place, yet we at the ſame time may gain ſome new Knowledge: Becauſe, if Water be of that Nature as to admit of Extenſion (tho' by Force) as Air does, we ſhall ſee the Stopper to deſcend. And if in the upper Part of the Glaſs we make a little jetting out, as V, in caſe any Air or other more ſubtile and more ſpirituous Matter ſhould penetrate thro' the Pores of the Glaſs or Wood, it would be ſeen to reſemble (the Water giving Way) in that Eminence V: Which Things, ſince they are not diſcover'd, we may reſt ſatisfied that our Experiment is made with due Caution; and thence conclude ſafely that the Water is not capable of Extenſion, nor the Glaſs penetrable by any Matter, tho' never ſo ſubtile.

SAGR. Well, this Diſcourſe has taught me the Reaſon of an Effect that I have for a long time wonder'd at, and never till now could underſtand: 'Tis this; I have obſerved a Ciſtern, wherein, to draw out the Water from it, there was made a Pump, by the Help whereof, I thought, but vainly enough, may be, might be raiſed the ſame or a greater Quantity of Water with leſs Labour, than by the ordinary Buckets; this Pump had its Sucker and Valve on high, ſo that the Water was raiſed by Attraction and not by Impulſe, as it is in thoſe Pumps which work below. This drew the Water in great Plenty, whilſt the Water was at a certain determinate Height in the Ciſtern; below which if the Water ſubſided, the Pump was uſeleſs. The firſt Time I obſerved this, I thought the Engine had been broke; I ſent for a Workman to mend it, who told

told me, that this Defect was owing to nothing but to the Water's being too low, and thence not ſuffering itſelf to be raiſed to ſuch a Height; and he added, that neither Pump nor any other Machine which raiſes Water by Attraction, could ever raiſe the Water a Hair's Breadth more than 18 Cubits; and let the Pump be big or little, that this is the utmoſt limited Height. Now although I knew that a Rope, a wooden Staff, and an iron Rod might be ſo lengthen'd, that lifting them upright, they would break by their own Weight; yet I was ſo ſilly, as never till now to conſider, that the ſame Thing would much more eaſily happen in a String or Thread of Water; and what elſe is that which is attracted within the Pump, but a Cylinder of Water, which, being more and more prolong'd, at length arrives at that Term, beyond which if it be elevated, it, like a Rope, is broke by its own exceſſive Weight?

SALV. 'Tis exactly as you ſay; and ſince the ſame Height of 18 Cubits, is the prefix'd Term to which any Quantity of Water can ſuſtain itſelf, be the Pump broad or narrow, or even no thicker than a Straw; whenever we weigh the Water contain'd in 18 Cubits of Pipe, be it broad or narrow, we ſhall find the Value of the Reſiſtance of a *Vacuum* in Cylinders, of any ſolid Matter whatſoever, of the ſame Thickneſs with the Bore of the propoſed Pipes. Now from what has been ſaid, may be ſhewn how eaſily to determine to what Length we may extend Cylinders, of any Thickneſs, made of Metal, Stone, Wood, or Glaſs, beyond which if they are extended, they cannot ſuſtain themſelves upright, but will break, being too much oppreſs'd with their own Weight:

Take, for Example, a Braſs Wire of any Thickneſs and Length, and fixing one of its Ends on high, hang more and more Weight continually to the other End, 'till at laſt it break; and let the greateſt Weight it can ſuſtain, be, for Example, 50 Pounds; it is manifeſt that 50 Pounds of Braſs more than its own Weight, which we will ſuppoſe an 8th of an Ounce, drawn out into a Wire of the ſame Thickneſs with the former, would be the utmoſt Length this Wire would be able to ſuſtain itſelf at: Then meaſure how long the Wire was which brake, and let it be, for Inſtance, one Cubit, which, ſince it weigh'd the 8th of an Ounce, and was able to ſuſtain itſelf and 50 Pounds over, which make 4800 8ths of an Ounce; we ſay, that all Braſs Wires, be their Thickneſs what it will, will be able to ſuſtain themſelves at the Length of 4801 Cubits, and no more: Wherefore, ſince a Braſs Wire of the Length of 4801 Cubits can ſuſtain itſelf, the Reſiſtance we find depending on a *Vacuum* in reſpect of the other that remains, is as much as is equivalent to the Weight of a Rope of Water 18 Cubits long, and of the ſame Thickneſs with the ſaid Braſs Wire. Now ſince Braſs is 9 times heavier than Water, the Power any Braſs Wire has to reſiſt Fraction depending on a *Vacuum*, is equivalent to the Weight of two Cubits of the ſame Wire. And arguing and working on after this Manner, we may find the utmoſt Length of Threads or Wires made of any ſolid Matter whatſoever, at which they can ſuſtain themſelves, as alſo what Share a *Vacuum* has in their Reſiſtance againſt Fraction.

SAGR. It

SAGR. It remains now that you explain to us in what conſiſts the Remainder of that Reſiſtance againſt Fraction, or of that Tenacity; *i. e.* What that Cement is which connecteth the Parts of a Solid; for I can't conceive what it can be that can't be burnt or conſum'd in a very hot Furnace in a Month's Time, no not in 10 nor 100 Months: For if Gold, Silver, or Glaſs continue ſo long melted or liquified, take them but out of the Furnace, and the Parts, as they cool, reunite, and are conjoin'd as before; ſo that the ſame Difficulty ariſes in relation to this Cement, as did in relation to the Connection of the Parts of the Glaſs, *i. e.* I can't imagine what it ſhould be that ſo cloſely connects its Parts together.

SALV. I myſelf have labour'd under, and entertain'd the ſelf-ſame Difficulties; but ſince the Excluſion of a *Vacuum*, without all doubt, is that which forbids the Separation of the two Plates, as alſo of the two great Pieces of the Pillar of Marble or Braſs, without great Force: Since this is ſo, I ſay, I can't ſee why it ſhould not in like Manner take Place, and be the Cauſe of the Coherence of the leſſer Parts, nay, of the moſt minute Particles of the ſame Matter: And ſince that of one Effect, there is but one only true and moſt potent Cauſe, if I can find no other Cement, why may I not enquire whether that which is deduc'd from a *Vacuum*, which really exiſts, be not ſufficient?

SIMP. But ſince you have already demonſtrated the Reſiſtance of the greater *Vacuum*, in the Separation of the

two great Parts of a Solid, to be but very ſmall in Compariſon of that which holds together its leaſt Particles, why don't you reſt aſſured that this is of a far different Nature from that?

SALV. To this *Sagredus* anſwers; Altho' a Million of Gold ſuffices not to pay a whole Army, that yet each ſingle Soldier may be paid with Monies collected from the general Tax or Subſidy of Shillings and Pence. And who knows for certain whether other the ſmalleſt *Vacuums* may not operate in like Manner amongſt the moſt minute Particles, whereby theſe Parts are connected and held together? I'll tell you what's juſt come in my Head, tho' not as an abſolute Truth, but as a Sort of an indigeſted Conjecture, ſubmitting it to be more exactly weigh'd and conſider'd of: Take what you like of it, and judge of the reſt as you pleaſe. When I have ſometimes conſider'd how Fire, penetrating between the minuteſt Particles of this or that moſt ſolid Metal, at length ſeparates and divides them; and how afterwards, the Fire being taken away, they are again as firmly connected as they were before, without loſing any thing of their Quantity, if in Gold, and but very little in other Metals, tho' they continue a long time melted; I have thought that this might happen from the moſt ſubtile Particles of Fire, penetrating thro' the narrow Pores of the Metals, (thro' which, by Reaſon of their Smallneſs, neither the leaſt Particles of Air, nor of other Fluids can enter) and filling up the moſt minute interpoſed Vacuities, might free the ſmalleſt Particles of the ſame, from that Violence wherewith the ſame Vacuities attract them, and hinder their Separation: And thus,

thus, being able to move freely their Maſs, may become fluid, and ſo continue as long as the Fire continues betwixt them: but this departing, and leaving the former Vacuities, their former Attraction may return, and conſequently the Coheſion of their Parts. And as to what *Simplicius* alledges, it may be thus, I think, anſwer'd; Altho' thoſe Vacuities were moſt minute, and conſequently each of them eaſy to be overcome, that nevertheleſs their innumerable Multitude (if I may ſo ſpeak) multiplies the Reſiſtances innumerably: And we have an evident Proof what, and how great the Power is, that reſults from the Conjunction of an immenſe Number of the weakeſt Moments, when we ſee the Weight of many thouſands of Pounds held by very great Cables, to yield, to be overcome and lifted by the Aſſault of the innumerable Atoms or Particles of Water; which (either driven by the South Wind, or elſe by being diſtended into a very thin Miſt, move to and fro in the Air) inſinuate themſelves between the Threads of the hardeſt twiſted Cables: Nor can the immenſe Force of the Weight which hangs to it, hinder their Entrance; ſo that perforating betwixt the narrow Paſſages, they ſwell the Ropes, and by Conſequence ſhorten them, whereupon the weighty Maſs is forcibly rais'd.

SAGR. There's no Doubt but any Reſiſtance, provided it be not infinite, may be overcome by a Multitude of the minuteſt Powers; inſomuch that even a Number of Ants, may be ſuppoſed able to carry on Shore a Ship Load of Corn; for we ſee by daily Examples, that one Ant can eaſily carry a ſimple Grain: and it's plain that the

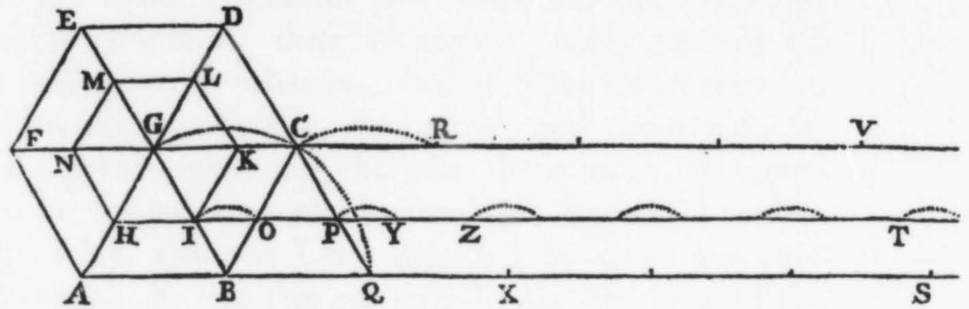
the Grains in the Ship are not infinite, but make up only ſome certain limited Number, than which another Number may be taken 4 or 6 times bigger, and a Number of Ants may be ſuppoſed equal to this laſt Number, and theſe, if they all ſet to work together, will be able to carry both Ship and Lading. It's true, indeed, the Number muſt be great, as, in my Judgment, thoſe Vacuities are, which hold together the ſmalleſt Particles of Metals.

SALV. What if theſe Vacuities ſhould be required to be infinite in Number? But may be this you think impoſſible.

SAGR. Not if the Maſs of Metal was infinite: But otherwiſe——

SALV. Otherwiſe, what? But now we are fallen upon Paradoxes, let's ſee whether we can any way demonſtrate, that infinite Vacuities may poſſibly be in a finite Extenſion; and by endeavouring this, tho' it fail, yet we ſhall, at leaſt, obtain a Solution of a moſt wonderful Problem, propoſed by *Ariſtotle* amongſt thoſe he himſelf calls Admirable; I mean, amongſt his Mechanical Queſtions, the Solution whereof may poſſibly be as plain and concluſive, as that he himſelf gives us, and different from that which the learned Monſig. *di Guevara* with great Subtilty diſcuſſeth. But I muſt firſt give you a Propoſition not touch'd by others, on which depends the Solution of the Queſtion, which afterwards, if I am not miſtaken, will draw after it other new and admirable Notions; for your better underſtanding whereof, take this Scheme: Conceive an

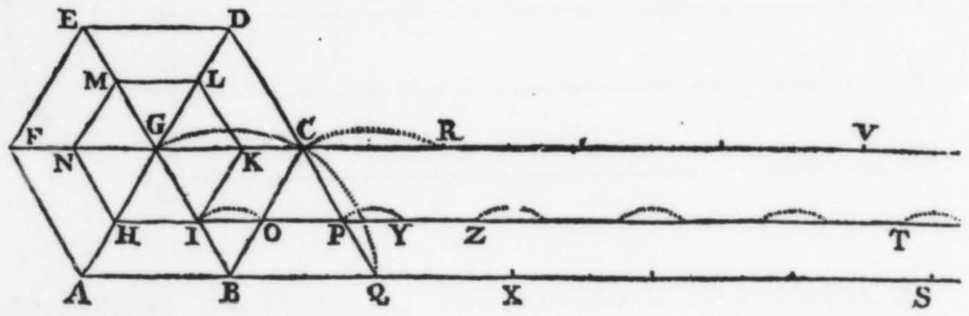
an equilateral and equiangled Polygon, of any Number of Sides, to be deſcrib'd about this Centre G; for Example, an Hexagon, as ABCDEF; ſimilar to which, and concentric with the ſame, deſcribe another leſſer Hexagon, as HIKLMN; and let one Side AB of the greater be indeterminately produc'd towards S, and let the correſpondent Side HI of the leſs be produc'd the ſame Way, by drawing



the Line HT parallel to AS, to which, thro' the Centre, draw the Line GV parallel. This done, ſuppoſe the greater Polygon to turn about upon the Line AS, and to carry with it the other leſſer Polygon; it is manifeſt, that the Point B, the End of the Side AB remaining fix'd in the Beginning of the Revolution, the Angle A muſt be rais'd, and, on the contrary, the Point C will deſcend, deſcribing the Arch CQ; ſo that the Side BC falls upon the Line BQ equal itſelf. But in this Converſion, the Angle (I) of the leſſer Polygon will be lifted up above the Line IT, becauſe the Line IB is oblique to AS; nor will the Point (I) return to the Parallel IT, till the Point C comes to Q: And then the Point (I) falls on O, having deſcrib'd IO without the Line HT; and the Side IK falls

at

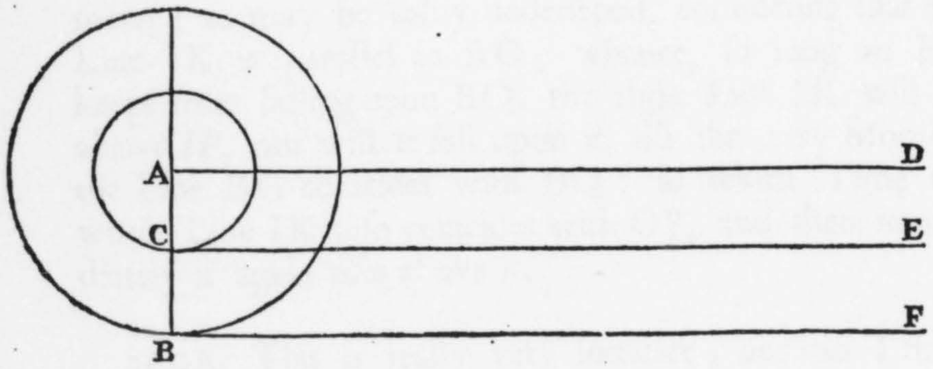
at the ſame time upon OP. But the Center G hath in the interim mov'd out of the Line GV, to which it ſhall not return, till it hath deſcrib'd the Arch GC. This being done, you ſee the Side BC of the greater Polygon lies upon BQ; but the Side IK of the leſſer, upon OP, having skipt all the Line IO without touching it, and the Centre G ſhall be got to C, its whole Courſe having been with-



out the Parallel GV: And at length the whole Figure will be in a Poſition like what it was at firſt. But now, if the Revolution be continu'd, the Side CD of the greater Polygon will fall upon QX, and the Side KL of the leſſer (having skipt over the Arch PY) will fall on YZ; and the Centre proceeding conſtantly without the Line GV, will at length return to it in R, (having made the great Skip CR); ſo that at laſt having finiſh'd an entire Revolution, the greater Polygon will have run over on its Line AS, ſix Lines without any Interpoſition, which together are equal to its Perimeter: And the leſſer Polygon ſhall have likewiſe trac'd ſix Lines, which together are equal to its Perimeter: But theſe Lines are diſcontinu'd by the Interpoſition of five Arches, whoſe Chords are the

Parts of the Parallel HT, not touch'd by the leſſer Polygon in its Revolution. And laſtly, the Centre G hath never touch'd the Parallel GV, except in ſix Points: From all which you may underſtand how the Space run over by the leſſer Polygon, is almoſt equal to that run over by the greater; I mean if you take the Line HT, together with the Spaces ſubtending the five Arches; *i. e.* the Line HT is nearly as long as AS, for it wants of it only the Length of one of the Chords of the 'fore-mention'd skipt Arches. Now you muſt underſtand that what has been ſaid and explain'd concerning theſe Hexagons, holds true in all other Polygons, of what Number of Sides ſoever they be, provided they are ſimilar, concentric, and conjoyn'd; ſo that the greater revolving, the other, how much leſs ſoever it be, be ſuppoſed to revolve with it: You muſt underſtand, I ſay, that the Lines deſcrib'd by them are very nearly equal, if, into that run over by the Perimeter of the leſſer Polygon, you take the Intervals under the little Arches, not touch'd by any Part of that Perimeter. Imagine now, a greater Polygon, of a thouſand Sides, ſuppoſe, to revolve, and conſequently to meaſure out a right Line, equal to its Perimeter; the other leſſer one will, at the ſame Time, paſs over a Line very nearly equal to the former; but a diſcontinu'd one, being compos'd of a thouſand ſmall Parts, equal to its thouſand Sides, together with the interpos'd thouſand void Spaces, for ſuch we may call them, in reſpect to the thouſand little Lines touch'd by the Sides of the Polygon:

What I have hitherto ſaid carries no Difficulty with it, nor admits of any Doubt. But now, if about any Centre,



as ſuppoſe the Point A, two concentric and conjoin'd Circles be deſcrib'd, and from the Points C and B be drawn the Tangents CE and BF, and from the Centre A the Parallel AD, and the greater Circle be imagin'd to revolve upon the Line BF, (which, as are the other two CE and AD, is ſuppos'd equal to its Circumference) pray tell me, when the greater Circle ſhall have perform'd one entire Revolution, what the leſſer Circle, and what the Centre ſhall have done? The Centre ſhall doubtleſs have run over, and touch'd the whole Line AD, and the Circumference of the leſſer Circle, ſhall, with its Touches, have meaſured the whole Line CE, after the ſame Manner as the above-mention'd Polygons did, differing only in this, that the Line HT was not touch'd, in all its Parts, by the Perimeter of the leſſer Polygon, but there were as many Parts left untouch'd, as there were touch'd by the Sides, occaſion'd by the interpos'd skipt Spaces: but here in the Circles,

Circles, the Circumference of the leſſer Circle never departs from the Line CE, ſo as to leave any of its Parts untouch'd; nor can there be any Touch made by the Circumference, which is not made in that right Line. Now the great Queſtion is, how is it poſſible that the leſſer Circle ſhould, without Skips, in one Revolution, run over a Line ſo much longer than its own Circumference?

SAGR. I was conſidering whether it mayn't be ſaid, that as only the Centre of the Circle is drawn along the Line AD, ſince its whole Contact is nothing but a Point, ſo likewiſe each Point of the leſſer Circumference, drawn by the Motion of the greater, might not only touch the Line CE in one ſingle Point, but might move over ſome ſmall Part of the Line CE.

SALV. This can't be, on a double Account; firſt, Becauſe there's no more Reaſon why one of thoſe like Touches (as, for Inſtance, C) ſhould move or glide along any Part of the Line CE, and the others not: and tho' it was ſo, yet ſince thoſe Touches (being Points) are infinite, thoſe Tracts or Glidings would be alſo infinite, and being ſo they would make an infinite Line; but, on the contrary, the Line CE here made, is finite. The ſecond Reaſon is this; Since the greater Circle, in its Revolution, continually changes its Contact, the leſſer Circle muſt needs do the like, there being no other Point but B, from which a right Line can be drawn to the Centre A, to paſs thro' the Point C: ſo that when the greater Circumference changes Contact, the leſſer alſo does ſo too: nor can any Point of the leſs touch more than one Point of its right

Line CE; becauſe in the Converſion of the Polygons, no Point of the Perimeter of the leſſer falls on more than one Point of that Line, which was traced by its Perimeter; as may be eaſily underſtood, conſidering that the Line IK is parallel to BC; whence, ſo long as BC keeps from falling upon BQ, the right Line IK will be above IP, nor will it fall upon it, till the very Moment the Line BC coincides with BQ: at which Time the whole Line IK alſo coincides with OP, and then immediately it again riſes above it.

SAGR. This is really very intricate; nor can I think of any Solution of it; therefore pray explain it to us.

SALV. To do this, I ſhould have Recourſe to the Conſideration of the 'fore-mention'd Polygons, the Effect of which is intelligible, and already known: Thus, as in the Polygons of an hundred thouſand Sides, the Lines run thro' and meaſur'd by the Perimeter of the greater Polygon, that is, by its hundred thouſand Sides directly and continually extended, is equal, or very nearly ſo, to that which is meaſur'd by the hundred thouſand Sides of the leſſer, together with the interjacent hundred thouſand void Spaces: So I would ſay in the Circles, (which are Polygons of an infinite Number of Sides) that the Line run over by the infinite Number of Sides of the greater Circle, contiguous to one another, is equal to the Line run over by the infinite Number of Sides of the leſſer Circle, together with as many interpoſed Vacuities betwixt them. And as the Sides, making up the Circumferences of theſe Circles, are not quantitative, but infinite, ſo alſo the

the interpoſed Vacuities are not quantitative, but infinite; *i. e.* the former Line is made up of infinite Points conjoin'd, the latter of infinite Points diſcontinu'd or diſjoin'd. And here I would have it obſerv'd, how any Line reſolved and divided into quantitative Parts, and conſequently of a finite Number, can't poſſibly be diſpos'd into a greater Extenſion than that which it poſſeſs'd whilſt the Parts were continued and connected, without the Interpoſition of a like Number of void Spaces; but imagining it to be reſolv'd into Parts not quantitative, *i. e.* into its infinite Indiviſibles, we may conceive it immenſely produc'd, not with the Interpoſition of quantitative, but of infinite indiviſible Vacuities. And what we have ſaid of ſimple Lines, may be underſtood as ſaid alſo of Surfaces and of Solids, conſidering them compos'd of infinite non-quantitative Particles. Theſe, indeed, if we ſuppoſe them divided into quantitative Parts, it's moſt certain we cannot diſpoſe them farther, or into larger Spaces than what the Solid occupied before ſuch Diviſion, except there be an Interpoſition of void Spaces; void, I ſay, at leaſt of the Matter of the Solid. But if we imagine an extreme and ultimate Reſolution to be made, that is, the Body to be divided into its infinite non-quantitative firſt-compounding Parts or Principles, we may eaſily conceive ſuch compounding Parts to be extended unto, or ſpread thro' an immenſe Space, without the Interpoſition of quantitative void Spaces, but only of infinite non-quantitative Vacuities. And thus, *v. g.* a very ſmall Ball of Gold may be drawn out to a vaſt Expanſion, without admitting any quantitative void Spaces, provided the Gold be ſuppoſed compos'd of its infinite Indiviſibles.

SIMP. Me-

SIMP. Methinks what you ſay has a direct Tendency to the diſſeminated Vacuities of a *certain antient Philoſopher*.

SALV. But you add not, *who denied a Divine Providence*; as, on ſuch another Occaſion, a certain Adverſary to our Academic ſubjoin'd, tho' very little to the Purpoſe.

SIMP. I ſee very well, and not without Indignation, the Envy of ſuch a malicious Opponent: But I'll ſay no more, not only on the Score of good Manners, but becauſe I know how diſagreeable ſuch Tenets are to the well-temper'd and well-diſpos'd Mind of a Perſon ſo pious, religious, very orthodox, and holy as you, Sir, are. But to return to our Purpoſe: I find many Difficulties to ariſe about your laſt Diſcourſe, and how to get clear of them I know not. And the firſt of them is this: If the Circumferences of two Circles be equal to the two right Lines C E and B F, this continued, and that interrupted by the Interpoſition of infinite void Points, how then can the right Line A D, deſcrib'd by the Centre, which is only a Point, be ſaid to be equal to the ſame, and how can it contain infinite Points? Again, the compoſing a Line of Points, a Diviſible of Indiviſibles, a Quantitative of Non-quantitatives, are all too great Difficulties for me to get over; and the very Neceſſity of admitting a *Vacuum*, ſo throughly confuted by *Ariſtotle*, is no leſs puzzling than the reſt.

SALV. Theſe

SALV. Theſe you have mention'd, and ſome others I could name, are, indeed, Difficulties: But let us remember, that we are amongſt Infinites and Indiviſibles, the former incomprehenſible by our finite Underſtandings, on Account of their Greatneſs, and the latter on Account of their Minuteneſs: but yet we ſee that human Reaſoning will find out ſome Way, tho' it be a round-about one, of conſidering them: wherefore, I will take the Liberty to give you a Conceit of my own, which, if nothing be neceſſarily concluded therefrom, yet, on Account of its Novelty, it will at leaſt excite Admiration in us. But maybe ſo long a Digreſſion may appear to you unſeaſonable and impertinent.

SAGR. Pray let us reap the Benefit and Privilege ariſing from talking freely with our living Friends, eſpecially concerning Things arbitrary, and of our own chuſing, which is vaſtly different from reading dead Authors, who ſtart a thouſand Difficulties, and ſolve not one of them. Make us therefore Partakers of thoſe Conſiderations, which the Courſe of our Conferences ſuggeſt to you, ſince we want not Time, being diſengaged from other urgent Buſineſs, to continue and diſcuſs the Thing you've juſt hinted to us; and pray endeavour in a particular Manner to clear thoſe Doubts which *Simplicius* has offer'd.

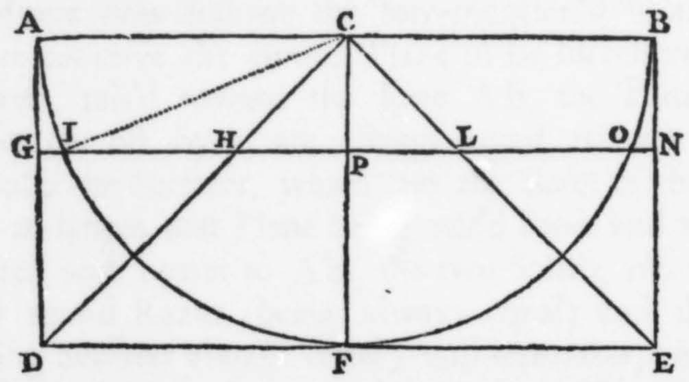
SALV. Be it ſo then, ſince you'll have it: And to begin with the firſt of them, *viz.* how it's poſſible to imagine one ſingle Point equal to a Line: Now as to this, ſince I can do no more at preſent, I will endeavour to explain

one

one Improbability, or at leaſt to ſoften it, by another like it, or by a greater; as ſometimes a Wonder is qualify'd or leſſen'd by a Miracle. And this I will do, by ſhewing you, that if there be two equal Surfaces, and at the ſame time two equal Bodies plac'd upon theſe Surfaces, as their Baſes; and theſe be imagin'd continually to move, ſo that both Surfaces and Bodies be equally diminiſh'd in the ſame Time, and their Remainders be always equal between themſelves: at length, I ſay, theſe Surfaces, as alſo the Solids, will terminate this their perpetual Equality, *viz.* one of the Solids with one of the Surfaces, in a very long Line, and the other Solid with the other Surface, in a ſingle Point, *i. e.* theſe two in a ſingle Point, and the other in Infinity.

SAGR. An admirable Propoſition this! pray explain and demonſtrate it to us.

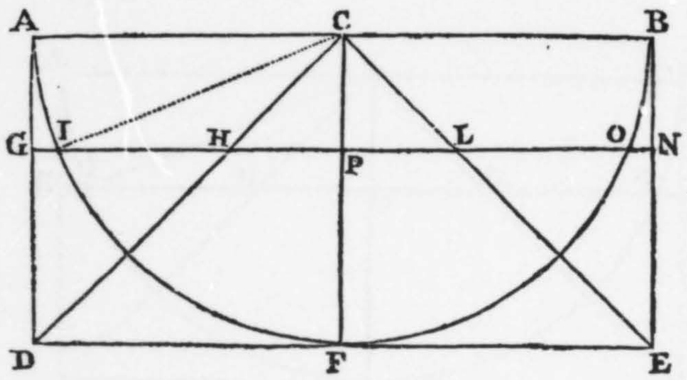
SALV. I muſt draw you a Figure then, becauſe the Demonſtration is purely Geometrical. Conceive a Semicircle



AFB, whoſe Centre is C; about this Semicircle deſcribe a right-angled Parallelogram ADEB, and from the Centre to

to the Points D, E, draw the right Lines CD and CE, as alſo the Radius CF, perpendicular to AB or DE, imagine this Radius CF to be immoveable, and about it this whole Figure to turn round: Then it's manifeſt, that by the Revolution of the Rectangle ADEB, a Cylinder is form'd, by that of the Semicircle AFB a Hemiſphere; and by that of the Triangle CDE a Cone. Theſe preſuppos'd, imagine the Hemiſphere to be taken away, in ſuch manner that there remains the Cone, and what is left of the Cylinder, which, becauſe it is of the Figure of a Diſh, we'll hereafter call *the Diſh*: and this we'll prove in the firſt Place to be equal to the Cone: And then a Plane being drawn parallel to the Circle, which is the Baſe of the Diſh, whoſe Diameter is the Line DE, and whoſe Centre is F; then I will demonſtrate that ſuch a Plane paſſing by, *v. g.* the Line HG, by cutting the Diſh in the Points G, I, O, N, and the Cone in the Points H, L, will cut off a Part of the Cone CHL always equal to a Part of the Diſh, whoſe Profile is repreſented to us by the Δ's GAI and BON; and we will further prove, that the Baſe of the ſame Cone (that is, the Circle whoſe Diameter is HL) is equal to that circular Superficies, which is the Baſe of Part of the Diſh, which is, as I may ſay, a Rim as Broad as GI. (Note here, by the way, what Mathematical Definitions are, *viz.* nought elſe but an Impoſition of Names, or, if you will, Abbreviations of Speech, ordained and introduced to prevent that tedious Prolixity which I and you meet with, merely for not having before agreed to call, *v. g.* this circular Superficies, a *circular Rim*, and that very acute Solid of the Diſh, a *round Razor*.) But now call them by what Name you pleaſe,

'tis all one; our Buſineſs is only to underſtand, that the Plane produc'd to any Diſtance at Pleaſure, ſo that it be always parallel to the Baſe, *i. e.* to the Circle whoſe Diameter is DE, always cuts thoſe two Solids, *i. e.* the Part of the Cone CHL, and the upper Part of the Diſh equal to each other, as alſo the two Surfaces which are the Baſes of theſe Solids; that is, the 'fore-mention'd Rim and the Circle HL are always equal to each other.



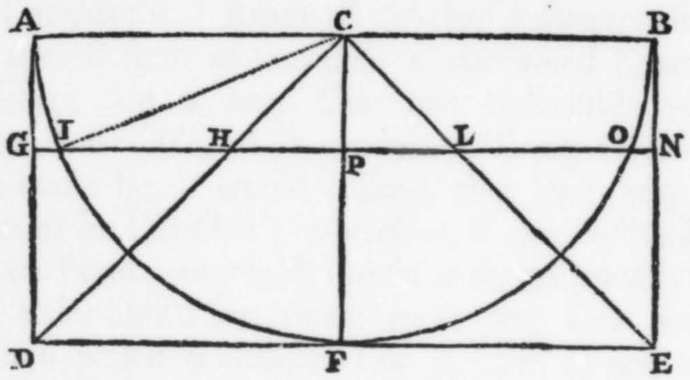
Hence now follows the 'fore-mention'd Wonder, *viz.* If we conceive the cutting Plane to be ſucceſſively, or by degrees, rais'd toward the Line AB, the Parts of thoſe Solids cut off by it, are always equal to each other, as are alſo the Surfaces, which are the Baſes of thoſe Solids, 'till at length that Plane being rais'd more and more, *i. e.* nearer and nearer to AB, the two Solids, *viz.* the Cone and round Razor (being always equal) and their Baſes (being Surfaces always equal) will terminate, *viz.* one Pair of them, (*i. e.* the Razor and its Baſe the Rim) in the Circumference of a Circle; and the other two, (*i. e.* the Cone and the Circle its Baſe) in a ſingle Point. Now, ſince in the

the Diminution of theſe two Solids, they, to the very laſt, keep their Equality to each other, I may, I think, aſſert, that the higheſt and ultimate Terms of ſuch Diminutions are equal, and not one infinitely bigger than the other: And if ſo, then the Circumference of an immenſe Circle may be ſaid to be equal to a ſingle Point; and this that happens in the two Solids, happens alſo in the two Surfaces, the Baſes of theſe Solids, *i. e.* theſe alſo in their common Diminution always preſerving an Equality at the Inſtant of their ultimate Diminution, end, the one (*i. e.* what we before call'd the Rim) in the Circumference of a Circle, and the other (*i. e.* the Baſe of the Cone) in a ſingle Point; And why may not theſe (*viz.* the Circumference of a Circle and a ſingle Point) be call'd equal, ſince they are the laſt Remainders, Tracts, Appearances, or Footſteps left by equal Magnitudes? And pray note farther, that were the Veſſels as large as the Celeſtial Hemiſpheres, their uppermoſt Rims and the Extremities of the contain'd Cones always preſerving an Equality between each other, would, after the very ſame manner, determine the former in Circumferences equal to thoſe of the greateſt Circles of the Celeſtial Orbs, and the latter in ſingle Points; whence methinks the Circumferences of Circles, however unequal among themſelves, may be ſaid to be equal to one another, and each of them equal to a ſimple Point.

SAGR. This Speculation ſeems to me ſo curions, that I would not contradict it, methinks, tho' I could; for I take it for a kind of Sacrilege to deface ſo fine a Structure, or to trace, purſue, or examine it with any pedantick Remarks. But, however, for entire Satisfaction, pray give us

the Demonſtration, which you ſay is Geometrical, of the continual Equality of theſe two Solids, as alſo of the two Surfaces, their Baſes, which I fancy muſt be extremely ſubtile; the Philoſophical Contemplation depending on ſuch a Concluſion, being ſo vaſtly ſubtile and pleaſant.

SALV. The Demonſtration is ſhort and eaſy: Let us reſume the foregoing Figure, in which the ∠ IPC being right, the Square of the Semidiameter IC is



equal to the two Squares of the Sides IP, PC. But the Semidiameter IC is equal to AC, and this to GP; and CP is alſo equal to PH: therefore the Square of the Line GP is equal to the two Squares of IP and PH; and its Quadruple to the Quadruple of theſe; *i. e.* the Square of the Diameter GN is equal to the two Squares of IO and of HL. And becauſe Circles are one to another as the Square of their Diameters, the Circle, whoſe Diameter is GN, will be equal to the two Circles, whoſe Diameters are IO, HL; take away the common Circle, whoſe Diameter is IO, and the Remainder of the Circle, whoſe Diameter is

is GN, or which before we call'd the Rim, will be equal to the Circle, whoſe Diameter is HL: ſo much for the firſt Part. And as for the latter, we'll omit its Demonſtration, as well becauſe the *Archimedes* of our Age, Signeur *Lucas Valerius*, who made uſe of it upon another Occaſion, has given it us in the 12th Propoſition of his 2d Book *De Centro Gravitatis Solidorum*, as becauſe in our Caſe it ſufficeth to have ſeen that the 'foremention'd Surfaces are conſtantly equal, and that they continually diminiſhing equally, at length terminate one in a ſingle Point, and the other in the Circumference of the greateſt Circle; for this was the Conſequence which raiſed our Wonder.

SAGR. The Demonſtration is as ingenious as the Reflection grounded upon it is wonderful. But now pray, if any thing particular occurs to you concerning the other Difficulty ſtarted by *Simplicius*, let's hear it. But I hardly believe this poſſible, it being a Controverſy already ſo canvaſs'd.

SALV. I will give you my particular Sentiments of that too; but firſt let me repeat what I juſt now told you, namely, that Infinity, as alſo Indiviſibility, from their very Nature, are Things incomprehenſible to us: What muſt they be then when join'd together? And certainly if we would compoſe a Line of Indiviſible Points, you muſt make them Infinite; and thus we muſt at the ſame time apprehend both Infinite and Indiviſible. Many Things on this Occaſion have often come into my Mind, ſome of which, and thoſe indeed of the greateſt Moment,

I can't

I can't perhaps recollect on a ſudden; but in the Progreſs of our Diſcourſe it may happen, as you both, but eſpecially *Simplicius*, ſhall find Occaſion of ſtarting Objections and Difficulties, that they may come into my Head, which otherwiſe might poſſibly be quite loſt: Wherefore, with my uſual Freedom, give me leave to produce wild Notions, and my own Conjectures, for ſuch may we juſtly call them, in Compariſon of ſupernatural Doctrines, the only true and certain Determiners of our Controverſies, and unerring Guides in our obſcure and doubtful Paths, or rather Labyrinths. Amongſt the chief Inſtances uſually produced againſt ſuch as compoſe a continued Quantity of Indiviſibles, this is one, That one Indiviſible added to another Indiviſible, can't produce a Thing Diviſible: For if this could be, it would follow, that that very Indiviſible muſt be Diviſible: wherefore, if two Indiviſibles, as, *e. g.* two Points conjoin'd, ſhould make a Quantity that ſhould be a diviſible Line, much more any Line would become ſuch which is compoſed of 3, 5, 7, or any other odd Number of Points: Whence, ſince theſe Lines may be cut into two equal Parts, the middle indiviſible Point of each will become diviſible. As to theſe and other Objections of this kind, we may ſatisfy the Propoſer of them, by telling him, that not only not two Indiviſibles, but not 10, nor 100, nor 1000, can compoſe a diviſible Quantity; but being Infinite they may.

Simp. Here already ariſes a Doubt which I think is not to be reſolv'd; and that is this: Since 'tis plain that one Line is given greater than another, and ſince both contain infinite Points, we muſt ſurely neceſſarily infer, that we have

have found in the ſame Species ſomething greater than Infinite, ſince the Infinity of Points of the greater Line exceeds the Infinity of Points of the leſſer. But now, to aſſign an Infinite greater than an Infinite, is what I can't poſſibly conceive.

Salv. Theſe are ſome of thoſe Difficulties which ariſe from Diſcourſes which our finite Underſtanding makes about Infinites, by aſcribing to them Attributes which we give to Things finite and terminate, which I think moſt improper, becauſe thoſe Attributes of Majority, Minority, and Equality, agree not with Infinities, of which we can't ſay that one is greater than, leſs than, or equal to another. For Proof whereof I have ſomething come into my Head, which (that I may be the better underſtood) I will propoſe by way of Interrogatories to *Simplicius*, who ſtarted the Difficulty. To begin then: I ſuppoſe you know which are ſquare Numbers, and which not?

Simp. I know very well that a ſquare Number is that which ariſes from the Multiplication of any Number into itſelf: thus 4 and 9 are ſquare Numbers, that ariſing from 2, and this from 3, multiplied by themſelves.

Salv. Very well; And you alſo know, that as the Products are call'd Squares, the Factors are call'd Roots: And that the other Numbers, which proceed not from Numbers multiplied into themſelves, are not Squares. Whence taking in all Numbers, both Squares and Not Squares, if I ſhould ſhould ſay, that the Not Squares are more than the Squares, ſhould not I be in the right?

Simp. Moſt

SIMP. Moſt certainly.

SALV. If I go on with you then, and ask you, How many ſquar'd Numbers there are? you may truly anſwer, That there are as many as are their proper Roots, ſince every Square has its own Root, and every Root its own Square, and ſince no Square has more than one Root, nor any Root more than one Square.

SIMP. Very true.

SALV. But now, if I ſhould ask how many Roots there are, you can't deny but there are as many as there are Numbers, ſince there's no Number but what's the Root to ſome Square. And this being granted, we may likewiſe affirm, that there are as many ſquare Numbers, as there are Numbers; for there are as many Squares as there are Roots, and as many Roots as Numbers. And yet in the Beginning of this, we ſaid, there were many more Numbers than Squares, the greater Part of Numbers being not Squares: And tho' the Number of Squares decreaſes in a greater Proportion, as we go on to bigger Numbers, for count to an Hundred you'll find 10 Squares, *viz.* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, which is the ſame as to ſay the 10th Part are Squares; in Ten thouſand only the 100th Part are Squares; in a Million only the 1000th: And yet in an infinite Number, if we can but comprehend it, we may ſay the Squares are as many as all the Numbers taken together.

SAGR. What

SAGR. What muſt be determin'd then in this Caſe?

SALV. I ſee no other way, but by ſaying that all Numbers are infinite; Squares are Infinite, their Roots Infinite, and that the Number of Squares is not leſs than the Number of Numbers, nor this leſs than that: and then by concluding that the Attributes or Terms of Equality, Majority, and Minority, have no Place in Infinites, but are confin'd to terminate Quantities. And therefore when *Simplicius* propoſes many unequal Lines, and asks me how it can be that there are not more Points in the greater than in the leſs? I anſwer, That in them there are neither more nor leſs, nor juſt ſo many, but that the Points in each are infinite: Or certainly, if I had anſwer'd how that in one Line there are as many Points as there are ſquar'd Numbers; in a greater, that there are as many as there are Numbers; and in the leaſt, that there are as many Points as there are cub'd Numbers; might not I, think you, have ſatisfy'd him by aſſigning more to one than to another, and yet to every one an infinite Number? Thus much to the firſt Difficulty.

SAGR. Hold a little, pray, and give me leave to add a Thought of my own, juſt come into my Head; 'tis this; That, granting what you have ſaid, methinks 'tis not only improper to ſay that one Infinite is greater than another Infinite, but alſo that an Infinite is greater than a Finite; for if an infinite Number be greater, *v. g.* than a Million, then it would follow, that, going on from a Million to other Numbers, and ſo to others continually greater, we

at length ſhould attain to Infinity : but now the contrary of this appears, for to how much greater Numbers we proceed, ſo far we recede from Infinite, becauſe by how much the greater the Numbers are taken, ſo much the fewer will the Squares be comprehended in them ; but in an infinite Number there can't be fewer than there are Numbers, as juſt now concluded ; and therefore the approaching to Numbers bigger and bigger, is going back from infinite Number.

SALV. And ſo from your ingenious Diſcourſe we may conclude, that the Attributes of greater, leſs, or equal, have no Place, not only among Infinites, but alſo among Infinites and Finites. I paſs on now to another Conſideration ; and that's this : Since it now is certain that a Line, and every continu'd Quantity, may be divided into Parts always farther diviſible, I don't ſee how we can avoid granting it to be compos'd of infinite Indiviſibles ; becauſe a Diviſion and Subdiviſion, that may be proſecuted perpetually, ſuppoſes that the Parts are infinite, ſince otherwiſe the Subdiviſion would at length be terminated. And ſince the Parts are infinite, we muſt conclude them non-quantitative, ſince infinite Quantitatives make an infinite Extenſion : And thus now we have a continued Quantity compos'd of infinite Indiviſibles.

SIMP. But if we may continually proſecute the Diviſion into quantitative Parts, why are we oblig'd, in that reſpect, to introduce Non-quantitatives ?

SALV. The

SALV. The very Poſſibility of continually proſecuting the Diviſion into quantitative Parts, introduces the Neceſſity of a Compoſition of infinite Non-quantitatives. Wherefore to come cloſer to you, pray tell me directly whether the quantitative Parts of a continu'd Quantity are, in your Opinion, Finite or Infinite ?

SIMP. They are both Infinite and Finite ; Infinite in Power, and Finite in Act. They are Infinite in Power, *i. e.* before Diviſion ; but Finite in Act, *i. e.* after they are divided ; becauſe the Parts are not actually conceiv'd to be in the Whole, 'till it is divided, or at leaſt till its Parts are continually mark'd, for otherwiſe they are ſaid to be in Power.

SALV. So that a Line, *v. g.* 20 Foot long, can't be ſaid to contain actually 20 Lines of one Foot a-piece, 'till it is divided into 20 equal Parts : But before Diviſion, 'tis ſaid to contain theſe Parts only in Power. Be it ſo as you would have it ; and now pray tell me whether this actual Diviſion into ſuch Parts being made, whether, I ſay, that firſt Whole is encreaſed or diminiſhed, or whether it continueth of the ſame Bigneſs.

SIMP. It is neither encreaſed nor diminiſhed.

SALV. So think I too : Therefore the quantitative Part in a continued Quantity, whether in Act or Power, neither augment nor diminiſh its Quantity : But 'tis manifeſt, that the quantitative Parts actually contain'd in their

Whole, if they be infinite, make it an infinite Magnitude; and therefore quantitative Parts, tho' infinite only in Power, can't be contain'd but in an infinite Magnitude: And therefore in a finite Magnitude, infinite quantitative Parts cannot be contain'd either in Act or in Power.

SAGR. How then can it be true, that a continu'd Quantity may be continually divided into Parts ſtill capable of new Diviſions?

SALV. That Diſtinction of Act and Powers ſeems to make that poſſible one Way, which another Way would be impoſſible: But I will endeavour to reconcile theſe Matters by ſome other Method. To the Queſtion then, *viz.* Whether the quantitative Parts in a terminated continued Quantity, are finite or infinite? And to this I anſwer directly, and quite contrary to *Simplicius's* Anſwer, That they are neither finite nor infinite.

SIMP. I never ſhould have thought of ſuch an Anſwer truly, having never ſo much as dreamt that there was any mean Term between Finite and Infinite: So then that Diviſion or Diſtinction which ſuppoſes any thing to be either finite or infinite, is lame and deficient.

SALV. In my Opinion it is; and if we talk of diſcrete Quantities, there ſeems to me to be a third mean Term between finite Ones and infinite, and this is agreeable to every aſſign'd Number. Thus, if in the preſent Caſe, it be ask'd, Whether the Parts of a continued Quantity be finite or infinite, the propereſt Anſwer is, That they are neither

neither finite nor infinite, but ſuch as may be agreeable, or may anſwer to any, or to every aſſign'd Number; that they may do which, 'tis neceſſary, that thoſe Parts be not comprehended within any limited Number, becauſe then they would not anſwer a bigger Number. Again, 'tis required that they be not infinite, becauſe no aſſign'd Number is infinite. And thus, at the Pleaſure of the Demander, in a propos'd Line, we may aſſign it an hundred quantitative Parts, or a thouſand, or a hundred thouſand, or any greater Number of ſuch Parts, provided they are not infinite. I grant therefore to the Philoſophers, that a continu'd Quantity contains as many quantitative Parts, as they pleaſe, and that either in Act or in Power, as they pleaſe: But I add, that in like manner, as in a Line of 10 Yards, there are contain'd 10 Lines of one Yard apiece, and 30 Lines of a Foot apiece, and 360 Lines of an Inch apiece; ſo it contains infinite Points, call it in Act or in Power, as you like beſt: And in this Particular I refer myſelf to your Opinion and Judgment, *Simplicius.*

SIMP. Why, I can't but commend your Diſcourſe; but yet I much fear that your concluding that Points are contain'd in a Line as quantitative Parts, comes up to the Hypotheſis of indiviſible Points. And that you can't ſo eaſily divide a propos'd Line into infinite Points, as thoſe Philoſophers can into 10 Yards, or 30 Feet: Nay, I don't think it poſſible to effect ſuch a Diviſion; wherefore this will be one of thoſe Powers which can never be reduc'd to Act.

SALV.

SALV. Tho' a thing can't be done without Pains and Diligence, or under a long Time, yet this renders it not impoſſible. You can't eaſily, I dare ſay, divide a Line into 1000 Parts; and if you are to divide it into 937, or ſome other great prime Number, you'll ſtill find it more difficult. But now, if I ſhould diſpatch that Diviſion, which you believe impoſſible, in as ſhort a time as another would take to divide it into 40, you'll the eaſier admit it, I hope, in our future Diſcourſe.

SIMP. I can't ſay but I am pleas'd with the Method you talk of; and to your Queſtion I reply, That I ſhould think it more than ſufficiently eaſy, if a Line could be as readily divided into Points, as into a thouſand Parts.

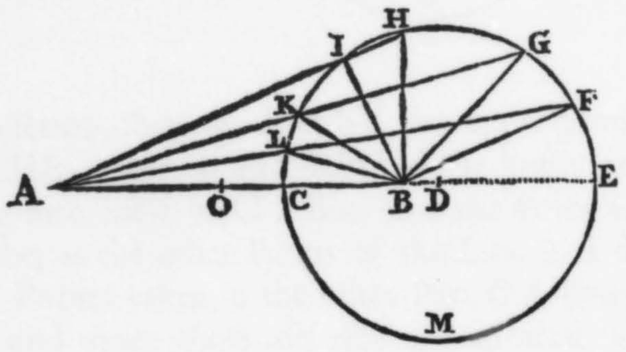
SALV. I'll tell you a Thing that perhaps will ſtartle you, at leaſt whilſt you preſuppoſe that you would, or are able to, reſolve a Line into its Infinites, by the Method others obſerve in dividing it into 40, 60, or 100 Parts; *i. e.* by dividing it firſt into 2, and then into 4, *&c.* For he that by ſuch a Proceſs propoſes to divide this Line into its infinite Points, would be greatly deceiv'd; for by this Progreſs, tho' he goes on to Eternity, he would never arrive at the Diviſion of all its quantitative Parts; and he is ſo far from ever arriving at the wanted Term of Indiviſibility by theſe Means, that, on the contrary, he goes farther and farther from it: And whilſt by continually repeating the Diviſion, and by multiplying the Multitude of the Parts, he thinks he is approaching to Infinity; I, for my Part, think that he recedes farther and farther from it; and my

my Reaſon is this: In the Diſcourſe we had juſt now, we concluded, That in an infinite Number there was of Neceſſity as many Squares, or as many Cubes, as there are Numbers in all, becauſe both thoſe and theſe are as many as their Roots are, and their Roots are as many as there are Numbers in all. After this we ſaw, that the greater the Numbers were taken, the fewer Squares, and ſtill the fewer Cubes, were found in them; and thence 'tis manifeſt, the greater Number you arrive at, the farther you recede from infinite Number: whence it follows, That by going Backwards, (ſince the 'foreſaid Progreſſion moves us farther and farther from the deſired Term) if any Number may be ſaid to be infinite, it is an Unit; and, indeed, in the Unit there are all the Conditions and neceſſary Requiſites of an infinite Number; I mean thus, it contains in it as many Squares, or as many Cubes, as there are Numbers.

SIMP. I don't underſtand this.

SALV. There's no Difficulty at all in it, for the Unit is a Square, a Cube, a ſquar'd Square, and every other Power: Nor is there any Particular whatſoever eſſential to the Square, or to the Cube, which is not found in the Unit: Thus, *v. g.* 'tis a Property of two ſquare Numbers to have between them a mean Proportional: Take any ſquare Number for one Term, and the Unit for the other, and you'll always find a mean Proportional between them. Let two ſquare Numbers be 9 and 4; then between 9 and 1 the mean Proportional is 3; and between 4 and 1 it is 2; and between 9 and 4 it is 6. And again,

again, 'tis a Property of Cubes to have neceſſarily between them 2 mean Proportionals. Suppoſe two Cubes be 8 and 27, then the Means between them are 12 and 18; and between 1 and 8 the Means are 2 and 4; as between 1 and 27 they are 3 and 9. Hence we conclude, that there is no infinite Number but the Unit. Theſe now are ſome of thoſe Wonders which ſurmount our Comprehenſion, and which ſhould teach us how exceedingly they err, who, reaſoning about Infinites, uſe the ſame Attributes as they do about Finites, which, in their Nature, are quite different from each other. Upon which I will not conceal from you a wonderful Accident, juſt come in my Mind, which will explain to you the infinite Difference, nay, the Contrariety of Nature that a terminate Quantity would incur, by paſſing to an infinite. Draw the right Line AB of any Length, and take in it any Point C, which may divide it into two unequal Parts; then, I ſay, if two Lines, keeping the ſame Proportion

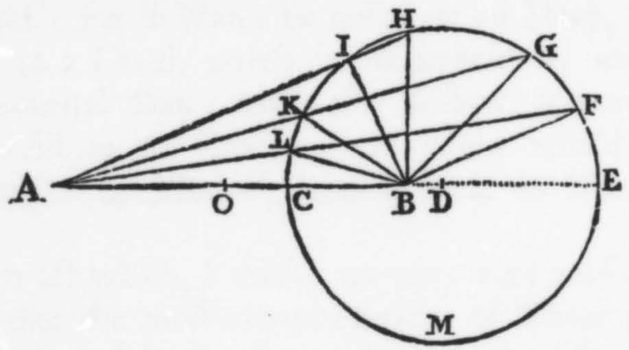


to each other, as the Parts AC and BC have, are drawn from the Extremities A and B, to meet one another, the Points

Points of their Interſection ſhall fall in the Circumference of one and the ſame Circle: As, *e. g.* if from the Points A and B, the right Lines AL and BL are drawn, which have the ſame Proportion to each other, as the Parts AC and BC have, and meet in the Point L: And if other two right Lines be drawn, keeping in like manner the ſame Proportion, and meet in K. And ſo on, if others as AI and BI, AH and BH, AG and BG, AF and BF, AE and BE; I ſay, that all the Points of their Interſections L, K, I, H, G, F, E, fall in the Circumference of one and the ſame Circle; ſo that if we conceive the Point C to be ſo mov'd continually, that the Lines drawn from it to the fix'd Terms A and B, retain always the ſame Proportion as there is between the firſt Parts AC and CB; then that Point C ſhall, by ſuch Motion, deſcribe the Circumference of a Circle, as I will preſently demonſtrate to you. And the Circle thus deſcrib'd will be greater and greater *ad infinitum*, as the Point C is taken nearer and nearer to the middle Point O; and leſs and leſs as the ſame Point C is taken nearer and nearer to the Point B, the Extremity of the ſaid firſt-given Line: So that from the infinite Points which may be taken in the right Line OB, (if they are ſuppos'd to move according to the 'fore-mention'd Law) there may be deſcrib'd Circles of any Magnitude; ſome leſs than the Pupil of the Eye of the leaſt Inſect, and others bigger than a great Circle of the *Primum Mobile*.

Now, ſince if any of the Points between the Terms O and B, be moved as aforeſaid, by every one of them a Circle will be deſcrib'd, and thoſe immenſe ones, deſcrib'd by the Points very near the Point O; thence, if

the Point O itſelf be in like manner mov'd, *i. e.* ſo that the Lines from it to the Terms A and B, may always keep the ſame Proportion to each other, as the firſt Lines AO and OB have; what Sort of a Line will it deſcribe? I ſay, that this Point O will alſo deſcribe, by ſuch Motion, the Circumference of a Circle, and of a Circle the greateſt of all, and therefore of an infinite one: Nay, indeed, it rather deſcribes a right Line perpendicular to AB from the Point O, and infinitely produc'd, ſo as never to return into itſelf as the others did; as thus, the Point C, by its



limited Motion, having deſcrib'd the upper Semi-circumference CHE, went on and deſcrib'd the lower one EMC, returning into itſelf in C: But now the Point O moving to deſcribe, as the other Points of the Line AB do, (ſince alſo the Points taken in the other Part OA deſcribe their Circles, and ſince thoſe deſcribe the greateſt which are neareſt to the Point O) its Circle; and that the greateſt of all, and conſequently infinite, it never can return to its firſt Term; and thence, in a Word, it deſcribes an infinite right Line for the Circumference of an infinite Circle.

Pray

Pray conſider now what Difference there is between a finite Circle and an infinite; which changeth its very Being in ſuch Sort as to loſe its very Exiſtence, nay, its very Power of exiſting, ſince we moſt evidently ſee that there can't poſſibly be an infinite Circle: whence we may fairly conclude, that much leſs can there be given in Nature an infinite Sphere, or any other Body, nor can there be an infinite Superficies made. What ſhall we ſay then to this Metamorphoſis in the Tranſition from Finite to Infinite? And why ſhould we meet with greater Oppoſition when, ſeeking an Infinite in Numbers, we conclude it to be in the Unit? And again, whilſt we breaking any Solid to Pieces, and endeavouring to reduce it into the ſmalleſt Duſt, when it is reſolv'd into Particles admitting no farther Diviſion; why may we not ſay, that it is return'd into one *Continuum*, but perhaps fluid, as Water is, or Mercury, or melted Metal? And do we not ſee Stones liquified into Glaſs, as alſo Glaſs itſelf, with a vaſt Heat, to become as fluid as Water?

SAGR. Muſt we then believe Fluids to be ſo call'd, becauſe they are reſolv'd into their firſt infinite, indiviſible, compoſing Parts?

SALV. I don't know how better to account for many ſenſible Appearances; of which this is one. When I take any hard Body, for Example, a Stone, or Piece of Metal, and, with a Hammer, or very fine File, divide it, as much as poſſible, into the fineſt and moſt impalpable Powder, 'tis manifeſt that its moſt minute Particles, altho' they are ſingly ſo ſmall as to eſcape our Sight, yet are they quan-

titative, figured, and numerable: And hence, if you heap them up, they continue in that Heap, or if you make a Hollow or Pit in that Heap, ſuch Pit continues, the Particles heap'd about it not falling in ſo as to fill it up; and if you ſhake or ſtir theſe Particles, yet as ſoon as the external moving Cauſe quits them, their Motion ceaſes: And the like Effects are ſeen in all the Aggregates bigger and bigger of ſmall Bodies, let them have what Figure they will, nay, tho' they are Spherical, as we ſee in Heaps of any ſmall round Seeds, as (Millet) Wheat, or any other Matter. But now none of theſe Accidents are met with in Water; for if Water be raiſed on an Heap, it inſtantly returns to a Level, unleſs it be upheld by any Veſſel or other external Stay: make this hollow, it preſently runs back to fill up the Cavity; and being mov'd it keeps a long while in Motion, and extends its Waves a vaſt Way.

From all which, I think, we may very reaſonably conclude, that the moſt minute Particles of Water of which it ſeems to be reſolved, (ſince it has leſs Conſiſtence than any the fineſt Powder, nay, ſince it has no Conſiſtence at all) are vaſtly different from the moſt minute Quantitatives and Diviſibles; and what can they be then but indiviſible? Again, I can't but think that its moſt exquiſite Tranſparency affords ſufficient Grounds for this Inference; for if we take the moſt diaphanous Cryſtal that is, and beat and pound it to Powder, it will loſe its Tranſparency, and the more, the finer 'tis made; but now ſtamp or pound Water as much as you will, yet it retains its Tranſparency. Gold and Silver, tho' reduced by *Aqua-Fortis* into a finer Powder than any File can make, yet they are

are but Powder, they acquire no Fluidity, nor can they be liquified, unleſs the Fire or the Sun-beams reſolve them, as I believe, into their firſt and higheſt infinite and indiviſible compoſing Parts.

SAGR. What you ſay of the Sun I have often obſerv'd with Admiration: For Inſtance; I have ſeen a concave *Speculum*, of a Foot Diameter, melt Lead in an Inſtant; whence I can't but think, that if the *Speculum* be large and well-poliſh'd, and of a parabolick Figure, it would in a very ſhort Time melt any other Metal, ſince I have ſeen the 'fore-mention'd one, which was but ſmall, not very well poliſh'd, and a ſpherical Concave, with ſuch Force melt Lead, and fire any other combuſtible Matter: Which Effects, I muſt own, make the Wonders reported of the *Specula* or Burning-Glaſſes of *Archimedes* credible to me.

SALV. As to the Effects of the *Specula* of *Archimedes*, all the Miracles met with concerning them in others, are render'd credible to me, by reading *Archimedes*'s own Book, which I have with infinite Amazement peruſed and ſtudied: And, indeed, if he had left me in any Doubt, yet that Treatiſe which not long ſince Father *Buonaventura Cavalieri* hath publiſh'd of Burning-Glaſſes, entitled, *Lo Specchio Uſtorio*, which I have read with Admiration, would have anſwer'd all Difficulties.

SAGR. I formerly knew the Author, and have ſeen and read the Treatiſe you ſpeak of with Pleaſure and great Admiration; and indeed it confirm'd me in the Opinion which

I before had conceiv'd of him, *viz.* That he was likely to become one of the greateſt Mathematicians of the Age. But to return to the admirable Effects of the Sun-beams in melting Metals: Muſt we think ſo fierce and violent an Operation is perform'd without Motion; or muſt we think it is perform'd by Motion, and that extremely ſwift?

SALV. We ſee other Burnings, and Diſſolutions, or Meltings, to be made by Motion, and that the ſwifteſt too. Obſerve the Operations of Lightening, of Gunpowder in Mines, and in Pieces of Ordnance; and laſtly, we ſee how, by quickning with Bellows the Flame of Coals, mixt with groſs and impure Vapours, its Force is increaſed in melting of Metals: Wherefore I can't conceive how the Action of Light, tho' moſt pure, can be without Motion, and that the ſwifteſt.

SAGR. But what, and how great may we judge this Operation of Light? Is it inſtantaneous, or is it like other Motions, ſome time in doing? Is it not poſſible, by ſome Experiment, to ſettle this Matter?

SIMP. Daily Experience ſhews the Expanſion of Light to be inſtantaneous: For when a Cannon is diſcharged a great Diſtance from us, we ſee the Flaſh inſtantly, tho' the Report reaches us not till ſome time after.

SAGR. Nay, *Simplicius*, nothing can be gather'd from this common Experiment but this, *viz.* Sound is longer in reaching our Ear, than Light the Eye. It does not aſſure me that the Acceſſion of Light is inſtantaneous; it may

may move very ſwiftly, and yet be ſome time in reaching us, for any thing your Experiment proves to the contrary. And another like Obſervation proves no more than this of yours; I mean, when we are told that as ſoon as the Sun reaches the Horizon, we ſee his Light; for pray who can aſſure me, that he does not reach the Horizon ſooner than we ſee him?

SALV. The weak Concluſions of theſe, and other Obſervations of the like Nature, occaſion'd me to think of ſome other Way, whereby I might be ſure whether the Illumination, *i. e.* whether the Expanſion of Light was really inſtantaneous or no; ſince the Motion of Sound aſſures us that of a Light is extreme ſwift. And the Experiment I hit upon was this: Let two Perſons take each of them a Light in a Lanthorn, ſo made that the Light may be hid from each other, by clapping the Hand before it, or by ſome other Means, and again diſcover'd at Pleaſure: then let them ſtand at a little Diſtance, with the Lights againſt one another, and let them practiſe the Diſcovery and Occultation of their Lights, from the Sight of each other; ſo that when one ſees the other's Light, he immediately diſcloſe his: In doing this mutually to each other ſeveral times, they will be ſo ready, that, without ſenſible Variation, the Moment the one opens his Light, the other ſhall anſwer him with a Diſcovery of the Light he holds: Having practiſed this ſufficiently at a ſmall Diſtance, let theſe two Perſons, with their two Lights, go from each other the Diſtance of two or three Miles; and repeating the ſame Experiment by Night, let them moſt carefully obſerve whether the mutual Diſcoveries and Occultations

cultations anſwer one another, according to the ſame Tenor which they did at that firſt ſmall Diſtance: If they do, we may certainly enough conclude, that the Expanſion of Light is inſtantaneous: But if it ſhould require Time at the Diſtance of three Miles, which, accounting the Going of one Light, and the Return of the other, make ſix, that Stay muſt be ſufficiently obſervable. And if this Experiment be made at a greater Diſtance; for Inſtance, at the Diſtance of eight or ten Miles, each Obſervator may make uſe of a Teleſcope fixt in that Place, where by Night he is to make the Experiment; and then, tho' the Lights are ſo ſmall as not to be diſcern'd at that Diſtance by the bare Eye, yet, by Help of theſe Teleſcopes, they may be eaſily ſeen.

SAGR. The Experiment ſeems to me capable of being made as exactly as the Invention of it is ingenious: But pray tell me what you have concluded from it.

SALV. Really I have tried it only at a ſmall Diſtance, leſs than a Mile; whence I could not ſafely gather, whether the Appearance of the oppoſite Light was inſtantaneous or no; but this I am ſure of, if it was not inſtantaneous, it was exceedingly ſwift, and, I may ſay, momentary. I would liken it, methinks, to that Motion we ſee a Flaſh of Lightning has among the Clouds eight or ten Miles off, the Beginning of which Flaſh, and, I may ſay, the Source and Riſe of it, we diſtinguiſh in a particular Place among thoſe Clouds; which Riſe its wide Expanſion immediately ſucceeds amongſt the adjacent Places: which ſeems to me a ſtrong Argument, that it is ſome ſmall Time in doing: For

For had the Illumination been made all at once, and not gradually, I can't think we could be able to diſtinguiſh its Riſe, and, I may ſay, Centre of its flaſhing Rays, and Extremities of their Dilatation. But we are plunging ourſelves inſenſibly into ſuch an Ocean of *Vacuums*, Infinites, Indiviſibles, and inſtantaneous Motions, that a thouſand Diſcourſes will hardly be able to ſet us on Shore.

SAGR. Things indeed far exceeding the narrow Bounds of our Underſtanding. Whilſt, pray mind, an Infinite ſought amongſt Numbers determines in the Unit: From Indiviſibles always ariſes a Diviſible: A *Vacuum* ſeems to reſide no where, unleſs indiviſibly mix'd with the *Plenum*. And, in ſhort, the Nature of Things commonly underſtood by us, may be ſo changed, that the very Circumference of a Circle may become an infinite right Line: Which is the Propoſition, if I remember aright, which you, *Salviatus*, promis'd to give us a Geometrical Demonſtration of. If you pleaſe, therefore, pray give it us without any farther Digreſſion.

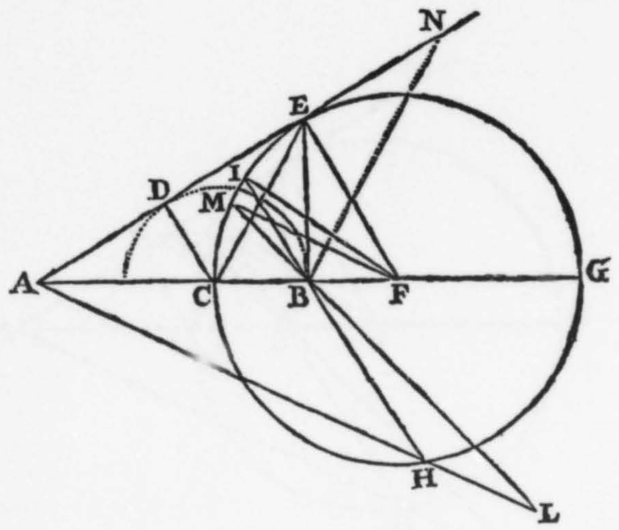
SALV. I will do it: but firſt, by way of Preparation, I muſt demonſtrate this Propoſition.

PROPOSITION.

Any right Line being given, cut into two unequal Parts in any Proportion, to deſcribe a Circle, that two right Lines being drawn from the Ends of the given right Line, to any Point of the Circumference of that Circle, may keep the ſame Proportion, as the Parts of the given Line

have to each other, ſo that thoſe Lines, which proceed from the ſame Ends, be homologous themſelves.

Let the given right Line be AB, unequally divided according to any Proportion in the Point C; it is requir'd to deſcribe a Circle, at any Point of whoſe Circumference

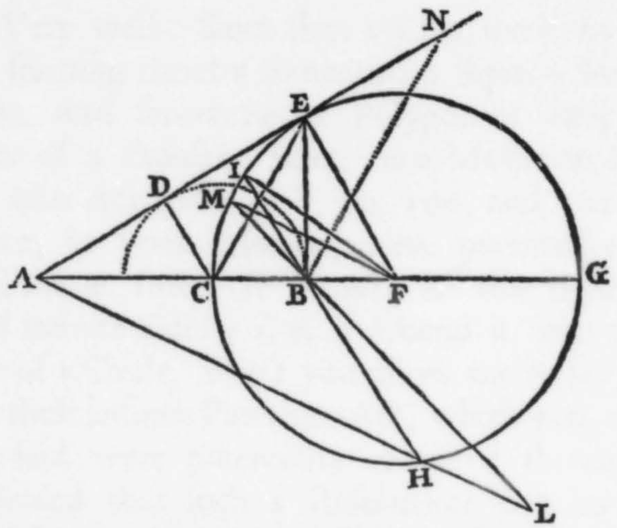


two right Lines, drawn from the Terms A and B, may meet, ſo as to have the ſame Proportion to each other, as the Parts AC and BC have; ſo that thoſe Lines which are drawn from the ſame Term are homologous. Upon the Centre C, at the Diſtance of the leſſer Part CB, let a Circle be imagin'd to be deſcrib'd; from the Point A draw to it the tangent Line AE indefinitely, and let the Point of Contact be D; and draw CD, which will be perpendicular to AE; upon AB erect the Perpendicular BE, which produced will meet the Line AE, the Angle A being acute; let them meet in E; from E erect a Perpendicular,

dicular, which produc'd, let meet AB indefinitely produc'd in F. Then I ſay firſt, that FE and FC are equal; for drawing EC, in the two Triangles DEC, BEC, the two Sides of the one DE, EC, are = two Sides of the other BE and EC, ſince DE and EB are Tangents to the Circle DB: and the Baſes DC and CB are alſo equal; wherefore the two Angles DEC and BEC will be equal. And ſince ∠ BCE wants of a right Angle the Quantity of the Angle CEB, and ∠ CEF wants of a right Angle the Quantity of ∠ CED, and theſe Defects are equal to each other; therefore ∠'s FCE and FEC will be alſo equal, and ſo conſequently are the Sides FE and FC: Wherefore if upon F as a Center with the Radius FE, a Circle be deſcrib'd, it will paſs thro' the Point C. Let this Circle be deſcrib'd, and let it be CEG; then, I ſay, this is the Circle ſought, that any two Lines drawn from the Terms A and B, and meeting each other in any Point of the Circumference whatſoever, will keep the ſame Proportion, as the two Parts AC and BC have to each other, which before did meet in the Point C. This is manifeſt of the two Lines, which meet in the Point E, *i. e.* of AE and BE; for in △ AEB, ∠ AEB is biſected by the Line CE, and thence 'tis AC: CB::AE:BE: For draw BN parallel to CE, cutting AE in N, then ∠ AEC being = CEB = EBN = ENB, the Line EN will be = EB; but now it is AC:CB::AE:EN, q^o' AC:CB::AE:EB. *Q. E. D.*

I'll demonſtrate the ſame of the two Lines AG and BG meeting each other in the Point G, becauſe (by Reaſon the Triangles AFE and EFB are ſimilar) it is AF:FE::FE:FB, *i. e.* AF:FC::FC:FB, it will be by Diviſion AF—FC:FC = FG::FC—FB:FB, *i. e.* AC:CF (*i. e.* FG)

(*i. e.* FG) :: CB : BF, *i. e.* AC : CB :: FG : BF; then by Compoſition AC + CB : FG + BF :: CB : BF, *i. e.* AB : BG :: CB : BF; and again by Compoſition, AB + BG : BG :: CB + BF : BF, *i. e.* AG : BG :: CF : BF, *i. e.* :: EF : BF, *i. e.* :: AE : EB; or laſtly, (by conſidering the 'foremention'd Proportion) :: AC : CB. *Q. E. D.*



Again, take any other Point in the Circumference of this Circle, as H, in which let the two right Lines AH and BH meet; I ſay likewiſe that AC : CB :: AH : HB, produce HB to the Circumference in (I), and join the Points I, F. Since now, as we have ſeen, 'tis AB : BG :: BC : BF, the Rectangle ABF will be = ▭ CBG, *i. e.* IBH; therefore 'tis AB : BH :: IB : BF, and ∠'s at B are equal : therefore 'tis AH : HB :: IF, *i. e.* EF : FB, *i. e.* :: AE : EB, *i. e.* :: AC : CB. *Q. E. D.*

And I ſay further, that 'tis impoſſible for Lines of ſuch Proportion, drawn from the Terms A and B, to meet

meet in any one Point, either within or without the Circle CEG. For if it be poſſible let two ſuch Lines meet in L, without the Circle, and let them be AL, BL; produce LB to the Circumference in M; join MF: If then it be AL : BL :: AC : BC. *i. e.* :: MF : FB, we ſhall have two Triangles ALB, MFB, which have the Sides Proportional about the two Angles ALB and MFB, with ∠'s at the Vertex or Point B equal, and the two remaining ones FMB and LAB leſs than right ones, (becauſe the right Angle at M has for its Baſe the whole Diameter CG, and not only Part of it BF: And the other at A is acute, becauſe the Line AL, homologous to AC, is greater than BL, which is homologous to the Part BC) and therefore the Δ's ABL and MBF are ſimilar; and g°' 'tis AB : BL :: MB : BF, whence ▭ ABF will be = ▭ MBL. But ▭ ABF was prov'd = ▭ CBG, g°' ▭ MBL is = ▭ CBG, which is impoſſible: therefore their meeting without the Circle can't poſſibly happen. In like manner it may be prov'd, that ſuch Meeting can't happen within the Circle; and therefore all theſe Meetings muſt fall in the Circumference of the Circle.

But it is time to return, and anſwer *Simplicius*'s Requeſt, by ſhewing him that to reſolve any Line into its infinite Points, is not only not impoſſible, but as eaſy as to divide it into its quantitative Parts; premiſing this, which I ſuppoſe, *Simplicius*, you will grant me, *viz.* you will not, I hope, require me to ſeparate the Points one from another, and ſhew you them one by one diſtinctly upon this Paper; for I ſhould think it ſufficient, that without actually ſeparating four or ſix Parts of any Line, you would ſhew

me

me the Diviſions mark'd, or at moſt ſhew them inclin'd one to another, making of them into a Square or Hexagon: If I do this, I perſuade myſelf you'll allow that they are ſufficiently and actually diſtinguiſh'd.

SIMP. I allow it.

SALV. Very well: Since then a Line made to incline at Angles, framing thereby ſometimes a Square, ſometimes an Octagon, and ſometimes a Polygon of forty, of an hundred, or of a thouſand Sides, be a Mutation ſufficient to reduce into Act theſe 4, 8, 40, 100, and 1000 Parts, which before, in your Opinion, were potentially in the ſaid right Line at firſt: If I make of that right Line a Polygon of infinite Sides; *i. e.* if I bend it into the Circumference of a Circle, won't you allow me to ſay, I have reduc'd all thoſe infinite Parts into Act, which you, whilſt it was ſtrait, ſaid were potentially contain'd therein? Nor can it be denied that ſuch a Reſolution into its infinite Points is made, any more than it can, that that Line is divided into four Parts, by forming it into a Square, or into a thouſand, by forming thereof a thouſand-ſided Figure; ſince in ſuch Reſolution there wants no one Condition found in the Polygon of a thouſand or of an hundred thouſand Sides: Which Polygon, if apply'd to a right Line, touches it with one of its Sides, *i. e.* with one of its hundred thouſand Parts: The Circle, which is a Polygon of infinite Sides, toucheth the ſame right Line with one of its Sides, which is one ſingle Point different from all its Collaterals, and therefore it is divided and diſtinct from them, no leſs than the Side of a Polygon is from all its Conterminals:

Conterminals: And as the Polygon, turn'd round upon any Plane, deſcribes, with the ſucceſſive Touches of its Sides, a right Line = its own Perimeter; ſo a Circle, mov'd round upon ſuch a Plane, deſcribes, by its infinite ſucceſſive Touches, a right Line = its Circumference. Now I know not, *Simplicius*, whether the Peripateticks, (to whom I freely grant, as a thing moſt certain, that a continu'd Quantity is diviſible into Parts always diviſible, ſo that continually repeating ſuch Diviſion and Subdiviſion, we ſhall never come to an End; whether theſe, I ſay) will grant me, that one of thoſe Diviſions is the Ultimate, as indeed it is not, ſince there always remains another; but the laſt and higheſt to be that which reſolves it into infinite Indiviſibles, to which, I own, we can never attain by dividing and ſubdividing it ſucceſſively into a greater and greater Number of Parts: But making uſe of the Method I propoſe of diſtinguiſhing and reſolving it by one only Tract, (an Artifice which ought not to be denied me) I ſhould think they ſhould be ſatisfy'd, and admit that a *Continuum* is compoſed of Particles abſolutely indiviſible: And eſpecially when by Help of this Method, more than by any other, we can diſingage or rid ourſelves of many intricate Labyrinths, ſuch as are, beſides that already mention'd concerning the Coherence of the Parts of Solids, the comprehending the Buſineſs of Rarefaction and Condenſation, without the Inconvenience of being forc'd to admit of void Spaces, and, on their Account, the Penetration of Bodies; which are two Inconveniencies, which, in my Opinion, may be ſhunn'd by admitting the afore-ſaid Compoſition of Indiviſibles.

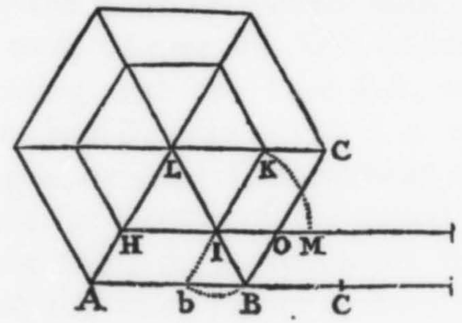
SIMP. I

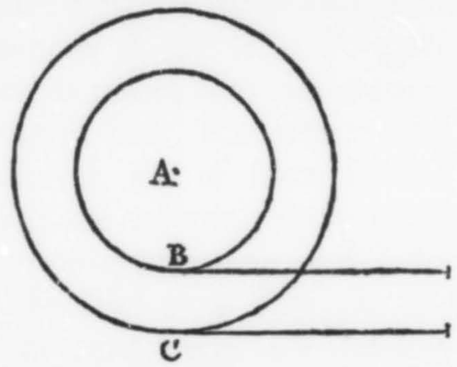
SIMP. I know not what the Peripateticks would ſay, ſince the Conſiderations you have offer'd, I believe, would appear to them moſtly new, and as ſuch they ought to examine them: And then, may be, they would find your Anſwers and Solutions ſufficient to explain thoſe Difficulties, which I have neither Time nor Ingenuity enough to ſolve. Wherefore, ſuſpending that for the preſent, I would willingly hear how the Introduction of theſe Indiviſibles facilitates the Conception of Condenſation and Rarefaction, by which we may avoid, at the ſame time, a *Vacuum*, and the Penetration of Bodies.

SAGR. I alſo greatly deſire to underſtand this, which at preſent is ſo obſcure to me, but upon this Condition, that I may not be denied the Hearing the Reaſons *Ariſtotle* brings to confute a *Vacuum*, and then thoſe you have to offer to prove the being of one.

SALV. I will do both: And as to the firſt; it's neceſſary that as on Account of Rarefaction, we make uſe of the Line deſcrib'd by the leſſer Circle, which is greater than the Circumference of that Circle, whilſt it was mov'd by the greater's revolving about; ſo alſo to our underſtanding of Condenſation, we'll ſhew how the greater, if mov'd by the leſſer Circle's turning about, deſcribes a right Line leſs than its Circumference: For the clearer Explication of which, let us conſider firſt what happens in the Polygons. About the common Centre L, deſcribe two ſimilar Hexagons, as ABCA and HIKH, with the parallel Lines HOM and ABC, upon which they are to make their

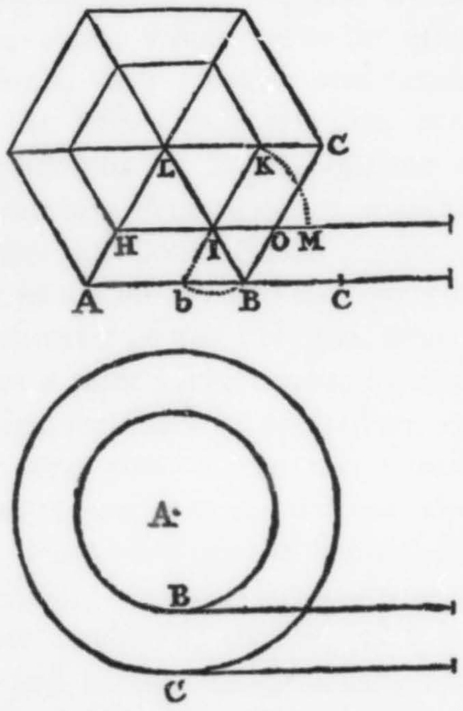
their Revolutions; and then the Angle (I) of the leſſer Polygon remaining fix'd, turn about the ſaid Polygon, till its Side IK fall on the Parallel HM, by which Motion the Point K will deſcribe the Arch KM, and the Side IK will coincide with the Part IM; now let's ſee what the Side BC of the greater Polygon does in the mean time. Becauſe the Revolution is made about the Point (I), the End B of the Line IB turning backward, will deſcribe





the Arch B*b*, under the Parallel CA; ſo that when the Side IK coincides with IM, the Side BC ſhall fall upon *bc*, advancing forwards only the Length B*c*, and retiring back, the Line ſubtending the Arch *b*B, and which falls upon the Line BA: And if we imagine the Revolution of the

leſſer Polygon to be continued after the ſame manner, this leſſer Polygon will, in turning once round, deſcribe on its Parallel H M, a Line = its Perimeter; but the greater ſhall deſcribe a Line leſs than its Perimeter, by the Quantity of ſo many times the Line B*b* as the Polygon has Sides, wanting one: And that Line is very nearly = that which is deſcrib'd by the leſſer Polygon, exceeding it only by the Quantity of B*b*. Here then, without any Trouble, you



ſee why the greater Polygon (carried by the leſſer) meaſures not out with its Sides a longer Line than the leſſer moves over, *viz.* becauſe one Part of each Side of the greater Polygon, falls upon its precedent contiguous Side.

But

But now if we conſider two Circles deſcrib'd about the Centre A, ſtanding upon their Parallels, ſo that the leſſer touches its Line in the Point B, and the greater its in the Point C: Here now, ſo ſoon as the leſs begins to revolve, the Point B will not remain, as before, for ſome time fix'd; whence the Line BC continually carries backward the Point C: As it happen'd in the Polygons, where the Point (I) remaining fix'd, 'till the Side KI falls upon the Line IM, the Line IB carried back the End B of the Line CB to *b*, whence the Side BC fell on *bc*, the Part *b*B coinciding with the Line BA, and advancing forwards by the Part B*c* only = IM, *i. e.* to one Side of the leſſer Polygon, by which Coincidencies or Applications which conſtitute the Exceſſes of the greater's Sides above the Sides of the leſſer, the Advances forwards, which are = the Sides of the leſſer Polygon, do compoſe in one whole Revolution a right Line = that trac'd and meaſur'd out by the Perimeter of the leſſer Polygon. But now, I ſay, if we come to reaſon after this manner of the Effects of Circles, we muſt own, that whereas the Sides of every Polygon whatſoever, are of ſome limited Number, the Sides of a Circle are infinite; thoſe are quantitative and diviſible, theſe non-quantitative and indiviſible: Again, the Terms of the Sides of a Polygon, in its Revolution, continue fix'd for ſome Time, *i. e.* for ſuch a Part of the Time of one whole Revolution, as that Side is of the whole Perimeter: But in Circles, the Stay of the Terms of their infinite Sides are momentary, ſince a Moment is ſuch a Part of quantitative Time, as a Point is of a Line which contains infinite Points. The Retrogreſſions of the Sides of the greater Polygon are not of the whole Side,

but only of its Exceſs above the Side of the leſſer Polygon, ſo that the Spaces advanc'd forwards, are only ſuch as are the Sides of the leſſer. In Circles, the Point or Side C, in the inſtantaneous Reſt of B, recedeth but as much as is its Exceſs above the Side B, advancing forwards as much as is the Quantity of the ſame B. And thus, at length, the infinite indiviſible Sides of their greater Circle, by their infinite indiviſible Retrogreſſions, made in the infinite inſtantaneous Stays of the infinite Terms of the infinite Sides of the leſſer Circle, and by their infinite Progreſſes or Advances, which are = the infinite Sides of the ſaid leſſer Circle, they compoſe and meaſure out a Line = that which the leſſer Circle deſcribes, containing in itſelf infinite Applications or Supra-poſitions non-quantitative, which, without any Penetration of quantitative Parts, make a Conſtipation and Condenſation; which cannot be conceiv'd to be in a Line divided into quantitative Parts, ſuch as is the Perimeter of any Polygon, which being diſtended at length into a right Line, cannot be contracted to a leſſer Length, unleſs the Sides be applied to, or fall upon, and ſo penetrate one another: In this Conſtipation of Parts not quantitative, but infinite, without the Penetration of quantitative Parts, and in the former Diſtraction of infinite Indiviſibles, with the Interpoſition of indiviſible Vacuities before explain'd, conſiſts, as I believe, the moſt that can be ſaid of the Condenſation and Rarefaction of Bodies, without the Neceſſity of introducing the Penetration of Bodies, or quantitative void Spaces. If there be any thing herein that pleaſes you, much good may it do you; if not, account it and my Diſcourſe vain, and ſeek ſome other Explanation more ſatisfactory to you. Let me

me only remind you of this one Thing, that we are talking of Infinites and Indiviſibles.

SAGR. I muſt confeſs this Invention of yours is very ſubtile, and to me wholly new and ſtrange; but whether or no Nature proceeds in this Order, I'm not able to ſay: But, however, until I meet with an Explanation more ſatisfactory, I'll content myſelf with this. But may be *Simplicius* may offer ſomething (which I have not yet met with) farther to explain the Explication given us by the Philoſophers on this abſtruſe Subject; for, indeed, what I have hitherto read about Condenſation, ſeems to me ſo denſe, and that of Rarefaction ſo rare and ſubtile, that my weak Sight can neither comprehend this, nor penetrate that.

SIMP. I am full of Confuſion, and find great Difficulties which Path ever I take, and more particularly in this new one: for according to this Rule, an Ounce of Gold may be rarefied and drawn out into a Maſs bigger than the whole Earth; and the whole Earth may be condenſed and contracted to a leſs Bulk than a Nut, which I neither believe, nor think that you yourſelf do; and the Reaſonings and Demonſtrations which you have hitherto brought, as they are abſtracted and purely Mathematical, I don't believe that, when they are applied to Things Phyſical and Natural, theſe Rules will take Place.

SALV. It is not in my Power, nor, I believe, do you deſire me, to make that Viſible which is Inviſible: But to appeal to our Senſes; do we not ſee an immenſe Extenſion

tenſion of Parts in Gold, ſince you have mention'd it? I can't tell whether you have ever ſeen the Method Artificers uſe in drawing out Gold Wire, the Outſide of which is only in Reality Gold, for the inward Matter is Silver; and the Way of ſtretching or diſgroſſing it is this: They take a cylindric Ingot, or Rod of Silver, about half a Yard long, ſuppoſe, and about three or four Inches thick; and this they gild with Leaf-Gold, which you know is ſo thin, that the leaſt Breath of Air will blow it about; and of theſe Leaves they lay on eight or ten thick, and no more. So ſoon as this Rod is gilt, they begin to draw with a vaſt Strength, and force it thro' the Hole of the drawing Plate, and then thro' ſeveral other Holes ſucceſſively narrower, till at length, after many ſuch Tranſitions, they bring it to the Fineneſs of an Hair, or finer, its Surface or Outſide continuing ſtill gilt over. Now I leave you to conſider to what a Fineneſs and Diſtenſion the Subſtance of the Gold is reduc'd.

SIMP. I do not ſee that from this Operation it follows, that the Subſtance of the Gold is ſo leſſen'd, as to effect thoſe Wonders which you ſpeak of: Firſt, becauſe the firſt gilding was with ten Leaves of Gold, which make a conſiderable Thickneſs: Secondly, becauſe, altho' in the Extenſion and leſſening that Silver, it increaſeth in Length, it yet decreaſes as much in Thickneſs; by which Compenſation the Superficies cannot be ſo enlarged, as to require, in covering the Silver with the Gold, the ſaid Gold to be reduc'd any thinner than the firſt Leaves were.

SALV. You're

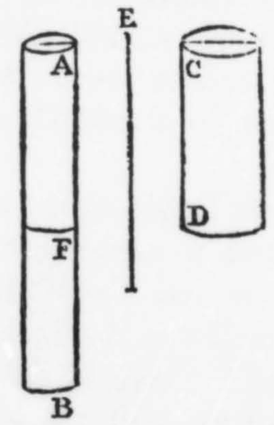
SALV. You're much deceived, *Simplicius*, for the Increaſe of the Surface is ſubduple the Extenſion or Prolongation, as I could demonſtrate geometrically to you.

SAGR. I beſeech you, both in Behalf of myſelf, and of *Simplicius*, to favour us with that Demonſtration, if you think we are capable of underſtanding it.

SALV. I will ſee whether I can, thus upon a ſudden, recal it to Mind ——— To begin then: 'Tis already manifeſt that the firſt thick Cylinder of Silver, and the Wire drawn out to ſo great a Length, are two equal Cylinders, for in each there's the ſame Silver: Now, if I ſhew you the Proportion the Surfaces of equal Cylinders have to one another, the Thing is done.

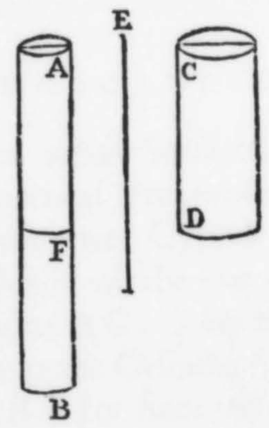
PROPOSITION.

The Surfaces of equal Cylinders, their Baſes omitted, are to one another in a ſubduple Proportion of their



Lengths: Let there be two equal Cylinders, whoſe Heights are AB and CD, between which let the Line E be a mean

mean Proportional: Then, I ſay, the Surface of the Cylinder AB is to the Surface of the Cylinder CD, the Baſes of both being omitted, as AB to E, which is ſubduple of the Proportion AB has to CD: Let the Cylinder AB be cut in F, and let the Height AF be = CD: Now becauſe that in equal Cylinders the Baſes and Heights are reciprocal; therefore the Circle the Baſe of the Cylinder CD, is to the Circle, the Baſe of the Cylinder AB, reciprocally as the Height BA to the Height CD: And becauſe Circles are to each other as the Squares of their Diameters, the ſaid Squares ſhall have the ſame Proportion as BA to CD: but as BA to CD, ſo the Square of BA to the Square of E; wherefore thoſe four Squares are proportional, and therefore their Sides will be proportional: And as the Line AB to E, ſo the Diameter of the Circle C, to the Diameter of the Circle A: but now as the Diameters are, ſo are the Circumferences, and as are the Circumferences, ſo alſo are the Surfaces of the Cylinders of



equal Height; therefore as the Line AB to E, ſo the Surface of the Cylinder CD to the Surface of the Cylinder AF:

AF: Therefore becauſe the Height AF is to the Height AB, as the Surface AF to the Surface AB, and as the Height AB to the Line E, ſo the Surface CD to the Surface AF: Thence by Perturbation of Proportion, as the Height AF to E, ſo the Surface CD to the Surface AB; and by Inverſion, as the Surface of the Cylinder AB to the Surface of the Cylinder CD, ſo the Line E to the Height AF, *i. e.* CD, *i. e.* as the Line AB to the Line E, which is a Ratio ſubduple of that which AB has to CD. *Q. E. D.*

Now if we apply what we have juſt now demonſtrated, to our Purpoſe, preſuppoſing that that Cylinder of Silver, which was gilt, when it was not more than half a Yard long, and four or five Inches thick, when leſſen'd to the Fineneſs of an Hair, acquires the Length of Twenty-thouſand Yards, for Inſtance, we ſhall find its Surface to become two hundred times greater than it was at firſt; and conſequently thoſe ten Leaves of Gold at firſt laid on, being diſtended on a Surface two hundred times bigger, aſſure us that the Gold which covereth the Surface of ſo many Yards of Wire, retains no more than the twentieth Part of the Thickneſs of an ordinary Leaf of Gold.

Pray conſider now what this Thinneſs muſt be, and whether it is poſſible to conceive it done, without a wonderful Diſtenſion of its Parts; and whether this ſeems to you an Experiment that tends to a Compoſition of infinite Indiviſibles, in Phyſical Matters: Tho' to aſſiſt in this there are not wanting other ſtronger and more concluding Accidents.

SAGR. The Demonſtration ſeems to me ſo ingenious, that, tho' it ſhould not be of Force enough to prove what was propoſed, and that on whoſe Account it was brought, (and yet in my Opinion it carries great Force with it) yet I think the Time well ſpent, which was employ'd in hearing it.

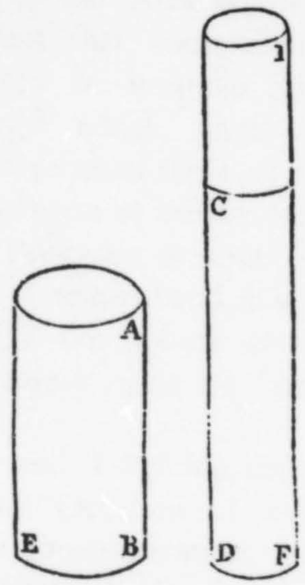
SALV. Since I ſee you are ſo well pleaſed with Geometrical Demonſtrations, which bring with them certain Profit; I will give you the Fellow of this, which will anſwer a very curious Queſtion often ask'd. ———

In the former we ſaw what happens in Cylinders that are equal, but of different Heights or Lengths: It will not be unpleaſant to hear what happens to Cylinders equal as to Surface, but of different Heights; note, I always mean thoſe Surfaces only which ſurround the Cylinders, *i. e.* not comprehending the Baſes upper or lower: I ſay then, That

PROPOSITION,

Upright Cylinders, whoſe Surfaces, omitting the Baſes, are equal, are in reciprocal Proportion to their Heights: Let the Surfaces of the two Cylinders AE and CF be equal, but let the Height of the one CD be greater than the Height of the other AC: I ſay the Cylinder AE has the ſame Proportion to the Cylinder CF, as the Height CD has to the Height AB: For ſince the Surface CF is = the Surface AE, the Cylinder CF will be leſs than the Cylinder AE: For if it was equal to it, its Surface by the laſt

laſt Propoſition would be greater than the Surface AE; and much more ſo, if the Cylinder CF were greater than AE: Let the Cylinder ID be ſuppoſed = AE; then by the preceding Propoſition, the Surface of the Cylinder ID will be to the Surface of the other AE, as the Height IF to a mean Proportional between IF and AB: But ſince the Surface AE is given = CF; and the Surface ID has the ſame Proportion to the Surface CF, as the Height IF



has to CD; therefore CD will be a mean Proportional between IF and AB: Moreover, ſince the Cylinder ID is equal to the Cylinder AE, each of them will have the ſame Proportion to the Cylinder CF; But ID is to CF as the Height IF to the Height CD; therefore the Cylinder AE has the ſame Proportion to the Cylinder CF, as the Line IF has to CD, *i. e.* as CD has to AB. *Q. E. D.*

Hence now an Accident, amazing to the Vulgar, is eaſily accounted for, *viz.* how it comes to paſs, if of the ſame Piece of Cloth or Sacking, longer one Way than the other, a Corn Sack be made, as the uſual Manner is, with a round Board for a Bottom, that it will hold more, if the Breadth of the Cloth be made the Depth of the Sack, (the Length encompaſſing the round boarded Bottom) than if the Sack be made up the other Way of the Sacking: Thus, for Example, ſuppoſe the Sacking was twelve Foot long, and ſix Foot wide; then, I ſay, it will hold more, if it be made into a Sack, by ſewing the two Ends together, by which Means the Sack will be ſix Foot deep, encompaſſing a Bottom twelve Foot round, than it will by ſewing together the two Sides, which will make it twelve Foot deep, encompaſſing a Bottom ſix Foot round: But farther, we do not only learn from the foregoing Propoſition, that the Sack holds more that Way than this, but we likewiſe are taught how much more; as thus, that the leſs Height it has, the more it will hold; and the greater Height, the leſs: And thus in the Meaſures aforeſaid, the Sacking being as long again as broad, it will hold but half the Quantity when ſewed lengthways, as it will if ſewed the other Way. In like Manner, if there be a Piece of Matt 25 Foot long, and ſeven broad, this ſewed together lengthways, will hold only ſeven Meaſures, ſuch as had it been ſewn together the other Way, that is, endways, it would have held 25.

SAGR. And thus, with great Pleaſure, we are continually diſcovering new and uſeful Knowledge. But as to the thing juſt now mention'd, I verily believe, that among ſuch

ſuch who are wholly ignorant of Geometry, there would not be found one in twenty who, at firſt Thinking, would not be pretty poſitive that thoſe Bodies which are contain'd under equal Surfaces, are in all Reſpects equal: As they run into the ſame Errors, ſpeaking of Surfaces, whilſt they are determining, as it often happens, the Magnitudes or Largeneſs of different Cities, they think they have done the Buſineſs, if they do but know how far they are about, not conſidering that one Circuit may be equal to another, and yet the Place, *i. e.* the Area contain'd by this, may be much greater than that comprehended by the other, as it happens, not only in irregular Surfaces, but alſo in regular ones, amongſt which, thoſe of more Sides are always more capacious than thoſe of fewer; ſo that the Circle, as being a Polygon of infinite Sides, is more capacious than any other Polygons of equal Perimeter: A Demonſtration whereof I remember I ſaw, with ſingular Delight, when I ſtudied the Sphere of *Sacroboſſo*, with a very learned Commentary upon the ſame.

SALV. It's true; and I lighting upon the very ſame Place, it gave me an Occaſion of inveſtigating how it may, with one ſole Demonſtration, be concluded, that the Circle is the greateſt of all the regular Iſoperimetral Figures, and of the reſt, thoſe of more Sides are greater than thoſe that have fewer.

SAGR. I am a great Lover of ſuch ſelect and uncommon Demonſtrations; therefore, pray Sir, (tho' I'm afraid I'm too troubleſome) make me a Partaker therein.

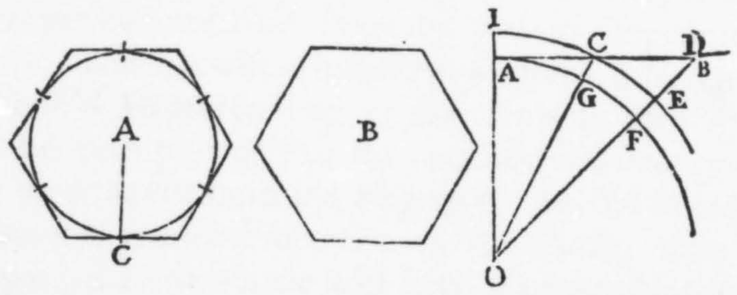
SALV. I'll

SALV. I'll do't in a very few Words, by demonſtrating the following Theorem.

PROPOSITION.

The Circle is a mean Proportional between any two Polygons ſimilar to each other, one of which is circumſcribed about it, and the other Iſoperimetral to it; ſo that it is leſs than all thoſe circumſcribed, and on the other hand, greater than all the Iſoperimetral.

Again, of the circumſcribed; thoſe that have moſt Angles, are leſs than thoſe that have fewer; and, on the other Side, of the Iſoperimetral, thoſe which have the moſt Angles are greateſt: Of the two ſimilar Polygons A and B, let A be circumſcribed about the Circle, and the other B, let be Iſoperimetral to it; then, I ſay, the Circle is a mean Proportional between them: For (having drawn the Semidiameter AC) the Circle being equal to a right-angled



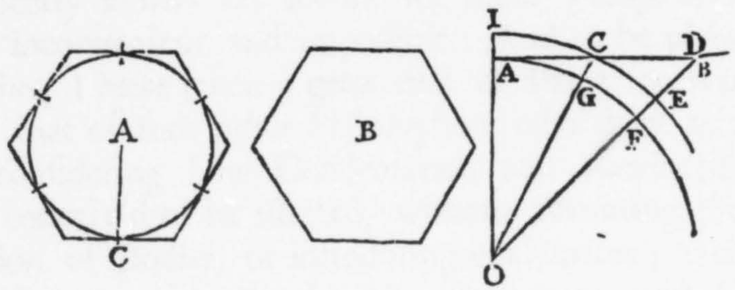
Triangle, one of whoſe Sides, containing the right Angle, is = the Semidiameter AC, and the other = its Circumference: And, in like manner, the Polygon A being = a right-angled

right-angled Triangle, which hath one Side containing the right Angle = the ſame right Line AC, and the other = the Perimeter of the ſame Polygon A; 'tis manifeſt that the circumſcrib'd Polygon A, has the ſame Proportion to the Circle, as its Perimeter has to the Circumference of the Circle, that is, to the Perimeter of the Polygon B, which is ſuppos'd = to the ſaid Circumference: But the Polygon A to the Polygon B (ſince they are ſimilar Figures) has a duplicate Ratio of that which the Perimeter of the Polygon A has to the Perimeter B; therefore the Circle A is a mean Proportional between the two Polygons A and B. And the Polygon A being greater than the Circle A, 'tis manifeſt that the ſame Circle A is greater than the Polygon B, its Iſoperimetral; and conſequently the greateſt of all regular Polygons that are Iſoperimetral to it. *Q. E. D.*

Now to the other Part of the Propoſition, which is to prove that of the Polygons circumſcrib'd about the ſame Circle, that which has the fewer Sides, is bigger than that which has more; And on the contrary, of Iſoperimetral Polygons, that which has more Sides is greater than that which has fewer. And theſe I prove thus;

To the Circle whoſe Centre is O, and Semidiameter OA, draw the Tangent AB, in which, for Example's Sake, ſuppoſe AD to be half the Side of a circumſcrib'd Pentagon, and AC half the Side of an Heptagon; and draw the right Lines OGC and OFD; and on the Centre O, with the Radius OC, deſcribe the Arch ECI; Now becauſe the Triangle DOC is greater than the Sector EOC, and the Sector COI greater than the Triangle COA, the Triangle DOC ſhall have a greater Proportion to the Triangle

Triangle COA, than the Sector EOC to the Sector COI, *i. e.* than the Sector FOG to the Sector GOA: And by Compoſition and Permutation, than the Triangle COA to the Sector GOA; and ten Triangles DOA ſhall have greater Proportion to ten Sectors FOA, than fourteen Triangles COA to fourteen Sectors GOA, *i. e.* the circumſcrib'd Pentagon ſhall have greater Proportion to the Circle, than the Heptagon hath to the ſame Circle: therefore the Pentagon is greater than the Heptagon.



Suppoſe now an Heptagon and a Pentagon Iſoperimetral to the ſame Circle, I ſay the Heptagon is bigger than the Pentagon: For the ſaid Circle being a mean Proportional between the circumſcrib'd Pentagon and its Iſoperimetral Pentagon; and likewiſe a mean Proportional between the circumſcrib'd Heptagon and its Iſoperimetral Heptagon; ſince it has been prov'd, that the circumſcrib'd Pentagon is greater than the circumſcrib'd Heptagon, the ſaid Pentagon ſhall have a greater Proportion to the Circle, than the Heptagon; *i. e.* the Circle will have a greater Proportion to its Iſoperimetral Pentagon, than to its Iſoperimetral Heptagon: And therefore the Pentagon is leſs than the Iſoperimetral Heptagon. *Q. E. D.*

SAGR. A

SAGR. A moſt elegant and curious Demonſtration! But how came we to change our Courſe, and plunge ourſelves thus into Geometry, when we were about conſidering, the Difficulties ſtarted by *Simplicius*, which are of great Moment? And in particular, that relating to Condenſation, is, in my Opinion, very abſtruſe.

SALV. If Condenſation and Rarefaction are oppoſite Motions, where there is found an immenſe Rarefaction, it can't be denied that there is alſo an extraordinary Condenſation: But immenſe Rarefaction, and, what increaſes the Wonder, as it were momentary, we ſee daily: For what a boundleſs Rarefaction is that of a little Quantity of Gunpowder, reſolv'd into a vaſt Maſs of Fire? And again, what is (I could almoſt ſay) the indeterminate Expanſion of its Light? If now that Fire and this Light ſhould reunite together, which is not impoſſible, ſince at firſt they lay in that little Room, what a Condenſation would this be? Upon conſidering, you'll find a thouſand ſuch Rarefactions, which, indeed, are much more obſervable than Condenſations, ſince denſe Matters are more tractable and more ſubject to our Senſes: thus we can handle Wood, and ſee it reſolv'd into Fire and Light; when, on the contrary, we cannot ſee Fire and Light condenſe to make Wood: So we find Fruits, Flowers, and a thouſand other ſolid Matters, to reſolve in a great meaſure into Odours; when, on the contrary, we never can obſerve thoſe odoriferous Effluvia to concur and make thoſe fragrant Solids. But when ſenſible Obſervation is wanting, Reaſon muſt ſupply its Place, by whoſe Help we ſhall be ſufficiently

able to apprehend the Motion of the Rarefaction and Reſolution of Solids; as alſo to the Condenſation of the fineſt and moſt rare Subſtances. But we not only enquire how the Condenſation and Rarefaction of Bodies, which are ſubject to Rarefaction and Condenſation, can be effected, but alſo how theſe can be conceived to be done, without introducing a *Vacuum*, and the Penetration of Bodies; which does not hinder but that in Nature there may be Matter which admits no ſuch Accident, and conſequently allows no Room for thoſe Things which you call inconvenient and impoſſible: And to be plain, *Simplicius*, I have taken a great deal of Pains for your Sake, and that of ſuch other Philoſophers, who think as you do, in conſidering how Condenſation and Rarefaction may be conceiv'd to be effected, without admitting the Penetration of Bodies, or introducing void Spaces; which two Effects you deny, and reject, as it were, with Horror; and which if you would grant me, I ſhould not ſo reſolutely oppoſe you as I do: Wherefore, either allow thoſe Inconveniencies, or acquieſce with my Speculations, or find out others yourſelf more to the Purpoſe.

SAGR. As to the Penetration of Bodies, I, with the Peripatetick Philoſophers, altogether deny it: But as to what relates to a *Vacuum*, I own I ſhould gladly hear *Ariſtotle's* Demonſtration, that oppoſes it, throughly examin'd, as alſo what you, *Salviatus*, can object to it. *Simplicius* will favour me punctually to recite the Philoſopher's Demonſtration; And I doubt not, *Salviatus*, but you will give us your Anſwer to it.

SIMP. *Ariſtotle*,

SIMP. *Ariſtotle*, as near as I can remember, inveighs againſt certain of the Antients, who introduc'd a *Vacuum*, as neceſſary to Motion, pretending that this could not be effected without that: *Ariſtotle*, oppoſing theſe Perſons, demonſtrates, that, on the contrary, the effecting of Motion (as we ſee) neceſſarily deſtroys the Suppoſition of a *Vacuum*. The Method he took was this: He ſuppoſes two Things; the one is touching Moveables of different Gravity, mov'd in the ſame *Medium*; the other is, concerning the ſame Moveable, mov'd in different *Mediums*. As to the former; He ſuppoſes Moveables of different Gravity, to be mov'd in the ſame *Medium* with unequal Velocities, which bear to each other the ſame Ratio as their Gravities: As for Example, Let one Moveable be ten times heavier than another, it ſhall move ten times more ſwiftly: In his other Suppoſition he aſſumes, that the Velocities of the ſame Moveable in different *Mediums*, do keep a reciprocal Proportion to that which the Thickneſs or Denſity of thoſe *Mediums* have: So that ſuppoſing, *v. g.* that the Denſity of Water be ten times as great as that of Air, he will have the Velocity in the Air to be ten times greater than in Water. And from this ſecond Suppoſition he deduces his Demonſtration in this Manner: Since the Tenuity of a *Vacuum* infinitely ſurpaſſes the Corporiety, tho' never ſo ſubtile, of any replete *Medium*, every Moveable that in the full *Medium* runs thro' a certain Space in a certain Time in a *Vacuum*, ought to run thro' the ſame Space in an Inſtant: But for a Motion to be made in an Inſtant is impoſſible; and therefore to introduce a *Vacuum* on Account of Motion is impoſſible.

 SALV. The

SALV. The Argument, you ſee, is *ad Hominem, i. e.* againſt ſuch as would have a *Vacuum* neceſſary to Motion: But now if I ſhould admit, that *in Vacuo* there would be no Motion, yet the Poſition of a *Vacuum* taken abſolutely, and without any Relation to Motion, is not thereby deſtroy'd; but to tell you what the Antients, may be, would anſwer, that thus you may the better ſee how far *Ariſtotle*'s Demonſtration is concluſive: 'Tis my Opinion, that both his Aſſumptions may be contradicted and denied. As to the former of them: I very much doubt whether *Ariſtotle* ever found, by Experience, how true it is, that two Stones, one ten times heavier than the other, let fall at the ſame Inſtant from an Height, *e. g.* of 100 Yards, differ ſo in Velocity, that when the greater comes to the Ground, the leſſer has not fallen above ten Yards.

SIMP. His Words ſeem to intimate that he has try'd it; for he ſays, *we ſee* that the heavier: now from theſe Words of his *we ſee*, we may conclude, that he had actually made the Experiment.

SAGR. Hold, *Simplicius*, for I myſelf have prov'd by Experience, that if a common Bullet, weighing 100, 200, or more Pounds, let fall from the ſame Height of 200 Yards, and at the ſame Inſtant, with a Musket Bullet that weighs but half a Pound; at the Arrival of the Cannon Bullet at the Ground, the other will not want a Hand's-breadth of it.

SALV. But

SALV. But without Experiments, it may be by ſhort and evident Demonſtration clearly prov'd, that it is not true that a Moveable heavier than another, moves or falls more ſwiftly than that other, the Moveables being ſuppos'd of the ſame Matter, and therefore ſuch as *Ariſtotle* ſpeaks of: For pray tell me, *Simplicius*, whether you admit that every heavy Body, in its Fall, has by Nature one determin'd Velocity, which can't be either accelerated or diminiſhed without ſome Force or Impediment?

SIMP. Without doubt the ſame Moveable, in the ſame *Medium*, hath a certain eſtabliſh'd, and, by Nature, determin'd Velocity; which cannot be increaſed without it ſuffer ſome new *Impetus*; nor diminiſhed, unleſs ſome Obſtacle retard it.

SALV. If therefore we take two Moveables, whoſe Velocities are unequal, 'tis manifeſt, that if we join the ſlower with the ſwifter, this would be ſomething retarded by the ſlower, and that ſomething accelerated by the ſwifter; don't you agree with me in this?

SIMP. This ſeems to me an undoubted Conſequence.

SALV. But now, if this be ſo, it will likewiſe be true, that if the greater Stone, moving, *e. g.* with eight Degrees of Velocity, and the leſſer with four, be joined together, that their compound Motion ought to be with a Velocity leſs than eight Degrees: But thoſe two Stones joined together, make a Stone greater than that which before mov'd with

with eight Degrees of Velocity; therefore this greater Stone moves with leſs Velocity than the leſſer, contrary to your Hypotheſis. You ſee, therefore, how from your Suppoſition, that the heavier Body is moved ſwifter than the lighter; I, on the contrary, conclude it to move ſlower.

SIMP. I find myſelf in a Snare, for certainly the leſſer Stone joined to the greater, muſt add Weight to it; and Weight being added to it, I don't ſee why its Velocity ſhould not be increaſed; at leaſt, certainly, it ought not to be diminiſhed.

SALV. Here now you run into another Error, *Simplicius*, for that leſſer Stone does not increaſe the Weight of the greater.

SIMP. No! I own this quite exceeds my Apprehenſion.

SALV. Your Surprize will be over, ſo ſoon as I ſhall have ſhewn you what's equivocal here, which occaſions your Doubt. Know then, that we muſt diſtinguiſh betwixt heavy Bodies *in Motion*, and the ſame *at Reſt*. A Stone put into a Ballance, certainly acquires, not only a greater Weight by laying another Stone upon it, but even by the Addition of a little Wool or Flax, it will in Weight acquire an Increaſe, as much as is the Weight of the Wool or Flax added; But yet if you ſhould let fall freely the Stone with the Flax tied about it from on high, do you think that in that Motion the Flax preſſes that Stone with its

its Weight, ſo as to accelerate its Motion; or do you think it in a manner ſuſtaining the Stone, will retard it? Thus, for Example, we feel indeed a Weight upon our Shoulders, ſo long as we oppoſe the Motion the Weight which lies upon our Shoulders would have, was it not ſo oppoſed: But if we alſo ſhould deſcend or fall with the ſame Velocity, wherewith that heavy Body would naturally deſcend; how do you think this Weight would preſs, gravitate, or bear upon us then? Is not this the ſame thing as a Man's ſtriving to wound another with a Lance, who runs before him with as much or more Speed than the other purſues with? You may conclude then, that in the free and natural Fall, the leſſer Stone doth not gravitate upon the greater, and conſequently doth not add to its Weight, as it does when they are at Reſt.

SIMP. But what if the greater be placed upon the leſſer?

SALV. It would increaſe the Weight of this, if its Motion were ſwifter: But it hath been already concluded, that in caſe the leſſer be more ſlow, it would in Part retard the Velocity of the greater, ſo that both together would move leſs ſwiftly, being greater than the other; which is contrary to your Hypotheſis: Therefore we conclude, that greater and leſſer Moveables, being of the ſame ſpecifick Gravity, move with equal Velocity.

SIMP. Your Diſcourſe is, I own, very ingenious; yet methinks 'tis hard to conceive that a ſingle Bird-ſhot ſhould fall with the ſame Velocity as a Cannon-Bullet.

SALV. You

SALV. You might have ſaid, that a Grain of Sand ſhould be carried with the ſame Celerity as a Mill-ſtone: But, *Simplicius*, I would not have you do as ſome others uſually do, I mean by diverting the Diſcourſe from the principal Deſign, to faſten upon ſome one Thing I have ſaid, that may want a Hair's-breadth, perchance, of the Truth, and under that Hair hide the Fault of another Man as big as a Cable. *Ariſtotle* ſays, an Iron Ball of an hundred Pounds Weight, falling from an Height of an hundred Yards, comes to the Ground before another of a Pound Weight has deſcended one Yard: And I ſay, that they'll both come to the Ground at the ſame Time. You find that the bigger out-ſtrips or gets before the other by two Inches; that is to ſay, when the greater reaches the Ground, the leſſer wants two Inches of it: But now you would, under theſe two Inches, hide *Ariſtotle*'s ninety-nine Yards; and only mentioning my very ſmall Error, would paſs over his great one in Silence.

Ariſtotle affirms, That Moveables of different Gravity move in the ſame *Medium* (as far as concerns Gravity) with Velocities proportional to their Weights, and exemplifies it by Moveables, wherein the pure and abſolute Effect of Weight may be conſider'd, omitting the other Conſiderations, as well of the Figures, as of the leaſt Moments, which Things ſuffer great Alteration from the *Medium*, by which the ſimple Effect of Gravity alone is alter'd: Whence we ſee Gold, that is heavier than any other Matter, beat out into a very thin Leaf, to fly to and again thro' the Air; and Stones beaten to very fine Powder to do the like: But if you would defend the univerſal Propoſition,

ſition, 'tis requiſite that you ſhew the Proportion of the Velocities to be obſerv'd in all grave Bodies, and that the Motion of a Stone of twenty Pounds Weight, is ten times ſwifter than that of a Stone of two Pounds: Which I tell you is falſe, ſince falling from the Height of 50 or 100 Yards, they both come to the Ground at once.

SIMP. Perhaps from very great Heights of Millions of Yards that might follow, which is not ſeen to happen from theſe leſſer Heights.

SALV. If this was *Ariſtotle*'s Meaning, he muſt be guilty of another Error; nay, thus you'll make him tell a Lye: For there being no ſuch perpendicular Height to be come at on Earth, *Ariſtotle* could not make the Experiment, which he would perſuade you he had, by ſaying that ſuch an Effect was *ſeen*.

SIMP. However *Ariſtotle* certainly made no uſe of *this*, but of *that other* Principle, which I don't believe is ſubject to theſe Difficulties.

SALV. You think ſo: But I ſay that's as falſe as this: And I admire you don't of yourſelf perceive the Fallacy; and ſee, that tho' it was true, that the ſame Moveable in *Mediums* of different Subtilty and Rarety, and therefore of a different Reſiſtance, as are, *e. g.* Water and Air, is mov'd with a greater Velocity in Air than in Water, according to the Proportion of the Air's Rarety to the Rarety of the Water, yet it does not follow, that all Moveables which deſcend in Air, do deſcend in like manner in Water;

which is ſo falſe, that very many Bodies deſcend in Air, that in Water are ſo far from deſcending, that, on the contrary, they always aſcend.

SIMP. I don't ſee the Neceſſity of your Conſequence: And I ſay farther, that *Ariſtotle* ſpeaks of ſuch heavy Bodies as deſcend in both *Mediums*, and not of ſuch as deſcend in Air, and aſcend in Water.

SALV. To defend this Philoſopher, you produce ſuch Pleas as he himſelf certainly never would, leſt he ſhould aggravate his firſt Miſtake: Pray tell me whether the Body and Subſtance of the Water, or of whatever it be which retards the Motion, hath any Proportion to the Craſſitude or Body of the Air that leſs retards it; and if it has, aſſign ſome Proportion at Pleaſure.

SIMP. It has: Let us ſuppoſe that Proportion to be decuple, and thence that the Velocity of any heavy Body that deſcends in both thoſe Elements, will be ten times ſlower in Water than in Air.

SALV. I'll take one of thoſe heavy Bodies that deſcend in Air, but not in Water; as for Inſtance, a Ball of Wood; Now I deſire you to aſſign it what Velocity you pleaſe, whilſt it deſcends thro' the Air.

SIMP. Let's ſuppoſe it to move with twenty Degrees of Velocity.

SALV. Very

SALV. Very well: Now 'tis manifeſt that that Velocity to ſome other leſſer may have the ſame Proportion, that the Solidity of the Water has to the Solidity of the Air; and this leſſer Velocity will be of two Degrees only: So that exactly and directly, according to *Ariſtotle*'s Hypotheſis, it ſhould be concluded, that the Ball of Wood, which in the Air, ten times more yielding than Water, deſcends, mov'd with twenty Degrees of Velocity, in the Water it ſhould deſcend with two, and not to aſcend from the Bottom to the Surface of the Water, as we ſee it does: unleſs you'll ſay, that the aſcending of the Wood in the Water to the Top, is the ſame with its ſinking towards the Bottom with two Degrees of Velocity; which I don't believe you'll venture to ſay: But ſeeing that this Ball of Wood tends not to the Bottom, I believe you'll grant me that ſome other Ball of other Matter, different from Wood, may be found to deſcend in Water with two Degrees of Velocity.

SIMP. Queſtionleſs there may; but it muſt be made of Matter conſiderably heavier than Wood.

SALV. That's what I deſir'd to know: But now this ſecond Ball, which in the Water deſcends with two Degrees of Velocity, with what Velocity will it deſcend in the Air? You muſt anſwer (if you'll follow *Ariſtotle*'s Rule) that it will deſcend with twenty Degrees; but you yourſelf have aſſigned twenty Degrees of Velocity to the Ball of Wood, therefore both this and the other, tho' much heavier, will move in the Air with equal Velocities:

But how will the Philoſopher reconcile this Concluſion with that other of his, which aſſerts, that Moveables of different Gravities move in the ſame *Medium* with different Velocities; and indeed as different as their different Gravities?

But omitting any farther deep Speculations; how comes it to paſs that you have not obſerv'd Accidents ſo very frequent and evident, and have not conſider'd two Bodies, one of which will move in Water 100 times more ſwift than the other; but in the Air, that that which is ſo much ſwifter in Water, will not out-go the other one hundredth Part in the Whole? Thus, *e. g.* a Piece of Marble in Form of an Egg, will ſink an hundred times ſwifter in Water than an Hen's Egg will; but in the Air, from the Height of twenty Yards, it will not out-fall it four Inches: And, in a Word, that heavy Body which is three Hours in falling to the Bottom of ten Fathom Water, will fall the ſame Depth in the Air in one or two Seconds; and ſuch another (as for Inſtance a Ball of Lead) will paſs thro' the ſame Space in half the Time: And here, *Simplicius*, I'm ſure you'll allow there's no Room left for any Diſtinction or Reply: We conclude therefore, that that Argument concludes nothing againſt a *Vacuum*; But if it ſhould, it would only deſtroy Spaces conſiderably great, ſuch as neither I, nor, as I take it, thoſe Antients ever ſuppoſed to be naturally allow'd or given, tho' perhaps with Violence they may be effected; as ſeems to be gather'd from ſeveral Experiments, too long to be inſiſted on here.

SAGR. Seeing

SAGR. Seeing *Simplicius* is ſilent, I'll take leave to ſpeak: Since you have with ſufficient Plainneſs demonſtrated that it is not true, that Moveables unequally heavy, move in the ſame *Medium* with Velocities proportional to their Gravities, but with equal Velocities; the heavy Bodies being ſuppoſed of the ſame Matter, or of the ſame ſpecifick Gravity, but not (I ſuppoſe) of different ſpecifick Gravities, (for I don't think that you pretended to prove to us, that a Ball of Cork moves with like Velocity with a Ball of Lead): Moreover, ſince you have alſo demonſtrated plainly that it is not true that the ſame Moveables in *Mediums* of different Reſiſtances, obſerve the ſame Proportion in their Velocities and Tardities, as their Reſiſtances have; you'd greatly oblige me to tell me what thoſe Proportions are, which are obſerv'd in both Caſes.

SALV. The Queſtions are very ingenious, and I have many times thought of them: I will relate to you the Contemplations I have had concerning them, and from thence what Concluſion I am come to about them.

After I had aſſured myſelf that it was not true, that the ſame Moveables in *Mediums* of different Reſiſtance, keep the ſame Proportion in their Velocities, with which thoſe *Mediums* give Way; nor yet again, that in the ſame *Medium* Moveables of different Gravity, keep in their Velocities, the Proportion of thoſe Gravities (ſpeaking of different ſpecifick Gravities); I bethought myſelf to join thoſe two Accidents together, and to conſider what would happen if the Moveables of different Gravity were put into *Mediums* of different Reſiſtance; and I found that the

the Inequalities of the Velocities were always greater in the more reſiſting *Mediums* than in the leſs; and that with this Diverſity, that of the two Moveables, which falling in the Air, differ very little in Velocity of Motion: One will, in the Water, move ten times faſter than the other; and that that which in the Air deſcends ſwiftly, in the Water not only will not deſcend, but will be wholly depriv'd of Motion; or, what is more, will move upwards: For ſometimes ſuch a kind of Wood may be found, or a Knot or Root of it, which in the Water will lie ſtill, but in the Air will deſcend ſwiftly.

SAGR. I have often, with the utmoſt Patience, ſtrove to bring a Ball of Wax, which of itſelf doth not ſink, to the ſame Gravity as Water, by adding to it Grains of Sand, ſo that it might ſtand ſtill in the Midſt of the Water; but I could never, with the utmoſt Care and Labour, ſucceed in the Attempt: Whence I doubt whether there be any Matter naturally ſo alike in Gravity to Water, as being put into it, to lie ſtill where-ever you put it.

SALV. In this, as well as in a thouſand other Operations, many Animals are more knowing than Men are: And in this Caſe of yours, the Fiſhes would have been able to have given you ſome Light, being ſo skilful in this Exerciſe, that at their Pleaſure they preſerve an Equilibrium, not only with one Sort of Water, but alſo with different, as ſuch as either are naturally or accidentally muddy, or elſe are ſalt, which makes a great Alteration: They, I ſay, ſo accurately preſerve an Equilibrium, that, without

without the leaſt Motion they can lie ſtill in every Place; and this they do, in my Opinion, by making uſe of an Inſtrument Nature has given them to that End; I mean a little Bladder which they have within them, and whoſe narrow Neck anſwers to their Mouth, thro' which, when they would lie ſtill, they ſend forth Part of the Air contain'd in the ſaid Bladder; and when they would riſe to the Surface, or Top of the Water, they draw more Air in: by this Artifice making themſelves at one time more, at another leſs heavy, and when they pleaſe they preſerve an Equilibrium.

SAGR. I deceiv'd ſome of my Friends with another Device, to whom I had boaſted, that I could reduce a Ball of Wax to an exact Equilibrium with the Water: Thus, *viz.* by putting ſome Salt-Water in the Bottom of the Veſſel, and then filling up the Veſſel with the like Quantity of freſh Water; which done, I ſhew'd them a Ball at reſt in the Midſt of the Water, which being thruſt to the Bottom, or raiſed to the Top of the Water, would continue in neither Place, but would always return to the Midſt.

SALV. This Experiment of yours has its Uſes: For when Phyſicians, in particular, treat of the different Qualities of Waters, and amongſt theſe of their Difference of Levity and Gravity, they make uſe of ſuch a Ball, ſo adjuſted, or poiſed (that it may reſt ambiguous, if I may ſo ſay) in the Water, between riſing and ſinking: If that Ball, upon the leaſt Difference of Weight, between two Waters, ſinks in one, in the other, which is more heavy, it will riſe.

riſe. And this Experiment is ſo exact, that if into ſix Pounds of Water, you put only two Grains of Salt, the ſame Ball, which before was ſunk to the Bottom, will now riſe to the Surface or Top of the Water. But farther, to confirm the Exactneſs of this Experiment, and clearly to prove the Non-reſiſtance of Water to Diviſion or Penetration; I affirm to you, that not only by mixing with the Water ſome heavier Matter, the Water becomes heavier, with ſo notable a Difference; but by warming or cooling it a little, the ſame Effect is produc'd, and with ſo ſubtile an Operation, that the infuſing into the ſix Pounds but four Drops of other Water, a little warmer or a little colder, ſhall cauſe the Ball to ſink or riſe; to ſink upon Infuſion of the Warm, and to riſe upon the Infuſion of the Cold. Now judge how much thoſe Philoſophers are deceiv'd, who would introduce in Water Viſcoſity, or other Conjunction of Parts, to make it reſiſt Diviſion or Penetration.

SAGR. I have ſeen many concluſive Diſcourſes concerning this Argument, in a Treatiſe of our Academick; yet there's one great Doubt reſts with me, which I can't get rid of; 'tis this: If there be nothing of Tenacity or Coherence amongſt the Parts of Water, how can ſuch large Heaps ſuſtain themſelves, as in a particular Manner are ſeen upon Cole-wort or Cabbage Leaves, without diſperſing or levelling?

SALV. Altho' it be true, that he that is Maſter of a true Concluſion, may reſolve all Objections that can be brought againſt it; yet will I not arrogate to myſelf the Power of

of doing ſo; nor ought my Inſufficiency to darken the Light of Truth. Firſt then, I freely confeſs that I know not how it comes to paſs, that thoſe Globes of Water ſuſtain themſelves at ſuch an Height and Bigneſs, altho' I certainly know that it does not proceed from any internal Tenacity that is between their Parts: whence I neceſſarily infer, that the Cauſe of that Effect is without the Water. That it is not internal, ſetting aſide the Experiments already produc'd, I can prove by another moſt convincing one; 'tis this: If the Parts of that Water which ſuſtains itſelf in Globules whilſt 'tis encompaſſed by the Air, had an internal Cauſe for ſo doing, they would much better ſuſtain themſelves encompaſſed by a *Medium*, wherein they would have leſs Propenſity to deſcend than they have in Air: But now ſuch a *Medium* would every Fluid heavier than Air, be; as, for Inſtance, *Wine*; and therefore if about that Globe of Water *Wine* be pour'd, this might raiſe itſelf on high about that, and yet the Parts of the Water, conglutinated by their internal Viſcoſity, would never be diſſolved: But now this does not ſo happen in theſe Globules, which no ſooner hardly doth the circumfuſed Liquor touch, but they diſſolve and become flat, reſting under the Wine, if it be Red: The Cauſe therefore of this Effect is from without, and perhaps from the ambient Air; and a great Diſſention obſervable between Air and Water, as I have found by another Experiment, and that is this; If I fill a round Cryſtal Bottle with Water, whoſe Mouth is no bigger than that of a Straw, and after this turn its Mouth downwards, yet will not the Water, altho' very heavy and prone to deſcend in Air, nor the Air, as much diſpoſed on the other Hand, as being

very light, to aſcend thro' the Water; yet will they not, I ſay, agree, that that ſhould deſcend, iſſuing out of the Mouth, and this aſcend, entering in at the ſame; but both keep their Places, and yield not to each other. But on the contrary, if I apply to the Mouth of this Bottle a little Veſſel of Red Wine, which is inſenſibly leſs heavy than Water, we ſhall ſee it in an Inſtant gently to aſcend by red Streams thro' the Water; and on the contrary, the Water, with the ſame Slowneſs, to deſcend thro' the Wine, without ever mixing with each other, till at length the Bottle will be full of Wine, and all the Water will ſink to the Bottom of the Veſſel that's underneath. What muſt we ſay now? or what Argument muſt we uſe, unleſs this, that there is a Diſſention, how I know not, between Water and Air? But perhaps———

SIMP. I can't help laughing to ſee the great Antipathy *Salviatus* has to Antipathy; that he does not think it worth naming, tho' it be ſo accommodate to reſolve theſe Difficulties.

SALV. Well, for your Sake, *Simplicius*, let this ſolve our Doubt; and leaving this Digreſſion, let us return to our Purpoſe.

We have found that the Difference of Velocities in Moveables of different Gravities to be more and more as the *Mediums* are more and more reſiſting: Thus in a *Medium* of Quickſilver, Gold does not only ſink to the Bottom more ſwiftly than Lead, but it is the only thing that will ſink in it, all other Metals and Stones moving upwards therein, and floating on its Surface; whereas between Balls

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of Gold, Lead, Braſs, Porphyry, or any other heavy Matter, the Inequality of their Motion in the Air ſhall be almoſt wholly inſenſible, ſo that indeed a Ball of Gold, falling from an Height of an hundred Yards, in the End of its Fall does not out-ſtrip one of Wax four Inches: And this being ſo, I have thought that if the Reſiſtance of the *Mediums* be wholly taken away, all Matter would deſcend with equal Velocity.

SIMP. How, *Salviatus*! You'll ne'er get me to believe, I think, that a Lock of Wool, and a Piece of Lead, would, even in a *Vacuum*, fall with the ſame Celerity.

SALV. Fair and ſoftly, *Simplicius*: Your Scruple is not ſo abſtruſe, nor I ſo haſty, as you think, in my Aſſertion, as not to be able to juſtify it, and to anſwer your Doubt. That you may underſtand my Explanation, pray give me the Hearing.

The Buſineſs we are now upon is to examine what would befal Moveables exceedingly different in Gravity, in a *Medium* of no Reſiſtance, ſo that all the Difference of Velocities, which is found between the ſaid Moveables, ought to be referred to the ſole Inequality of Gravity: And becauſe only a Space altogether void of Air, and of every other the fineſt and moſt yielding Body, would be fit ſenſibly to ſhew us what we ſeek, and ſince we want ſuch a void Space, let us obſerve what happens in the more ſubtile and leſs-reſiſting *Mediums*, and compare them with what we find happens in others leſs ſubtile and more reſiſting: For if we ſhould really find Moveables different in Gravity, to differ leſs and leſs in their Velo-

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cities, according as the *Mediums* are leſs and leſs reſiſting; and if at length that tho' they are extremely different in Gravity, the Difference of Velocities is found next to nothing, and in a manner inobſervable, when they move in a *Medium* more ſubtile than any other, tho' not in a *Vacuum*, we may, methinks, reaſonably conjecture that in a *Vacuum* their Velocities would be exactly equal: Let us conſider, therefore, what happens in the Air; and that we may have a Figure of an uniform Superficies, and of very light Matter, let us take a blown Bladder, in which the included Air, in a *Medium* of the Air itſelf, will weigh little or nothing, ſince therein it cannot much compreſs itſelf; ſo that all its Gravity conſiſts only in that of the little Skin, which is not a thouſandth Part of the Weight of a Lump of Lead of the ſame Bigneſs with the blown Bladder. Now if theſe two, *Simplicius*, be let fall from the Height of four or ſix Yards, how far do you think would the Lead out-ſtrip the Bladder in the Fall? You may aſſure yourſelf that that would not fall three times, no nor twice, ſo faſt as this, altho' you would have that it falls a thouſand times as ſwift.

SIMP. It may poſſibly happen, that at the Beginning of their Fall, *i. e.* in ſo ſmall a Length as four or ſix Yards, it may be as you ſay; but in a long Continuation, I dare ſay, the Lead would leave the Bladder behind ſix, eight, if not ten Parts in twelve.

SALV. I believe ſo too, not doubting but that from very great Heights the Lead will have fallen a hundred Miles, before the Bladder has fell one: But this Effect which

which you propoſe, is ſo far from contradicting my Propoſition, that 'tis the greateſt Confirmation of it imaginable. My Intent is (to repeat it once more) to explain to you, that in Bodies of different Gravities the Cauſe of their different Velocities does in nowiſe conſiſt in their different Gravities, but depends on exterior Accidents, and indeed, in particular, on the Reſiſtance of the *Medium* they fall thro', ſo that this Reſiſtance being taken away, all Moveables fall with the ſame Degrees of Velocity. And this I deduce chiefly from what you have juſt now admitted, and which is very true, namely, that of Moveables very different in Gravity, the Velocities differ more and more, according as the Lines of their Fall are greater and greater; which Effect would by no Means follow, if they depended on the different Gravities: For the Gravities being always the ſame, the Proportions betwixt the Lines of Deſcent would then alſo be always the ſame: But now we ſee this Proportion in the Continuation of the Fall conſtantly to increaſe, inaſmuch as the heavieſt Moveable in a Fall of one Yard will not leave the lighteſt behind the tenth Part of a Yard; but in a Fall of twelve Yards, it will leave it four Yards; and in one of a hundred Yards, it will leave it behind ninety Yards.

SIMP. Very well: Then (to make uſe of your own Way of arguing) if the Difference of Gravity in Moveables of different Gravity, does not cauſe the Difference of Proportion in their Velocities, their Gravities continuing the ſame; then neither can the *Medium*, which, in our Caſe, is ſuppoſed always to continue the ſame, cauſe any Alteration in the Proportion of the Velocities.

SALV. Seem-

SALV. Seemingly you have contradicted me; but I'll endeavour to set this right, before I proceed to any thing else. I say then, that every heavy Body has by Nature an intrinsick Principle of tending towards the common Centre of heavy Things; namely, to that of our Terrestrial Globe, and that with a Motion continually accelerated, and indeed always equally accelerated, so as in equal Times are equally added new Moments and Degrees of Velocity: And this must be understood to hold true when all accidental and external Impediments are remov'd, amongst which there is one we can't remove, I mean the Impediment of a full *Medium*, which must be cut and mov'd aside by the falling Body; to which transverse Motion the *Medium*, tho' yielding and calm, opposes itself with a Resistance at one time less, at others greater and greater, according as it is more slowly or hastily to separate or open to give Passage to the Moveable, which because, as I have said, it of its own Nature moves with a continual Acceleration, consequently it continually encounters greater Resistance in the *Medium*, and therefore meets with Hinderances and Letts, in acquiring new Degrees of Velocity; so that at length the Velocity arrives to that Pitch, and the Resistance of the *Medium* to that Strength, that ballancing each other they take away all further Acceleration, and bring the Moveable to an equable and uniform Motion, with which it afterwards continually goes on: There is therefore in the *Medium* an Augmentation of Resistance, not that it changes its Essence, but because the Velocity is chang'd, wherewith it ought to open

open and be mov'd laterally, to give Passage to the falling Body, which is continually accelerated.

Since now the Resistance of the Air to the *Momentum* or *Impetus* of the Bladder is very great, and to the great Weight of the Lead but very little, I am induc'd to conclude for certain, that if this Resistance should be wholly removed, (a great Advantage to the Bladder, and but little to the Lead) their Velocities would become equal; Taking this Principle therefore for granted, that in that *Medium* wherein either because of a *Vacuum* or some other Reason, there is no Resistance to abate the Velocity of the Motion, so that the Velocities of all Moveables become equal; we might aptly enough assign the Proportion of the Velocities of Moveables in the same, and in different, full, and therefore resisting, *Mediums*. And this we may obtain by considering how much the Gravity of the *Medium* takes from the Gravity of the Moveable, which Gravity is the Instrument with which the Moveable makes its Way, by forcing the Parts of the *Medium* aside: an Operation that can't take Place in a *Vacuum*, or empty *Medium*; wherefore no Difference is to be expected from their different Gravities: And because 'tis manifest that the *Medium* takes away from the Gravity of the Body therein contain'd, as much as is the Weight of such another Mass of the *Medium's* own Matter; if now in such Proportion we diminish the equal Velocities of Moveables, which (as it has been suppos'd) they would have in a non-resisting *Medium*, we shall obtain our Desire. Thus, for Example, suppose Lead to be ten thousand times heavier than Air, and Ivory only a thousand times; Now as to the Velocities of these two Matters, which, absolutely taken

taken, that is, all Reſiſtance taken away, would be equal, the Air would take from the ten thouſand Degrees of the Lead one Degree; and from the thouſand Degrees of the Ivory likewiſe one Degree; or, if you will, from its ten thouſand, ten. If therefore the Lead and the Ivory ſhould fall from any Height in the Air, which they would do in the ſame Time, all Reſiſtance of the Air being taken away, the Air will abate one Degree of the ten thouſand of Velocity of the Lead: But of the ten thouſand Degrees of Velocity of the Ivory, it will abate ten, that is, that Altitude from which the Moveables fall, being divided into ten thouſand Parts, the Lead will come to the Ground, leaving the Ivory beind ten, or at leaſt nine Parts of thoſe ten thouſand Parts. And what elſe is this, I pray, than that a Ball of Lead falling from a Tower two hundred * Cubits high, ſhall out-ſtrip an Ivory one leſs than four Digits: The Ivory weighs, ſuppoſe, a thouſand times more than Air, but the Bladder ſo blown but four times more; wherefore the Air takes away one Degree of a thouſand from the intrinſick and natural Velocity of the Ivory; but from that of the Bladder (which taken abſolutely would have been the ſame with that of the Ivory) the Air takes one Part of four: When therefore the Ball of Ivory, falling from the Tower, reaches the Ground, the Bladder has fall'n but three Quarters of the Way down. Lead is twelve times heavier than Water, but Ivory only twice as heavy; the Water therefore, from their abſolute Velocities which would be equal, takes from that of the

* One Cubit = 1 Foot 6 Inches. 1 Digit = 0 Foot 0 Inches $\frac{3}{4}$.

Lead

Lead a twelfth Part, but from that of the Ivory the half; When therefore the Lead ſhall have ſunk in the Water eleven Parts, the Ivory ſhall have ſunk but ſix: And following this Rule, we ſhall find, I believe, that ſuch a Computation will be more agreeable to Experience than that *Ariſtotle* has given us.

And by the like Method we may find the Proportion between the Velocities of the ſame Moveable in *different* fluid *Mediums*, not by comparing the different Reſiſtances of the *Mediums*, but by conſidering the Exceſs of the Gravity of the Moveable above the Gravities of the *Mediums*: Thus, for Example, Tin is a thouſand times heavier than Air, and ten times heavier than Water; wherefore the abſolute Velocity of Tin divided into a thouſand Degrees it ſhall move in the Air which takes away a thouſandth Part from it, with Nine hundred ninety-nine Degrees, but in the Water with Nine hundred only, the Water taking off the tenth Part of its Gravity, and the Air the thouſandth.

Again, ſuppoſe a Solid ſomething heavier than Water, for Inſtance, Oak; and let a Ball of it weigh, *e. g.* a thouſand Drachms, and let an equal Maſs of Water weigh Nine hundred and fifty, and an equal Maſs of Air two: 'Tis manifeſt now, that if we ſuppoſe its abſolute Velocity to be a thouſand Degrees, in Air there would remain only Nine hundred ninety-eight, and in Water only fifty; *viz.* becauſe the Water takes off from the thouſand Degrees of Gravity Nine hundred and fifty, leaving only fifty: Wherefore ſuch a Body will move twenty times as ſwift in Air as in Water, as the Exceſs of its Gravity above that

of the Water is = the twentieth Part of its own *. And here I would have it conſider'd, that ſince in Water no Matter can ſink, unleſs they are ſpecifically heavier than Water, and conſequently many hundreds of times heavier than Air; when we are enquiring into the Proportion of thoſe Velocities in Air and in Water, we may, without ſubjecting our Calculation to any conſiderable Error, conſider the Air as taking nothing at all from the abſolute Gravity, and conſequently nought from the abſolute Velocities of ſuch Matter; wherefore having found with Eaſe the Exceſs of their Gravities above the Gravity of Water, we may ſay that their Velocities in the Air, to their Velocities in Water, have the ſame Proportion as their whole Gravity has to the Exceſs of this above the Gravity of Water: For Example, If a Ball of Ivory weigh twenty Ounces, and an equal Maſs of Water weigh ſeventeen Ounces; then the Velocity of the Ivory in the Air, to its Velocity in Water, will be very nearly as 20 to 3.

SAGR. I have long buſied myſelf in a Matter of itſelf curious enough, wherein, tho' without much Benefit, I have often thought till I've been weary; tho' I muſt own there's nothing wanting to reduce theſe Speculations of yours to Practice, could we but find what Gravity the Air has, in Compariſon to that of Water, and conſequently to thoſe of other Matters.

* *i. e.* As 1000 — 950 = 50 is = 1000 ÷ 20 = 50.

SIMP. But

SIMP. But if we ſhould find that the Air, inſtead of Gravity, has Levity; what muſt we ſay of the foregoing Diſcourſes, otherwiſe very ingenious?

SALV. Why, if ſo, then what I have been ſaying may be truly accounted airy, light, and vain. But can you doubt that the Air has Weight, when *Ariſtotle* expreſly affirms it? Who ſays, that all the Elements, even the Air itſelf, has Gravity; and ſubjoins this as a Proof, *viz.* a * Bladder after it's blown, weighs more than before.

SIMP. I don't believe that the Gravity of a blown Bladder proceeds from the Gravity that is in the Air, but from the very many thick Vapours intermix'd with it, in theſe our lower Regions: What if I ſhould ſay, that hence may be deduc'd the additional Gravity of the Bladder?

SALV. I would not have you ſay ſo, much leſs introduce *Ariſtotle* as ſaying ſo; becauſe if he, (ſpeaking of the Elements) in order to perſuade me that the Element of Air has Weight, and in order to prove this by Experiment, ſhould bid me take a Bladder and fill it with groſs Vapours, and I ſhall find the Weight of the Bladder will be encreaſed; I might anſwer him thus, That the Bladder would weigh more if he filled it with Bran; and might tell him farther, That theſe Experiments prove indeed that Bran and groſs Vapours have Weight; but as to the Element of Air, I ſhould ſtill be in the ſame Doubt. The

* Un Otro Gonfiato.

Experiment therefore of *Ariſtotle* is good, and his Propoſition true. But I can't ſay ſo much of another Reaſon I have met with in another Philoſopher, whoſe Name I have forgot; but I am ſure I have read it: who argues, that the Air is rather heavy than light, becauſe it more eaſily preſſes heavy Bodies downwards, than it lifts the light ones upwards.

SAGR. Very good, truly! So, according to this Way of Reaſoning, the Air muſt be much heavier than Water, ſince all heavy Bodies are carried more eaſily downwards thro' the Air than in Water; and all light ones more eaſily upwards in This than in That; nay, infinite Matters aſcend in Water which in Air deſcend. But to go on: The Weight of the Bladder, *Simplicius*, whether it be aſcrib'd to groſs Vapours, or to pure Air, is nothing to our Purpoſe, who are ſeeking what happens to Moveables, that are moved in this our vaporous Region. To return then to that which more concerns me, that I may be fully and abſolutely inſtructed in this Matter, I would be throughly aſſured, not only that the Air has Gravity, which I believe it has, but I would, if poſſible, know what Gravity it has. Therefore, *Salviatus*, if you can ſatisfy me in this Point, I beg you will.

SALV. That the Air has poſitive *Gravity*, and not, on the contrary, as ſome have thought, *Levity*, which perhaps is found in no Matter whatſoever, the Experiment of the * blown Bladder produc'd by *Ariſtotle* demonſtrates

* Del Pallone Gonfiato.

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to us; for if the Quality of abſolute and poſitive Levity was in the Air, then the Air being multiplied, if I may ſo ſay, and compreſſed, its Levity would, by ſo doing, increaſe, and conſequently its Propenſity to aſcend: but Experience ſhews the contrary.

As to the other Part of the Queſtion, *viz.* To find what the Gravity of the Air is; I'll tell you the Method I have taken to do it. I took a pretty big Glaſs Bottle, with a narrow Neck, and tied very cloſe to its narrow Neck a Leathern Cover; to this Cover, and within Side, I put a Valve; thro' this with a Syringe I forced a great Quantity of Air, of which, becauſe it admits of great Condenſation, twice or thrice as much may be forced in as the Bottle naturally holds: Then I very carefully weigh'd in a moſt exact Ballance the Bottle with the compreſſed Air within it, adjuſting their Weight by very fine Sand; then opening the Valve, I let out the Air which was violently contain'd in the Veſſel; I put the Bottle again into the Scales, and finding it much lighter than before, I took out of the other Scales ſo much Sand (keeping it by itſelf) until the Sand and the Bottle were in *Equilibrio*: Now there can be no Room to doubt but that the Weight of the Sand taken out is = the Weight of the Air which was violently forced into the Bottle, and which afterwards was let out.

But this Experiment aſſures me of no more than this, *viz.* that the Weight of the Air violently compreſſed in the Bottle, is equal to the Weight of that reſerved Sand: But I have not yet determin'd how much the Air abſolutely weighs in reſpect of Water, or any other heavy Matter; nor can I know this, unleſs I meaſure the Quantity of the

compreſſed

compreſſed Air; which may be done by either of the two Ways following.

The former is this: Take ſuch another Bottle, with a Neck exactly of the ſame Size with that of the former; round which Neck tie very faſt another Leather, the other End of which tie alſo cloſely over the former Bottle's Neck. Now the Bottom of this ſecond Bottle muſt be drill'd or bor'd thro', ſo that thro' the Hole a Wire may be put, wherewith at Pleaſure the Valve of the former Bottle may be open'd, to let out the ſuperfluous Air after it hath been weigh'd: But now this ſecond Bottle muſt be fill'd with Water. All things thus prepar'd, and the Valve open'd by Help of the Wire, the Air iſſuing out with Impetuoſity, and entering the Bottle of Water, ſhall drive the Water out by the Hole at the Bottom. Now 'tis manifeſt that the Quantity of Water forc'd out in this manner, is = the Bulk and Quantity of Air that iſſued out of the other Bottle; Wherefore keeping that Water, let the Bottle, now lighten'd of the compreſſed Air, be again weigh'd, (for I ſuppoſe it to have been weigh'd before, together with the compreſſed Air) and then 'tis manifeſt that the ſuperfluous Sand, reſerv'd as before order'd, is exactly equal to the Weight of ſuch a Maſs of Air, as is the Maſs of Water forc'd out and preſerv'd, which, if weigh'd, we ſhall ſee how many times its Weight ſhall contain the Weight of the reſerv'd Sand; and we may ſafely affirm, that the Water is ſo much heavier than the Air, which will not be only ten times, as ſeems to be *Ariſtotle*'s Opinion, but nearly 400 times, as this Experiment ſhews us.

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The other Method is more expeditious, and by making uſe of one Veſſel only; and that is the former fitted as before directed, excepting that I would have no more Air in it than what is naturally in it; but I would have Water injected without letting out the leaſt Air, which being forced to give Way to the ſupervenient Water, muſt of Neceſſity be compreſs'd: Having gotten in as much Water as poſſible, (which without much Force may be about three Fourths of what the Bottle will hold) put it into the Scales, and weigh it very exactly; which done, holding the Veſſel with the Neck upwards, and opening the Valve, let the Air out, of which there will iſſue ſuch a Quantity as there is Water in the Bottle. The Air being gone out, put the Veſſel again into the Scales, which, becauſe of the Air let out, will be found lighter; and taking out of the other Scale the Over-weight, it will give us the Weight of ſuch a Maſs of Air as is that of the Water in the Bottle.

SIMP. I muſt confeſs your Contrivances are very ſubtile and ingenious: But whilſt, on one Hand, they ſeem throughly to ſatisfy my Underſtanding, yet, on the other, they really confound me. For it being undoubtedly true, that the Elements in their proper Regions are neither light nor heavy; I can't conceive how and whence that Portion of Air, which weigh'd, for Inſtance, four Drachms of Sand, ſhould afterwards have that ſame Gravity in the Air, in which the Sand, which is weigh'd againſt it, eaſily retains it; Wherefore methinks this Experiment ought not to be made in the Element of Air, but in ſome *Medium*

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where the Air itſelf might exerciſe its Quality of Gravitation, if it really have ſuch Quality.

SALV. A ſmart Objection, I muſt own, *Simplicius*, which ſhould be throughly anſwer'd, or own'd unanſwerable. That that Air which, compreſſed, I demonſtrated to weigh as much as that Sand, when ſet at Liberty in its Element, does no longer gravitate, as, on the contrary, the Sand does, is a thing moſt manifeſt: Wherefore to make ſuch an Experiment, we muſt chuſe a Place and *Medium* where the Air, as well as the Sand, may do ſo; becauſe, as has been often ſaid, the *Medium* takes off from the Weight of every Matter immers'd therein, ſo much as is the Weight of a Maſs of the ſame *Medium*, which is = the immerſed Maſs; ſo that I can't deny but that the Air depriveth the Air of all its Gravity. The Experiment therefore ought to be tried in a *Vacuum*, where all Bodies would exerciſe their Faculties without any Diminution: Wherefore, *Simplicius*, if we weigh any Portion of Air in a *Vacuum*, would you then be convinc'd in this Matter?

SIMP. Yes, truly; but this is to deſire or expect an Impoſſibility.

SALV. Surely then you'll own yourſelf highly oblig'd to me, if for your Sake I do that which is impoſſible: But I won't offer to ſell now what I have already given you: Know then, that in the foregoing Experiment we actually weigh'd Air in a *Vacuum*, and not in Air, or any other full *Medium*. But that from the Gravity, *Simplicius*, of a Maſs immers'd in a fluid *Medium*, is

t

is taken ſomething by that *Medium*, comes to paſs from hence, that it reſiſts its being open'd and driven aſide, and at length its being rais'd and held up, as we plainly ſee by its Proneneſs to return and fill the Space up again, which the immerſed Maſs occupied, as ſoon as ever the Maſs is taken out, when otherwiſe, if that *Medium* ſuffer'd nothing from ſuch Immerſion, it would never operate againſt, or contrary to it.

But now when you hold in the Air, the aforeſaid Bottle fill'd with the Air which is naturally contain'd therein, tell me, pray, what Diviſion, Impulſe, or, in a Word, what Mutation the external ambient Air receives from that ſecond Air, which was newly forc'd into the Veſſel? Doth it enlarge the Bottle whereby the ambient Air muſt recede to give it Place? No, certainly. Therefore we may certainly ſay, that that ſecond Air is not immerſed in the ambient Air, it not occupying any Space therein, but is as if in a *Vacuum*: nay, it is really ſo, interpoſing itſelf in Vacuities, not quite fill'd up with the former uncondenſed Air. And I know not how to diſcern any Difference between the two Conſtitutions of the Incloſed and Ambient, becauſe in this the Ambient does no-ways preſs the Incloſed, and in that the Incloſed doth not in the leaſt oppoſe the Ambient: And ſuch are the Diſpoſitions of any Matter *in Vacuo*, and of the ſecond Air compreſſed in the Bottle: Wherefore the Weight which is found in ſuch condens'd Air is the ſame, which that ſame Air would have, were it freely diſtended in a *Vacuum*. 'Tis true, indeed, that the Weight of the Sand which is weigh'd againſt it, inaſmuch as 'tis in the open Air, would *in Vacuo* weigh ſomething more; ſo that we may ſay that

the Air weigh'd is in reality ſomething lighter than the Sand which counterpoiſes it, *i. e.* ſo much as an equal Maſs of Air would weigh *in Vacuo*.

SIMP. I really thought that there had been ſomething wanting in the foregoing Experiment; but now I am throughly ſatisfied.

SALV. The Things I have hitherto alledg'd, and particularly this, That the Difference of Gravity, tho' never ſo great, has not the leaſt to do in altering the Velocities of Moveables, but that all things would move with equal Celerity, let their Difference of Gravity be what it will, is ſo new, and, at the firſt thinking, ſo remote from all Probability, that could I not make it apparent, and as clear as the Sun itſelf, I had better never have mention'd it: Wherefore ſince I have mention'd it, I think I ſhould omit neither Experiment nor Reaſon to eſtabliſh it.

SAGR. Not only this, but many other alſo of your Propoſitions, are ſo remote from the Opinions and Doctrines commonly receiv'd, that if they were publiſh'd they would gain you many Adverſaries; the innate Diſpoſition of Men being ſuch, that they look with an envious Eye upon ſuch as diſcover Truth or Fallacies which they themſelves could not find out, calling them Innovators of Doctrines, a Character but few like; endeavouring thus to cut the Knots they can't unty, and with ſubterraneous Mines to blow up thoſe Structures which have been erected with ordinary Tools by induſtrious Architects: But with us here, who are far from any ſuch Thoughts, your

your Experiments and Arguments are fully ſatisfactory: But nevetheleſs if you can produce any Experiments more palpable, or Arguments more convincing, we would gladly receive them.

SALV. That Experiment made with two Moveables as different as may be in Weight, by letting them fall from a Place on high, in order to obſerve whether their Velocities are equal, is ſubject to ſome Difficulty; becauſe if the Height be great, the *Medium* which muſt be open'd, and laterally repell'd by the *Impetus* of the falling Body, will more reſiſt the ſmall *Momentum* of a very light Moveable, than the Violence of a very heavy one; whence in a long Fall the heavy one will leave the light one behind; and in a ſhort Fall there is either in Reality no Difference, or if any, 'tis next to inobſervable. Wherefore I have often been thinking to repeat that Deſcent from a ſmall Height ſo often, and to collect together thoſe very ſmall Differences of Times which are betwixt the heavy Body and the light one's coming to the Ground, until theſe ſo conjoined would make a time not only obſervable, but very eaſily ſo.

Moreover, that I might ſubject to Obſervation, Motions as ſlow as poſſible, in which the Reſiſtance of the *Medium* operates leſs as to the Change of the Effect, that depends on ſimple Gravity, I reſolv'd to let go the Moveables upon a declining Plane, not much rais'd above the Plane of the Horizon; for upon this, no leſs than perpendicularly, we may find that which is done by heavy Bodies different in Weight: And proceeding farther, that I might free myſelf from any Impediment which might ariſe from the Contact of thoſe Moveables, and the declining Plane, I took two

Balls, one of Lead, and the other of Cork; That above a hundred times heavier than This, and faſten'd them on high with two equally fine Threads, each equally four or five Yards long; then moving each Ball from their perpendicular State, I let them both go at the ſame Moment; and they deſcending by the Circumferences of Circles deſcrib'd by the equal Threads as Semidiameters, preſently return the ſame Way back, till at length it doth, in nearly a hundred Vibrations to and fro, ſenſibly ſhew that the heavier Ball moves ſo exactly in the time the lighter one does, that it does not in a hundred, no nor in a thouſand Vibrations out-run it by one Moment of Time, but they keep moving by equal Paces: and here may be noted the Operation of the *Medium*, which in ſome meaſure hindring the Motion, doth much more diminiſh the Vibrations of the Cork than of the Lead, not that it makes more or leſs of them in Number; but altho' the Arches run thro' by the Cork are not above five or ſix Degrees, but thoſe deſcrib'd by the Lead are fifty or ſixty Degrees, yet they both deſcribe them in the ſame time.

SIMP. If this be ſo, how can it be that the Velocity of the Lead is not greater than that of the Cork, when that runs a Courſe of ſixty Degrees, in the ſame time that this runs barely ſix?

SALV. But what would you ſay, *Simplicius*, if they ſhould vibrate in the ſame time, when the Cork removed thirty Degrees from its Perpendicular, ſhould paſs over an Arch of ſixty Degrees; and the Lead removed from the ſame Point only two Degrees, ſhould paſs over an Arch

Arch of four Degrees; would not then the Cork be ſo much ſwifter than the Lead? And yet Experience ſhews this to be the Caſe: Pray obſerve, therefore, that the *Pendulum* of Lead being mov'd, *v. g.* fifty Degrees from the Perpendicular, and then let go, it ſwings nearly fifty Degrees beyond the Perpendicular, deſcribing an Arch in the whole of nearly an hundred Degrees; and then returning of itſelf back again, it deſcribes an Arch a little leſs; and ſo by continually repeating thoſe Vibrations, after a great Number have been made, it at length is reduced to reſt. Now all theſe Vibrations are made in the ſame time, as well thoſe of ninety Degrees, as thoſe of fifty, twenty, ten, or four; and conſequently the Velocities of the Mover conſtantly leſſen, becauſe in equal times it doth ſucceſſively run thro' Arches leſſer and leſſer.

The like way the ſelf-ſame Effect is produc'd by the Cork hanging by a String of the like Length, excepting that after a leſs Number of Vibrations it returns to Reſt, as being, on account of its Levity, leſs fit to overcome the Reſiſtance of the Air; and yet all the Vibrations great and ſmall, are made in equal times, which are alſo equal to the times of the Vibration of the Lead: Wherefore it is true, that if whilſt the Lead runs thro' an Arch of fifty Degrees, the Cork moves thro' one of but ten Degrees, the Cork is then ſlower than the Lead: But, on the other hand, it will alſo happen, that the Cork moveth thro' an Arch of fifty Degrees, whilſt the Lead runs thro' an Arch of ten or ſix; and ſo the Lead ſhall be ſwifter one while than the Cork, and at another the Cork ſwifter than the Lead: But if the ſame Moveables ſhall alſo in the ſame

times

times move thro' equal Arches, we may, without any Scruple, conclude their Velocities to be equal.

SIMP. This Diſcourſe of yours ſeems to me at the ſame time to demonſtrate the thing, and not to demonſtrate it: And I find ſuch a Confuſion in my Mind, ariſing from the ſame Moveables being mov'd one while ſwiftly, another ſlowly, and another while extremely ſlowly; that I own I'm in Doubt whether it be true or no that their Velocities are always equal.

SAGR. Pray, *Salviatus*, let me put in a Word or two, and tell me, *Simplicius*, whether you grant that it may be ſaid with abſolute Verity, that the Velocities of the Cork and Lead are equal, in caſe that both of them departing at the ſame Moment from Reſt, from the ſame Diſtances from the Perpendicular, do run over equal Spaces in equal Times?

SIMP. This admits of no Doubt; nor can it be contradicted.

SAGR. It happens now in the *Pendulums* that each of them moves thro', now ſixty Degrees, now fifty, now thirty, now ten, eight, four and two Degrees: And when each of them moves over an Arch of ſixty Degrees, they do it in the ſame time, and each Moveable ſpends the ſame time in paſſing thro' an Arch of fifty Degrees; and ſo on in Arches of thirty, ten, and the reſt Degrees: Whence we may conclude, that the Velocity of the Lead in the Arch of ſixty Degrees, is equal to the Velocity of the

the Cork in the ſame Arch of ſixty Degrees: And that the Velocities of each in the Arches of fifty Degrees, are alſo equal to each other, and ſo on in the reſt. But it cannot be ſaid that the Velocity which is exerciſed in an Arch of ſixty Degrees, is equal to that obſerv'd in the Arch of fifty Degrees, nor this equal to that which is an Arch of thirty: But in leſſer Arches the Velocities will always be leſs, as is collected from our ſeeing the ſame Moveable, as to Senſe, ſpend exactly the ſame time in running thro' the greater Arch of ſixty Degrees, as it requires to run thro' the leſſer Arch of fifty, or a leſs of ten Degrees; and, in a Word, in their being all run thro' always in equal times. It is therefore true, that both the Lead and Cork continually retard their Motions according to the Diminution of the Arches, but that their Harmony is not alter'd in keeping an Equality of Velocity thro' all the ſame Arches run thro' by them.

Thus much I thought fit to ſay, more that I might hear whether I have rightly underſtood *Salviatus*'s Meaning, than that I imagined *Simplicius* to ſtand in need of any clearer Explication than what *Salviatus* gave; which is (as indeed all that he advances are) as clear as poſſible; and when he often reſolves Queſtions, not only apparently obſcure, but even ſeemingly repugnant to Nature and Truth, he does it by Reaſons, Obſervations, or Experiments ſo common and familiar to every one, that it has given Occaſion to one of the moſt celebrated Profeſſors of our Age, to eſteem leſs his Novelties as depending on too mean and common Fundamentals; as if the moſt admirable and moſt valuable Property of the demonſtrative Sciences, was not their ariſing and flowing from moſt known Principles,

ſuch as every Perſon underſtands and grants. But let us go on and feaſt upon this Diet ſo light of Digeſtion. And ſuppoſing *Simplicius* to be convinced, that the internal Gravity of different Moveables has no Part in altering their Velocities, but that all Moveables, for any thing that has to do with it, would move with the ſame Velocities; tell us, *Salviatus*, in what you place the ſenſible and apparent Inequalities of Motion, and pray anſwer to that Inſtance which *Simplicius* produces, and which I agree with him in; I mean when we ſee a Cannon Ball move more ſwiftly than a Bird Shot, that that Difference of Velocities is but ſmall, in reſpect of that which I object againſt you, in Moveables of the ſame Matter, of which ſome of the greater will deſcend in one Second that Length which others will not run thro' in the ſame *Medium* in one, in four, or in twenty Hours; ſuch are Stones and ſmall Grains of Sand, eſpecially that very fine Sand which makes Water muddy: In which *Medium* it will not deſcend two Yards in many Hours, which Stones, and thoſe not very large ones, will paſs thro' in one Second of Time.

SALV. What Effect the *Medium* produces in retarding Moveables, more according as they are amongſt themſelves of leſs ſpecifick Gravity, has been already declared, by ſhewing it to be done by Subtraction or Diminution of Weight: But to explain how the ſame *Medium* can with ſo much Difference diminiſh the Velocity in Moveables, differing only in Magnitude, altho' they are of the ſame Matter and Figure, requires a more ſubtile Diſcourſe than that which ſuffices to make us underſtand how the more dilated

dilated Figure of a Moveable, or the contrary Motion of the *Medium* to the Moveable, retards the Velocity of the ſaid Moveable.

And for my Part I take the Reaſon of the preſent Problem to be the Scabroſity and Poroſity, which is commonly and generally found in the Surfaces of ſolid Bodies, and which, whilſt the Body keeps moving, continually puſh and thruſt againſt the Air, or other ambient *Medium*: An evident Sign of which we have when we hear Bodies, tho' made as round as poſſible, to hum as they paſs ſwiftly thro' the Air; nay, which indeed we hear not only hum, but alſo to hiſs or whiſtle, if any more than ordinary Cavity or Prominence be found in them. Thus all round Bodies, in their turning round, we ſee excite a little Wind. But what need of many Words? Don't we hear a *Top* to make a notable humming, and in a very acute Tone, whilſt it turns round on the Ground the ſwifteſt? The Acuteneſs or Shrilneſs whereof becomes continually flatter or lower as the Velocity of its Whirling about doth gradually ſlacken, which is an Argument that the Scabroſities, tho' very ſmall on their Surfaces, do neceſſarily encounter and ſtrike againſt the Air. Thus 'tis not to be doubted but theſe, in the Deſcent of Moveables, rubbing againſt the fluid *Medium*, muſt retard their Velocities, and ſo much the more, by how much the Surface is greater; as are the Surfaces of leſſer Solids when compared with thoſe of greater.

SIMP. Hold a little, I pray, for here I find myſelf at a Loſs; for tho' I underſtand and grant that that mutual Confrication of *Medium* and Surface muſt retard the

Motion of the Moveable, and more ſo where, *Cæteris Paribus*, the Surface is greater; yet I don't comprehend upon what Grounds it is that you aſſert, that the leſſer Solids have the greater Surfaces. And furthermore, if, as you affirm, the greater Surface infers the greater Retardation, the greater Solids, I ſhould think, ſhould be the ſlower; the contrary of which is true: But this may be eaſily ſolv'd by ſaying that altho' a greater Solid has a greater Superficies, yet it hath alſo a greater Gravity, and therefore the Impediment which the greater Surface oppoſes to the greater Gravity, does not exceed the Impediment which the leſſer Surface objects to the leſſer Gravity; ſo that the Velocity of the greater Solid becomes not leſs. Therefore I ſee no Reaſon why the Equality of the Velocities ſhould be alter'd, ſince ſo much as the moving Weight is diminiſh'd, ſo much the Faculty of the retarding Surface is alſo diminiſh'd.

SALV. I'll anſwer all your Objections at once. You'll admit, *Simplicius*, without Controverſy, that if of two equal Moveables of the ſame Matter, and alike in Figure, (which Bodies, without doubt, would move with equal Velocity) as well the Gravity as Surface of one of them be diminiſhed, (retaining their Likeneſs in Figure) that yet the Velocity does not decreaſe in that which is diminiſh'd.

SIMP. I think it ought to follow as you ſay, ſtill having regard to our Doctrine, which teaches that neither the greater nor the leſſer Gravity operates at all in the Acceleration or Retardation of Motion.

SALV. Right;

SALV. Right; whence it ſeems to me to follow, that if the Gravity diminiſhes more than the Surface, in a Moveable ſo diminiſhed there may be ſome Retardation of Motion introduc'd, and that greater and greater, by how much in Proportion the Diminution of the Weight was greater than the Diminution of the Surface.

SIMP. I don't at all queſtion it.

SALV. Now you muſt know, *Simplicius*, that the Surfaces of Solids can't be diminiſhed ſo much as the Weights, provided they keep the ſame Likeneſs of Figure: For it being manifeſt, that when a heavy Body is diminiſhed, it loſes as much of its Weight as of its Bulk, whenever the Bulk is diminiſhed more than the Surface, (the Similitude of Figure being retain'd) the Weight is diminiſhed alſo more than the Surface. But *Geometry* teacheth us, that the Proportion in like Solids, between Bulk and Bulk, is greater than that between Surface and Surface: which, that you may the better ſee thro', I will explain by the following Example.

Conceive a Cube, the Length of each of whoſe Sides is two Inches, one of its Faces then will be four ſquare Inches; and thence all the ſix Faces, that is the whole Surface of the Cube will be twenty-four ſquare Inches. If then you imagine the ſame Cube to be cut by three Sections into eight leſſer Cubes, the Length of the Side of any one of them will be one Inch, and each of its Faces one ſquare Inch; and therefore the whole Superficies of each of theſe will be ſix ſquare Inches; of which the

whole Cube contained in its Superficies twenty-four. Hence you ſee now, ſince the ſuperficial Content of the leſſer Cube is a fourth Part of the Surface of the greater, (ſix being the fourth Part of twenty-four) but the ſolid Content of the ſaid Cube is only the eight Part; that therefore the Bulk, and conſequently the Weight, is much more diminiſhed than the Superficies.

But now if you ſubdivide that Cube into eight others, for the whole Surface of one of theſe we ſhall have one ſquare Inch and a half, which is but the ſixteenth Part of the Surface of the firſt Cube; but its Bulk is only the ſixty-fourth Part of the Bulk of that. You ſee, therefore, how that in only theſe two Diviſions the Bulks are diminiſhed four times more than their Surfaces are: And if we ſhould proſecute this Subdiviſion until the firſt Solid be reduced into fine Duſt, we ſhould find the Gravity of the moſt minute Particles to be diminiſhed above a hundred times more than their Surfaces.

And this which I have exemplified in Cubes, happens in all like Solids, the Bulks of which are in a *ſeſquialteral* Proportion of their Surfaces. You ſee therefore in how much a greater Proportion the Impediment which ariſes from the mutual Contact of the Moveable's Surface and the *Medium*, increaſes in leſſer Moveables than in greater: And if we add that the Scabroſities in the very ſmall Surfaces of thoſe Corpuſcles are not, may be, leſs than thoſe which are in the Surfaces of greater Solids, which are carefully poliſh'd; you ſee how fluid that *Medium* muſt be, to give Paſſage to ſo feeble a Power. Obſerve therefore, *Simplicius*, that I did not equivocate when juſt now

I ſaid that the Surfaces of leſſer Solids, are greater in Compariſon to thoſe of greater Solids.

SIMP. I am throughly ſatisfied, and, believe me, were I to begin my Studies again, I would follow *Plato*'s Advice, and begin with Mathematicks, which I ſee proceed with the utmoſt Caution, and refuſe to admit any thing as certain but what they demonſtrate.

SAGR. This Diſcourſe has given me great Pleaſure; but before we go any farther, I ſhould be glad to underſtand a *Term* you us'd, *Salviatus*, juſt now, when you ſaid that ſimilar Solids are to one another in a *ſeſquialteral Ratio* of their Surfaces: I have, indeed, ſeen and underſtood that Propoſition and its Demonſtration, wherein 'tis prov'd that the Surfaces of ſimilar Solids are in a *duplicate Ratio* of their Sides; and that other alſo which proves the ſame Solids to be in a *triplicate Ratio* of the ſame Sides: But I don't remember that I ever till now heard ſo much as mention'd the Proportion between Solids and their Surfaces.

SALV. You have anſwered yourſelf, and cleared your own Doubt: for that which is the *Triple* of any thing whereof any other is *Double*, doth not that *Triple* become the * *Seſquialter* of that *Double*? Certainly it does. Now if Surfaces are in a double Proportion of thoſe Lines of

* One and a half: thus in the Numbers 4, 2, and 6; 6 is the Triple of 2, whereof 4 is the Double, then 6 is Seſquialter, or 1 ½ of 4.

which the Solids are in triple Proportion, may not we ſay that Solids are in a ſeſquialteral Proportion of their Surfaces?

SAGR. I underſtand you now very well: And tho' I have ſome other Enquiries to make, relating to ſome particular Matters already treated of, yet by running from Digreſſion to Digreſſion, we ſhould make it too long before we come to the Queſtions principally intended, which concern the Diverſity of Accidents of the Reſiſtances of Solids againſt Fraction. If you pleaſe, therefore, let us reſume the firſt Thread, or what was propoſed at the Beginning of this Diſcourſe.

SALV. Well mention'd. But ſo many and ſuch different Things as we have examin'd, have ſtolen away ſo much of our Time, that there's but little of the Day left for our principal Buſineſs; wherein many Geometrical Demonſtrations are neceſſary, and which muſt be conſider'd with Attention: wherefore I ſhould think it better to adjourn our Meeting till to Morrow, as well on the aforeſaid Account, as becauſe then I might bring with me ſome Papers, wherein I have ſet down in order thoſe Theorems and Problems, wherein the different Paſſions of that Subject are propoſed and demonſtrated, which may be would not in a proper Method come now into my Mind.

SAGR. I very gladly comply with you, and ſo much the more willingly, in that, for a Concluſion to this Day's Conference, I ſhall have Time to hear you explain ſome Doubts which remain with me, concerning the Matter laſt

laſt handled. One of which is this, Whether we are to eſteem the Impediment of the *Medium* ſufficient to aſſign Bounds to the Acceleration of the greateſt and heavieſt Bodies, which are of a ſpherical Form: I inſtance in ſuch as are of a ſpherical Form, as that which is comprehended under the leaſt Surface, and thence the leſs ſubject to Retardation. Another ſhall be touching the Vibrations of *Pendulums*; and this hath many Heads: One ſhall be, Whether all great, mean, and very ſmall, are made really and exactly in equal Times: And another, What the Proportion is of the Times in Moveables, when faſten'd to unequal Strings; I mean the Times of their Vibrations.

SALV. Theſe are Queſtions of great Moment; and, as it happens in all Truths, I doubt not but which ever of them we handle, it will draw after it ſo many other Truths and curious Conſequences, that the Remainder of the Day will hardly ſuffice for the diſcuſſing them all.

SAGR. If they are but as pleaſant as the foregoing ones, I could with all my Heart ſpend as many Days as we have Hours to Night: And I dare ſay *Simplicius* will be no more tired with ſuch Reaſonings than I ſhall be.

SIMP. No indeed; eſpecially when the Queſtions treated on are Phyſical, and which no other Philoſophers have touch'd, or given us their Opinions about.

SALV. To the firſt then: I poſitively affirm that there is no Sphere ſo big, or of Matter ſo heavy, but that the Renitence

Renitence of the *Medium*, tho' never ſo thin, checks its Acceleration, and, in the Continuation of the Motion, reduces it to Equability; an Argument of which we may learn from Experience. For if any falling Moveable, in the Continuation of its Motion, be able to attain to any Degree of Velocity, no Velocity conferred upon it by the external Mover could be ſo great, but it would get rid of and free itſelf from it by means of the Impediment of the *Medium*. Thus a Cannon Ball, that hath deſcended in the Air, *e. g.* four Yards, and hath acquired, for Inſtance, ten Degrees of Velocity, with which it enters the Water; if the Impediment of the Water were not able to reſtrain its *Impetus*, it would acquire a greater Velocity, or with the ſame would move to the Bottom, which yet we don't ſee to follow: But the Water, altho' but a few Fathoms deep, does reſtrain and weaken it, ſo that it hits the Bottom of the River or Lake very gently: 'Tis therefore manifeſt, that that Velocity which the Water had Power to deprive it of, in ſo little a Depth, it would never ſuffer it to acquire again, tho' in the Depth of a thouſand Fathoms: For why ſhould it permit it to acquire that in a thouſand Fathoms, which afterwards it will take from it in four? But what need of more? Don't we ſee the immenſe *Impetus* of the Ball ſhot from the Cannon itſelf to be ſo deaden'd by paſſing thro' a few Fathoms of Water, that, hardly reaching the Ship, it hits it without any manner of Damage? Nay, even the very Air, altho' very yielding, doth nevertheleſs repreſs the Velocity of the falling Moveable, tho' it be very heavy, as we may collect from like Experiments: For if from the Top of a very high Tower, we ſhould diſcharge a Musket downwards, this

this will make a leſs Impreſſion on the Ground, than if it ſhould be diſcharged at the Height of but four or ſix Yards above the Plane; an evident Sign that the *Impetus* wherewith the Bullet ruſhes from the Gun, diſcharged on the Top of the Tower, in its Deſcent thro the Air, is continually diminiſhed: Therefore the Deſcent from any great Height whatſoever, will not ſuffice to make it acquire that *Impetus*, of which the Reſiſtance of the Air depriv'd it, and which had been by any Means whatſoever conferr'd upon it. In like manner I don't believe the Blow of a Bullet, ſhot perpendicularly from an immenſe Height, will be with that Force as it would was the Blow of the ſame Bullet to ſtrike at the Diſtance of twenty Yards. I believe therefore there is a Bound or Term in every natural Moveable that departs from Reſt, and that the Impediment of the *Medium* at length reduces it to Equability, which afterwards it always maintains.

SAGR. Theſe Experiments are truly, in my Opinion, much to the Purpoſe: And nothing remains, unleſs the Adverſary ſhould ſtoutly deny that they will hold true in very great and very heavy Maſſes; and that a Cannon Ball coming from the Concave of the Moon, or from the upper Region of the Air, would make a greater Percuſſion than if ſhot from a Cannon.

SALV. No doubt but many Things may be objected, which Experience cannot ſolve: Yet in this Objection, methinks, there is ſomething may fall under Conſideration; to wit, that it is very likely that a heavy Body falling from ſo great a Height, by that Time it reaches the Earth,

has got ſuch an *Impetus* as is required to throw it to that Height it fell from, as is plainly ſeen in a *Pendulum* reaſonably heavy, which, being mov'd fifty or ſixty Degrees from the Perpendicular, acquires ſuch a Velocity and Power as exactly ſufficeth to carry it again to the ſame Height, abating only that little which is taken from it by the Impediment of the Air. To place then a Cannon Ball at ſuch a Height, in falling from which it may acquire ſuch an *Impetus* as it receives when fired out of the Cannon, it would ſuffice to ſhoot it upwards perpendicularly from the ſame Cannon, and then obſerving whether in its Fall it makes an Impreſſion equal to that Percuſſion made near at Hand on its iſſuing forth from the Piece. But indeed I believe it would be much weaker; Wherefore I hold that the Velocity with which the Ball is carried at its being juſt ſent out of the Piece, is ſuch as the Impediment of the Air will never ſuffer it to acquire whilſt it deſcends with a natural Motion, leaving the State of Reſt from any very great Height.

I come now to the other Queſtions relating to *Pendulums*, Matters which to ſome would ſeem very frivolous, and more eſpecially to thoſe Philoſophers who are continually employ'd in the more profound Queſtions of natural Philoſophy: Yet I am far from contemning them, encouraged by *Ariſtotle*'s Example, in whom I can't help admiring this, *viz.* that he hath left nothing unhandled, as one may ſay, that's worth one's Conſideration. And as to your Queſtions, I think I can give you a Conceit of my own, relating to ſome Problems concerning *Muſick*, a noble Subject, of which ſo many great Men, and even *Ariſtotle* himſelf, have written; and touching which he conſiders

conſiders many curious Problems: And if I, from eaſy and ſenſible Experiments, can deduce Reaſons of the wonderful Accidents of Sounds, I may hope my Diſcourſe may be grateful to you.

SAGR. Not only grateful, but by me in particular moſt deſireable; for I take great Delight in all Muſical Inſtruments: And altho' I have thought much concerning Conſonances, yet I never could comprehend whence it ariſes, that one ſhould more pleaſe me, and give me greater Delight than another; that ſome others do not only not delight me, but, on the contrary, highly offend me. And again, that common Problem concerning two Strings ſet to an Uniſon, one of which actually ſoundeth upon ſtriking the other, I, as yet, cannot ſolve; nor do I clearly underſtand the Forms, and ſome other Particulars relating to Conſonances.

SALV. I will ſee whether from theſe our *Pendulums* I can deduce any thing to anſwer all theſe Difficulties.

And firſt as to your Queſtion, *viz.* Whether the ſame *Pendulum* doth really and exactly perform all its Vibrations, greateſt, mean, and leaſt, in Times preciſely equal, I refer myſelf to that which I have long ſince learnt of our *Academic*, who plainly demonſtrates, that a Moveable deſcending along the Chord ſubtending any Arch, paſſes them all in equal Times, *i. e.* as well the Subtenſe of 180°, (*i. e.* the whole Diameter) as the Subtenſe of 100°, or 60, 10, 2, ½ a Degree, or of four Minutes, ſuppoſing them all to terminate in the loweſt Point, which touches the horizontal Plane: Again, concerning Deſcendents by

the Arches of the ſame Chords, elevated above the Horizon, Experience likewiſe ſhews that they all are paſſed over in equal Times, but in ſhorter than the Times in which the Chords are run thro'; an Effect ſo wonderful, that at firſt View one would think the contrary ſhould happen; for the Terms of the Beginning and Ends of the Motion being common, and a right Line being the ſhorteſt of thoſe which are comprehended between the ſame Terms, it ſeems reaſonable that the Motion made thro' is ſhould be perform'd in the ſhorter Time; which yet it quite otherwiſe, and the ſhorteſt Time, and conſequently the ſwifteſt Motion, is that which is made thro' the Arch of which that right Line is the Chord.

In the next Place: As to the Times wherein the Vibrations of Moveables, ſuſpended by Strings of different Lengths are performed, they are in ſubduple Proportion of the Lengths of the Strings; or, if you will, the Lengths are in a duplicate Ratio of the Times, *i. e.* as the Squares of the Times; ſo that ſuppoſing, for Example, the Time of a Vibration of one *Pendulum*, to be double the Time of a Vibration of another *Pendulum*, it neceſſarily follows, that the Length of the String of that *Pendulum* be quadruple of the Length of the String of this: And then in the Time of one Vibration of that, another will make three Vibrations, when the String of that is nine times as long as that of the other: From whence it follows, that the Lengths of the Strings have the ſame Proportion to one another, as have the Squares of the Numbers of the Vibrations made in the ſame Time.

SAGR. So

SAGR. So that, if I underſtand you aright, I may eaſily know the Length of a String hanging from any never ſo great a Height, tho' I can't ſee its upper End, by which it is faſten'd on high, but only its lower End: For if I faſten Weight enough to the ſaid String here below, and ſet it a vibrating to and fro, letting ſome Friend number ſome of its Vibrations. Then if I at the ſame time count the Vibrations of another Moveable hung by a String exactly a Yard long; then from the Number of Vibrations which thoſe *Pendulums* make in the ſame time, I ſhall find the Length of that String: Thus, for Example's Sake, ſuppoſe that in the time that my Friend hath counted 20 Vibrations of the longer String, I have counted 240 of my String that is a Yard long: The Numbers 20 and 240 being ſquar'd, which are 400 and 57600, I can pronounce the longer String to contain 57600 of thoſe Meaſures whereof my String contains 400; and becauſe this is but one Yard in Length, I will divide 57600 by 400, and the Quotient will be 144: And ſo many Yards I affirm that String is in Length.

SALV. And you'll not be miſtaken one Inch, eſpecially if you take a ſufficient Number of Vibrations.

SAGR. You give me frequent Occaſion to admire the Riches, and withal the extraordinary Bounty of Nature, whilſt from Things ſo common and mean, you deduce, one way or other, many curious and new Notions, and ſuch oftentimes as are remote from all Imagination. I have very often carefully conſider'd the Vibrations in particular

ticular of the Lamps, which in ſome Churches hang by very long Ropes, and which have been by Chance ſtirr'd or mov'd by any one: But the moſt that I inferr'd from that Obſervation was this, that their Opinion was very improbable, who will have that ſuch Motions are maintain'd and continu'd by the *Medium*, *i. e.* by the Air: For I ſhould think the Air endued with great Judgment, and with the greateſt Eaſe, to cauſe an hanging Weight to vibrate for Hours together with ſo much Regularity: But I ſhould never have learnt from hence, that if the ſame Moveable, ſuſpended by a String a hundred Yards long, now elevated above the loweſt Point ninety Degrees, now only one, or half a one, that, I ſay, it ſhould ſpend as much time in moving thro' the leaſt Arch, as it does in moving thro' the greateſt; I don't think this would ever have entered into my Head, and, to be plain, I ſtill think it next to impoſſible. But now I ſtand prepar'd to hear how from theſe the moſt ſimple and plain Things can be aſſign'd me ſuch Reaſons concerning thoſe Muſical Problems, as in ſome meaſure to give me Satisfaction about them.

SALV. Above all things you muſt know that every *Pendulum* hath the Time of its Vibrations ſo limited and prefixed, that it is impoſſible to make it vibrate in any other Period than that which is natural to it: For let any one take the String the Weight is faſten'd to in his Hand, and let him try to increaſe or leſſen the Number of its Vibrations, and he ſhall find his Labour to be in vain: But we may, on the contrary, to a *Pendulum*, tho' it be heavy, and at Reſt, give a Motion by only blowing upon it,

it, and, repeating the Blaſt, a Motion conſiderable; but this ſecond Blaſt muſt be made at at Inſtant ſuitable to the Motion of the Vibrations; ſo that if at the firſt Blaſt we move it half an Inch from the Perpendicular, and then again, if when it has returned to us, and is beginning its ſecond Vibration, we add a ſecond Blaſt, we ſhall confer a new Motion to it; and thus by ſucceſſively repeating the Blaſts at proper Times, but not when the *Pendulum* is moving towards us, (for ſo we ſhould check and not help the Motion) we may confer upon it ſuch an *Impetus*, that a much greater Force than our Breath will be required to ſtop it again.

SAGR. I have obſerv'd, when I was a Boy, that one Man alone by ſuch Impulſes, given at right Times, has been able to raiſe a very great Bell; and when four or ſix have taken hold of the Rope to ceaſe the Bell from Motion, they have been all raiſed from the Ground; they all being not able to withſtand the *Impetus*, which one Man alone, with regular Pulls, had conferred upon the Bell.

SALV. An Example that declares my Meaning as fully as my foregoing one is ſuitable to render the Reaſon of that admirable Problem of the Strings of the Viol or Lute, which moves and makes not that only to ſound, which is tun'd to an Uniſon with it, but that alſo which is ſet to an Eighth, and to a Fifth: The String being touch'd or ſtruck, its Vibrations begin and continue ſo long as its Sound laſts; theſe Vibrations give a Vibration and Tremor to the adjacent Air, whoſe Tremors and Circulations, extending

extending themſelves a great Way, ſtrike upon all the Strings of the ſame Inſtrument, as alſo upon the Strings of any others near it: That String that is ſet to an Uniſon with that touched, being diſpoſed to make its Vibrations in the ſame Time, at the firſt Impulſe begins to move a little; and upon the Addition of a ſecond, a third, and twentieth, and many more, all made in fit and periodic Times, it receives at laſt the ſame Tremulation with that firſt touched, and, as plainly may be ſeen, it dilates its Vibrations exactly according to the Dilatation of thoſe of its Mover.

This Undulation that extends itſelf thro' the Air, moves and makes to vibrate, not only the Strings, but likewiſe any other Body diſpoſed to Tremulation, and to vibrate in the ſame time with the trembling String; wherefore if upon the Sides of the Inſtrument, ſeveral ſmall Particles of Hair, or Briſtles, or any other flexible Matter, be laid, ſounding the Viol, we ſhall ſee now this, now that Corpuſcle tremble, as this or that String is ſtruck, whoſe Vibrations are made in the ſame Time; but the others will not move at the Striking or Sound of this String; nor will that Corpuſcle move at the Sounding of any other String.

If the Baſe String of a Viol be ſmartly ſtruck with a Bow, and a thin and ſmooth Drinking-Glaſs be ſet by it; if the Tone of the String be an Uniſon with the Tone of the Glaſs, the Glaſs will tremble and ſenſibly reſound: Again, we plainly ſee the Circulation of the *Medium* about the reſounding Body, to diffuſe to a large Space, by making a Glaſs to ſound, that has ſome Water in it, by rubbing the Rim or Edge of it with the Tip of one's Finger:

Finger: For we ſhall thus ſee the Water in the Glaſs to undulate in a moſt regular Order; which Effect will yet more clearly be ſeen, if we put the Foot of the Glaſs in the Bottom of a Veſſel of reaſonable Bigneſs, and fill it with Water nearly to the Glaſs's Rim, and then make it ſound by rubbing it round, as before, with the Tip of one's Finger, for then we ſhall ſee the Circulations in the Water to be moſt regular, and with great Velocities to ſpread to a great Diſtance round about the Glaſs: Nay, I have often happen'd to ſee, in making a pretty big Glaſs, almoſt full of Water, to ſound as before, the Waves form'd with an exact Equality; but the Tone of the Glaſs happening ſometimes to riſe an Eighth higher, I have ſeen at that very Inſtant every one of the ſaid Waves to divide themſelves into two: which Accident moſt plainly proves the Form of the Octave to be the Double.

SAGR. The ſame Thing has happen'd to me more than once, to my great Delight and Advantage too: For I ſtood a long time in Doubt concerning the Forms of Conſonance, not thinking the Reaſons commonly brought by the learned Authors, who have hitherto wrote of Muſick, ſufficiently demonſtrative. They tell us that the *Diapaſon*, that is the *Octave*, is contain'd by the *Double*; and that the *Diapente*, which we call *the Fifth*, is contain'd by the *Seſquialter*: For if a String, ſtretch'd upon the marine Trumpet, be ſounded open, and afterwards placing a Bridge under the Midſt of it, its half only be ſounded, you'll hear an Eighth: And if the Bridge be placed under one third of the String, and you then ſtrike the two thirds open, it ſoundeth a Fifth to that of the whole String ſtruck

when open : whereupon they infer that the Eighth is contain'd between two and one, and the Fifth between three and two. But I don't think we can conclude from hence, that the *Double* and *Seſquialteral* can naturally aſſign the Forms of the *Diapaſon* and *Diapente* : And my Reaſon for't is this : There are three Ways by which we may ſharpen the Tone of a String, *viz.* by ſhortning it, by ſtretching it, or by making it thinner : If now, retaining the ſame Tenſion and Thickneſs, we would hear an Eighth, we muſt make it ſhorter by half, *i. e.* we muſt firſt ſound the whole String, and then its half. But if keeping the ſame Length and Thickneſs, we would have it riſe to an Eighth from its preſent Tone, by ſtretching it, or ſcrewing it higher, 'tis not ſufficient to ſtretch it with a Double, but with four times the Force : Thus, if at firſt it was ſtretch'd by a Weight, ſuppoſe of one Pound, we muſt hang a four Pound Weight to it, in order to raiſe its Tone an Eighth. And laſtly, if keeping the ſame Length and Tenſion, we would have a String to ſound an Eighth, this String muſt be but one fourth of the Thickneſs of the other it muſt ſound an Eighth to. And this that I ſay of the Eighth, *i. e.* that its Form taken from the Tenſion or Thickneſs of the String in a duplicate *Ratio* to that which it receiveth from the Length, I would have underſtood of all other Muſical Intervals ; becauſe if that which the Length gives us in a *Seſquialter* Proportion, *i. e.* when firſt by ſtriking it open, and then its two thirds, we would do the ſame from the Tenſion, or Tenuity, or thinning the String, we muſt double the *Seſquialter* Proportion, by taking the double *Seſquiquartan* : And if the grave String be ſtretch'd by a four Pound Weight to the ſharp one, we muſt hang not ſix

ſix but nine Pounds : And as to the Thickneſs ; we muſt make the grave String thicker than the acute, in the Proportion of nine to four, to gain the Fifth.

Theſe Things being really ſo in Fact, I ſaw no Reaſon why theſe ſage Philoſophers ſhould rather conſtitute the Form of the *Eighth Double* than *Quadruple* ; and the Form of the *Fifth* rather *Seſquitertian*, than the *double Seſquiquartan:* But becauſe 'tis impoſſible to number the Vibrations of the String, which, while it ſounds, are made very ſwiftly ; I ſhould always have been in Doubt whether or no it were true, that the String ſounding the higher *Eighth* did perform double the Number of Vibrations, in the ſame Time to thoſe of the graver String, unleſs the Undulations, which may be continued as long as you pleaſe, by keeping the Glaſs ſounding and vibrating, had ſenſibly ſhewn me, that in the very Inſtant wherein ſometimes we hear a Tone to jump to an *Eighth*, there are ſeen to ariſe other leſſer Undulations, which, with infinite Accuracy divide each of the former into two.

SALV. An excellent Obſervation this ! whereby we can diſtinguiſh one by one the Undulations produced by the Tremor of the ſounding Body ; which are thoſe which, diffuſing themſelves thro' the Air, cauſe a Titillation on the Drum of our Ear, whence in our Soul is produc'd a Sound. But ſince we can't ſee and obſerve thoſe Undulations in the Water, any longer than that Rubbing with the Tip of the Finger laſteth, and they being not at a Stay, but in conſtant Motion all that Time, wou'd'nt it be a great Thing to contrive ſuch as we might exquiſitely

obſerve, as long as we pleaſe, for Months if we will, nay, for Years, whence we ſhould be able exactly to meaſure, and eaſily to count them?

SAGR. I aſſure you I ſhould highly eſteem ſuch an Invention.

SALV. The Diſcovery was merely accidental; the Obſervation and applying it to Uſe was my own: I own I like the Thing, and have often conſider'd it with great Satisfaction, tho' the Experiment at firſt View may ſeem to be of little or no Account or Worth.

Scraping a Copper Plate with an Iron Chizel to take out ſome Spots in it, upon moving the Chizel quick to and fro, amongſt the many Attritions or Rubbings, I, more than once, heard it ſend forth a whiſtling Noiſe or Sound; and then looking upon the Plate, I eſpied a long Row of ſmall Streaks, parallel to one another, and exactly equidiſtant. Returning to my Scraping again, I found, by ſeveral Trials, that the Chizel left thoſe Streaks upon the Plate, at thoſe Scrapings, and thoſe only that yielded the aforeſaid Noiſe: But when the Attrition was made without ſuch Whiſtling, there appear'd not the leaſt Sign of any ſuch Streaks: Repeating the Experiment, for my farther Satisfaction, ſeveral times afterwards, ſcraping one while with greater, another with leſs Velocity, the Whiſtling I found to have one while a ſharper Tone, and another a graver; and I obſerv'd the Marks made at the ſharper Sounds, to be more, and cloſer together; and fewer, and farther aſunder, at the grave Sounds: And furthermore, as the Swiftneſs towards the End of the ſelf-ſame Scrape grew

grew greater than that at the Beginning, the Sound was heard continually to grow ſharper, and the Streaks were obſerv'd to ſtand thicker, but always made with the greateſt Regularity, and exactly equidiſtant from one another: And furthermore, when in ſcraping ſuch Sound was heard, I felt the Chizel to tremble in my Hand, and a certain Shivering to run along my Arm; and, in ſhort, I obſerv'd the very ſame Thing effected by this Tool, which may be obſerv'd in ourſelves when we whiſper, and afterwards ſpeak aloud; for when we ſend forth our Breath without forming a Sound, we do not perceive any Motion in the Throat or Mouth, in Compariſon of that Tremor which we feel in the Wind-Pipe and Jaws, upon ſending forth the Voice, eſpecially if the Tones emitted be grave and ſtrong. I have, moreover, at ſuch a time obſerv'd, amongſt the Strings of the Viol, two to ſound Uniſons, to two of the Sounds made by ſcraping after the Manner aforeſaid, and exact Fifths to one another: then meaſuring the Intervals of the Streaks of both the Attritions or Scrapes, I found forty-five Spaces of the one to be equal to thirty of the other, which, indeed, is the Form attributed to the *Diapente*.

But now, before I proceed any farther, I muſt tell you, that of the three Ways of making a Tone more acute, that which you refer to the Slenderneſs or Fineneſs of the String, may with more Truth be attributed to its Weight: The Alteration taken from the Thickneſs, anſwers indeed when the Strings are of the ſame Matter, and ſo a String of Gut to ſound an Eighth, muſt be four times as thick as the other Gut-ſtring, below which it is to ſound an Eighth; and one of Wire alſo four times thicker than another

another of Wire: but if betwixt two Strings, one of Wire, and the other of Gut, I would get an Eighth, I muſt not make it four times thicker, but four times heavier; ſo that, as to the Thickneſs, the Wire one ſhall not be four times thicker, but four times heavier than the other; for then the Wire one will be finer than that of Gut which ſounds an Eighth above it. Whence it comes to paſs, that ſtringing one Inſtrument with Gold Strings, and another with Braſs ones, provided the Strings of the one Inſtrument be of the ſame Length, Thickneſs, and Tenſion with thoſe of the other, the Tone of the Gold Strings ſhall be about a Fifth lower than that of the Braſs ones, Gold being nearly double the Weight of Braſs. And here I would have you obſerve, that the Gravity of the Moveable more reſiſts the Velocity of the Motion, than the Thickneſs does, contrary to what one at firſt would think; for, indeed, in Appearance, 'tis more reaſonable that the Velocity ſhould be retarded, by the Reſiſtance of the *Medium*, againſt dividing or opening, more in a thick and light Moveable, than in a heavy and ſmall one: and yet in this Caſe it happens quite contrary. But to purſue our firſt Purpoſe; I ſay, that 'tis neither the Length, nor the Tenſion, nor the Thickneſs, that conſtitutes the neareſt immediate Reaſon of the Forms in Muſical Intervals, but the Proportion of the Number of Vibrations and Percuſſions in the Undulations of the Air, ſtriking the Drum of our Ear, which itſelf alſo doth tremulate under the ſame Meaſures of Time.

Having eſtabliſh'd this Point, we may perhaps be able to aſſign a juſt Reaſon whence it comes to paſs, that of Sounds differing in Tone, ſome Pairs are heard with great Delight,

Delight, others with leſs; and that others are very offenſive to the Ear. This we may do, I ſay, by ſeeking the Reaſon of the more or leſs perfect Conſonances, and alſo of Diſſonances. The Offence theſe give, proceeds, I believe, from the diſcordant and jarring Pulſations of two different Tones, which, without any Proportion, ſtrike the Drum of the Ear: And the Diſſonances will be extreme harſh, in caſe the Times of the Vibrations are incommenſurable. And of ſuch this is one; When one of two Strings, Uniſons, is ſounded with ſuch a Part of another, as is the Side of a Square of its Diagonal; which Diſſonance is like to the *Tritone* or *Semi-diapente*.

Thoſe Pairs of Sounds ſhall be Conſonances, and will be heard with Pleaſure, which ſtrike the *Timpanum* in ſome Order; which Order requires, in the firſt Place, that the Percuſſions made in the ſame Time be commenſurable in Number, that the Cartilage of the *Timpanum* or Drum may not be ſubject to a perpetual Torment of bending itſelf two different Ways, in Submiſſion to the ever diſagreeing Percuſſion.

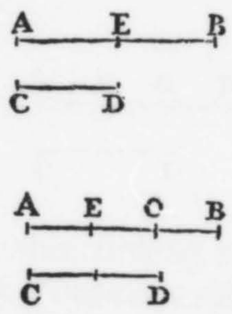
The firſt and moſt grateful Conſonance will be the *Eighth*, ſince for every Stroke wherewith the grave String ſtrikes the Ear, the acute one gives two; ſo that *both*, in every ſecond Vibration of the ſharp String, ſtrike at once, or together; wherefore half the whole Number of Strokes agree in ſtriking together; but the Strokes of the Strings that are *Uniſons*, are always ſtruck together, and therefore are but as one only String, and make no Conſonance.

The *Fifth* alſo is delightful, where, for every two Strokes of the grave String, the ſharp one gives three; whence it follows, that if we count the Vibrations of the acute String,

String, we ſhall find the third Part of the whole Number to agree, and ſtrike together; that is, between each Pair of Conſonances, there are two ſingle Sounds: and in the *Diateſſeron* there are three ſingle ones interpos'd. In the ſecond, that is, in the *Seſquioctave* Tone in nine Pulſations there's only one that ſtrikes in Concord with the other of the graver String: and all the reſt are Diſcords, and fall upon the *Timpanum* irregularly and troubleſomely, and thence by the Ear are eſteem'd as Diſſonances.

SIMP. I wiſh you would explain this Diſcourſe a little more.

SALV. Let the Line AB be the Space and Dilatation of one Vibration of the grave String, and the Line CD of the acute one, which are Eighths to each other: Biſect AB in E; then 'tis manifeſt that if the Strings begin to



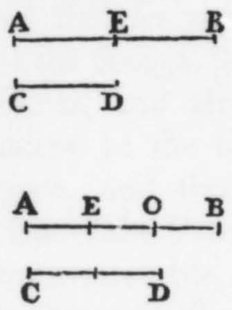
move in the Terms A and C, that when the acute Vibration ſhall be come to the Term D, the other will be extended only to its half, or to its middle Point E, which not being the Term or Bound of the Motion, it doth not yet ſtrike, but yet a Stroke is made in D. Then whilſt the

the Vibration returns from D to C, the other paſſes from E to B, upon which the two Percuſſions of B and C now ſtrike both together upon the *Timpanum*; and becauſe they continue to repeat in like manner the ſubſequent Vibrations, we may conclude that the Union of the Percuſſions of the Line CD, with the Percuſſions of the Line AB happen every other Time: But the Pulſations of the Terms AB are always accompanied, and are always the ſame with one of CD; which is manifeſt from hence, that ſuppoſing A and C to ſtrike together, whilſt A paſſes to B, C goes on to D, and goes back to C; ſo that the Strokes B and C are made together.

But now let AB and CD be two Vibrations which produce the *Diapente*, whoſe Times are in a *Seſquialteral* Proportion, and divide the Line AB of the grave String into three equal Parts, by the Points E and O; and ſuppoſe the Vibrations to begin at the ſame Time from the Terms A and C; then its manifeſt, that when the Stroke is made in D, the Vibration of AB is got only to O; wherefore the *Timpanum* receives the Pulſation D only. Then whilſt it returns from D to C, the other Vibration is paſs'd from O to B, and return'd to O, firſt making a Pulſation in B, which is alſo made alone, and in Countertime (a Thing to be conſider'd) for ſince we ſuppos'd the firſt Pulſations to be made at the ſame Moment in the Terms A and C, the ſecond which was made alone from the Term D was made ſo long after as is the Time of Tranſition thro' CD, that is AO: but the following one which is in B, is diſtant from the other, as much as is the Tranſition OB only, which is the half: but it goes on from O to A, in the Time the other paſſes from C to D,

and thus are two Pulſations made together in A and D: There afterwards follow other Periods like theſe, that is, with the Interpoſition of two ſingle Pulſations of the acute String by itſelf; and one ſingle Pulſation of the grave String, alone or by itſelf, between thoſe two ſingle Strokes of the acute one.

Wherefore ſuppoſing the Time divided into Moments, *i. e.* into its leaſt equal Particles, and ſuppoſing that in the two firſt Moments, the concordant Pulſations made in A and C, to paſs to O and D, and that in D a Stroke is made; and that in the third and fourth Moment, one paſſes from D to C, and ſtrikes in C; and the other goes on from O to B, and returns to O, ſtriking in B: and in the fifth and ſixth Moment the one paſſes from C to D, and the



other from O to A; then ſtriking together we ſhall receive the Pulſations on the *Timpanum* diſtributed in this Order, that ſuppoſing the Pulſations of the two Strings to be made in the ſame Moment, two Moments after it ſhall receive one ſolitary Percuſſion, in the third Moment another, alſo ſolitary, in the fourth, another; and two Moments after, that is, in the ſixth, two together; and here ends the Period, and *Anomaly*, if I may ſo ſay; which Period is often, after the ſame Manner, repeated.

SAGR. I

SAGR. I can hold no longer! but muſt loudly expreſs the Pleaſure I have to hear ſuch appoſite Reaſons aſſigned of Effects, concerning which I have been ſo long in the Dark. Now I underſtand how it comes to paſs, that an *Uniſon* differs not from a ſingle Tone: Now I ſee why the *Octave* is the principal Conſonance, and ſo like to the *Uniſon*, that it is taken and conjoin'd with others as an *Uniſon*. It reſembles an *Uniſon*, becauſe, as the Pulſations of Strings ſet to an *Uniſon* all ſtrike together, the Pulſations of the grave String, which ſounds an Eighth, are all accompany'd with Beats of the acute String, and of theſe one ſolitary one is interpos'd, and in equal Diſtances, and in a certain Order, without any Variety; whence ſuch Conſonance, without the leaſt Allay, becomes moſt melodious.

But the *Fifth*, with thoſe its Counter-times, and which the Interpoſitions between the Pairs of conjoin'd Pulſations of two ſolitary ones of the acute String, and one ſolitary one of the grave one, and theſe three in ſuch Interval of Time, as is its half, produces ſuch a Titillation upon the Cartilage of the *Timpanum*, that, allaying the Sweetneſs by a Mixture of Tartneſs, it ſeems at one and the ſame Time to kiſs and bite.

SALV. Since I ſee you are ſo pleas'd with theſe Novelties, I'll ſhew you a Method whereby not the Ear only, but the Eye alſo, may be recreated, in beholding the ſame Sports the Ear feels or hears.

Hang Balls of Lead, or other heavy Matter, by three Strings of different Lengths, in ſuch Manner, that in the Time the longeſt makes two Vibrations, the ſhorteſt makes

four, and the middle one three; which will happen when the longeſt contains ſixteen Feet, or other Meaſures, ſuch as the middle one contains nine, and the ſhorteſt four. And removing them all at once from Perpendicularity, and then letting them go, you'll ſee a various Intermixture of thoſe *Pendulums* with various Accidents, but ſo that at every fourth Vibration of the longeſt, all the three will concur together in one and the ſame Term; which they afterwards leave and then again repeat the ſame Period; And this Mixture of Vibrations is the ſame with that which being made in Strings of Inſtruments, preſents to the Ear an *Eighth*, with an intermediate *Fifth*. And if we order the Length of other Strings in the like Diſpoſition, ſo that their Vibrations anſwer to thoſe of other muſical conſonant Intervals, we ſhall ſtill ſee new Connections, and thoſe always ſuch as all the Strings (be they three or four) will, in determinate Times, and after determinate Numbers of Vibrations, concur in the ſame Moment, at the Term of their Vibrations, and thence they again begin another like Period; But if the Vibrations of two or more Strings, are either incommenſurable, ſo that by vibrating to and fro, they never harmoniouſly perform a determin'd Number of Vibrations; or if they are not incommenſurable, and yet return not till after a long Time, and after a great Number of Vibrations, then the Sight is confounded by the irregular and confus'd Order of irregular Intermixtures, as the Ear with Regret receives the intemperate Impulſes of the Air's Tremulations, which, without Order or Rule, ſucceſſively ſtrike its *Timpanum* or *Drum*.

But whither, Gentlemen, have we been tranſported for ſo many Hours, by various Problems, and unlook'd for Diſcourſes?

Diſcourſes? We have made it Night, and yet have handled few or none of the Points firſt propos'd; but, on the contrary, have ſo loſt our Way, that I hardly remember where we begun, and what that ſmall Introduction was, which, by Way of *Hypotheſis*, we laid down as a Principle to build upon in the Demonſtrations which were to follow.

SAGR. True, it will be convenient, therefore, that we put an End to this Day's Conference, and refreſh our Minds by the Night's Repoſe, and To-morrow (if you pleaſe ſo far to favour us) we will reſume the Diſcourſes deſir'd, and at firſt chiefly intended.

SALV. I'll not fail to be here at the uſual Hour, to enjoy your Company, and do you what Service I can.

The END of the FIRST DAY'S CONFERENCE.

GALILÆUS

GALILÆUS

HIS

DIALOGUES.

DIALOGUE II.

Concerning the CAUSE *of the* COHERENCE *in* SOLIDS.

SALVIATUS, SAGREDO, *and* SIMPLICIUS.

SAGR. WHILST *Simplicius* and I waited your coming, we have been recollecting that laſt Conſideration, which, as the Principle and Suppoſition whereon you ground the Concluſions which you propoſed to demonſtrate to us, was that Power by which all Bodies reſiſt *Fraction*, depending on that Cement which connects and holds together their Parts, ſo as not to ſuffer them to give Way and ſeparate without a ſtrong Attraction : Afterwards we began an

an Enquiry into the Cauſe of that Coherence, which in ſome Solids is very ſtrong: and I remember that was propoſed as the Chief, which is deduced from a *Vacuum*; which afterwards occaſion'd ſo many Digreſſions, as imploy'd us the whole Day, and carried us ſo far from the Matter firſt propoſed, which was the Contemplation of the Reſiſtances of Solids to Fraction.

SALV. I remember all this very well: To return then to the Place where we began: Whatever we ſuppoſe this Reſiſtance of Solids to Fraction, by a violent Attraction, to be; it is ſufficient to us that there is undoubtedly in Solids ſuch a Thing, and which, tho' it be very great, againſt the Strength of one that draws them directly, or ſtrait out, yet 'tis obſerv'd to be leſs, if the Force be applied tranſverſely or ſideways.

Thus we ſee, for Example, a Rod of Steel or Glaſs, to ſuſtain lengthways a Weight of a thouſand Pounds, which, faſten'd at right Angles to a Wall, will break with a Weight of fifty Pounds only. Now this ſecond Reſiſtance ſhall be the Subject of our preſent Diſcourſe, where we ſhall enquire in what Proportion it is found in Priſms and Cylinders of like or unlike Figure, Length and Thickneſs, provided they be of the ſame Matter: In which Enquiry, I take for a Maxim that, which in *Mechanics* is demonſtrated amongſt the Properties of the *Vectis*, which we call the *Leaver*; namely, that in the Uſe of the Leaver, the Force is to the Reſiſtance in a reciprocal Proportion of their Diſtances, *i. e.* the Diſtances of the Force and Reſiſtance from the *Fulcrum*.

SIMP. *Ariſtotle*

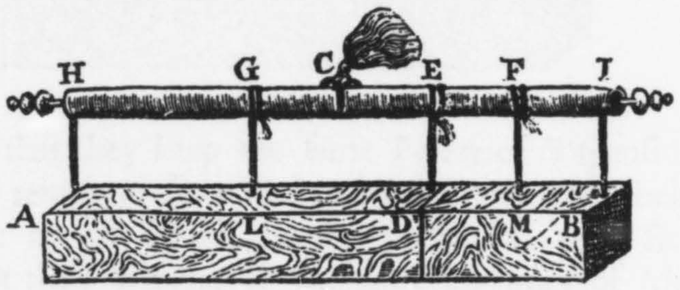
SIMP. *Ariſtotle* firſt of all demonſtrated this in his *Mechanics*.

SALV. I allow the Precedency to *Ariſtotle*, as to Time, but as to the Firmneſs of Demonſtration, I think *Archimedes* ought to be preferr'd before him: and that on Account of a ſingle Propoſition by him demonſtrated in his Book *De Equiponderantibus*, whereon depends the Reaſons, not only of the Leaver, but alſo of the greater Part of the other Mechanical Inſtruments.

SAGR. But ſince this Principle is the Foundation of all that you intend to demonſtrate to us, it would be requiſite, and very kind of you, if it ben't too much Trouble, to give us its Demonſtration, and thereby a full and perfect Satisfaction concerning it.

SALV. However neceſſary this is, yet it will be better that I introduce you into the Field of all our future Speculations, by an Entrance different from that *Archimedes* uſed: and here I ſhall take nothing for granted, but that equal Weights put into a Ballance of equal Arms, make an *Equilibrium* (a Principle likewiſe ſuppoſed by *Archimedes* himſelf); afterwards, I may demonſtrate to you, that it is no leſs true, that unequal Weights make an *Equilibrium* in a Ballance of Arms unequal, according to the Proportion of thoſe Weights reciprocally ſuſpended: and that it is the ſame thing to place equal Weights at equal Diſtances, as to place unequal Weights at ſuch Diſtances, which are in a reciprocal Proportion to their Weights.

Now to demonſtrate theſe Things clearly, Deſcribe a Cylinder or Priſm as AB, hung by its Ends to the Line HI, and ſuſtain'd by the Cords HA and IB. Now 'tis manifeſt, that if I ſuſpend the Whole by the Cord C, plac'd in the Midſt of the Ballance HI, the Priſm AB will remain in *Equilibrio*, one half of its Weight being on one Side, and the other half on the other of the Point of Suſpenſion C, by the Principle by us preſuppoſed. Now ſuppoſe the Priſm to be divided into unequal Parts, by a Plane paſſing thro' D, and let DA be the greater Part, and DB the leſſer: And that ſuch Diviſion



being made, the Parts of the Priſm may remain in the ſame Situation and Conſtitution in reſpect of the Line HI, apply the Cord ED, which tied in the Point E, may ſuſtain the Parts DA and DB of the Priſm: Then there can be no Doubt, ſince there is no Local Mutation made of the Priſm, in reſpect of the Ballance HI, but it will remain in the ſame State of Equilibration: But it will reſt in the ſame Situation, if the Part AD of the Priſm, which is now ſuſpended at the two Extremes by the two Cords AH and DE, be hang'd at one ſingle Cord placed in the Middle; And in like manner the other Part DB will

not

†

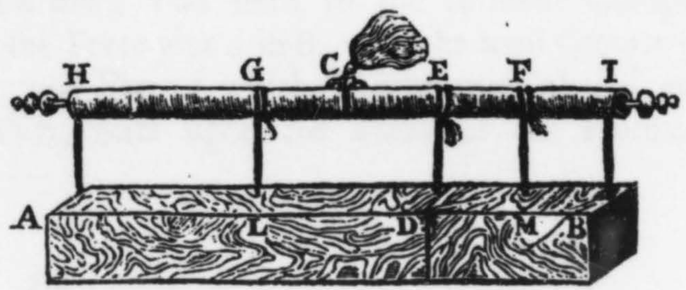
not change its State, if ſuſpended by the Middle, it be ſuſtained by the Cord FM: So that the Cords HA, ED, and IB being untied, and only the two Cords GL and FM being left, the *Equilibrium* will ſtill remain; as if the Suſpenſion was ſtill made at the Point C.

Here now let us conſider that we have two heavy Bodies AD and DB, hanging from the Terms G and F of the Beam GF, in which the *Equilibrium* is kept at the Point C, in ſuch manner that the Diſtance of the Point of Suſpenſion G, from which the Weight AD hangs from the Point C, may be the Line CG; and the other Part CF be the Diſtance at which the other Weight DB hangs. It remains then only to be demonſtrated, that thoſe Diſtances have the ſame Proportion to one another, as the Weights themſelves have, but reciprocally taken; that is, that the Diſtance GC is to the Diſtance CF, as the Priſm DB to the Priſm DA: Which I thus demonſtrate;

Since the Line GE is half the Line EH, and EF the half of EI, therefore GF will be the half of the whole Line HI, and thence will be = CI; and taking away the common Part CF, the Remainder GC will be = the Remainder FI, that is to FE: Then adding the common Part CE, the two Lines GE and CF will be equal to each other: And therefore it will be GE: EF:: FC: CG; but as GE to EF, ſo the Double to the Double; that is, ſo is HE to EI, that is the Priſm AD to the Priſm DB. And therefore by Equality and Converſion of Proportions, as the Diſtance GC to the Diſtance CF, ſo is the Weight BD to the Weight DA. *Q. E. D.*

If you underſtand what has hitherto been ſaid, I believe you will eaſily admit, that the two Priſms AD and DB

make an *Equilibrium* in the Point C, becauſe one half of the whole Solid AB, is on the right Hand of the Point of Suſpenſion C, and the other half to the left Hand; and that in this manner there are exhibited two equal Weights, diſpoſed and extended at two equal Diſtances. Again, that thoſe two Priſms AD and DB, afterwards reduced into two Cubes, or two Balls, or into two any other Figures,



(provided that they keep the ſame Points of Suſpenſion G and F) do retain an *Equilibrium* in the Point C, I believe you don't at all doubt, ſince 'tis manifeſt that Bodies change not their Weight, if the ſame Quantity of Matter is retain'd: From which we may gather this general Concluſion, That any two Weights whatever make an *Equilibrium*, at Diſtances anſwering reciprocally to their Gravities.

This Principle then being eſtabliſh'd, before I proceed, I am to examine how theſe Forces, Reſiſtances, *Momenta*, and Figures, may be conſider'd abſtractedly, or ſeparate from Matter; as alſo in Concrete, or conjoin'd with Matter: and by theſe Means, thoſe Accidents that agree with Figures conſider'd as immaterial, ſhall receive certain Modifications, when we join Matter to them, and conſequently Gravity.

Gravity. As for Example; ſuppoſe a Leaver as BA, apply'd to the *Fulcrum* E, in order to raiſe the heavy Stone D.

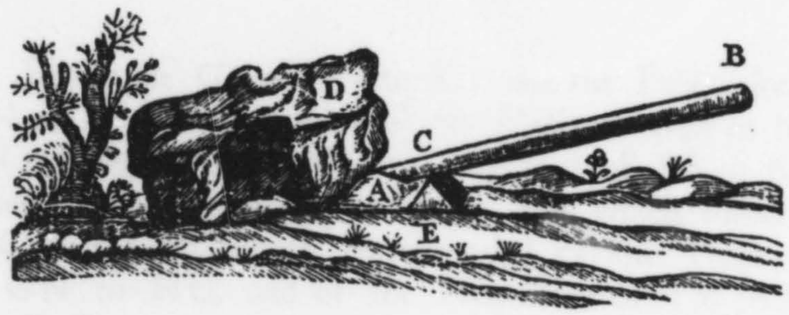
Now 'tis manifeſt, from the Principle before demonſtrated, that the Power plac'd at the Extremity B, is ſufficient to equal the Reſiſtance of the Weight D, if ſo be its *Momentum* have the ſame Proportion to the *Momentum* of the ſaid Weight D, as the Diſtance AC has to the Diſtance CB: and this is as true, if we conſider no other *Momenta* than thoſe of the ſimple Power in B, and of the Reſiſtance in D; as if the ſaid Leaver was immaterial



and deſtitute of all Gravity. But if at the ſame time we conſider the Gravity of the Inſtrument or Leaver itſelf, which may be made either of Wood or Iron, 'tis manifeſt that the Weight of the Leaver, join'd with the Power in B, alters the Proportion; which it will be requiſite to declare in other Terms: Wherefore, before we go any farther, 'tis neceſſary that we diſtinguiſh between theſe two Ways of Conſideration; calling that a taking it abſolutely, when we underſtand the Inſtrument in Abſtract, that is, ſeparate from the Gravity of its own Matter. But if with the ſimple and abſolute Figures we conjoin Matter and Gravity, theſe

theſe Figures ſo join'd with Matter, we'll call the *Momentum* or compounded Power.

SAGR. I muſt of neceſſity break the Reſolution I had taken, *viz.* to give you no Occaſion of Digreſſion; for I ſhall not be able to apply myſelf with Attention to what follows, unleſs you remove a Scruple that juſt offers itſelf: 'Tis this; You ſeem to me to make Compariſon between the Force plac'd in B, with the total Gravity of the Stone D, one Part of which Weight, methinks, if not the greater Part, reſts upon the Plane of the Horizon, ſo that ——

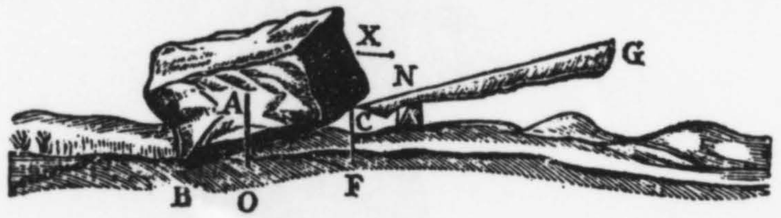


SALV. I underſtand your Meaning, ſo that you need ſay no more; only take Notice that I nam'd not the total Gravity of the Stone, but ſpake of the *Momentum* that it has and exerciſes at the Point A, the extreme Term of the Leaver BA, which is always leſs than the whole Weight of the Stone, and is variable according to the Figure of the Stone, and according as it is more or leſs rais'd.

SAGR. I

SAGR. I am ſatisfied in that Particular; but I have another Requeſt, and that is this, that for my entire Satisfaction, you'll pleaſe to demonſtrate to me, if it can be done, how I may find what Part of the whole Weight that is which is ſuſtained by the ſubjacent Plane, and what that which gravitates upon the Leaver at the Extremity A.

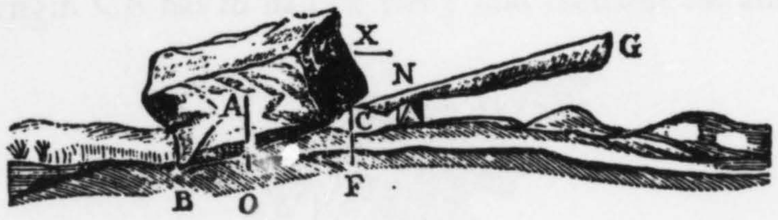
SALV. Since I can ſatisfy you in a few Words, I'll do it: In the annexed Figure ſuppoſe the Weight, whoſe Centre of Gravity is A, to reſt upon the Ground with one



End B, and at the other End lifted up by Help of the Leaver CG, upon the *Fulcrum* N, by a Power plac'd in G; then from the Centre A and Term C let fall to the Horizon two Perpendiculars AO and CF: Then, I ſay, that the *Momentum* of the whole Weight has to the *Momentum* of the Power in G a Proportion compounded of the *Ratio* of the Diſtance GN to the Diſtance NC, and of the *Ratio* of FB to BO.

Make as the Line FB to the Line BO, ſo NC to X; And ſince the whole Weight A is ſuſtain'd by the two Powers plac'd in B and C, the Power B will be to the Power C, as the Diſtance FO to the Diſtance OB; and by Compoſition, the two Powers B and C together; that is,

is, the whole *Momentum* of the whole Weight A, is to the Power in C, as the Line FB to the Line BO, that is, as NC to X: but the *Momentum* of the Power in C, is to the *Momentum* of the Power in G, as the Diſtance GN to the Diſtance NC: Therefore by Perturbation of Proportion, the whole Weight A will be to the *Momentum* of



the Power in G, as GN to X. But the Proportion of GN to X, is compounded of the *Ratio* of GN to NC, and of the *Ratio* of NC to X, that is of FB to BO: Therefore the Weight A, to the Power which ſuſtains it in C, has a Proportion compounded of the Proportion of GN to NC, and of the Proportion of FB to BO. *Q. E. D.*

And now to return to our firſt Purpoſe: If the Things hitherto declared are well underſtood, it will be eaſily conceiv'd, That,

Prop. I.

A Priſm or Cylinder of Glaſs, Steel, Wood, or any other frangible Matter, which, ſuſpended lengthways, will ſuſtain a very great Weight hang'd thereat, will, ſideways

ways or tranſverſely (as we juſt now ſaid) be broken in Pieces by a far leſs Weight, according as its Length ſhall exceed its Thickneſs.

Imagine the Priſm ABCD to be fix'd into a Wall at the End AB, and at the other End imagine the Weight E to be hang'd, (the Wall here I underſtand to be perpendicular to the Horizon, and the Priſm or Cylinder to be perpendicular to the Wall) then 'tis manifeſt, if it muſt break,



it will be in B, where the Mortiſe in the Wall is in the Room of a *Fulcrum*, and the Line BC that Part of the Leaver where the Force is plac'd, and BA, the Thickneſs of the Solid, is the other Part of the Leaver, wherein is

the Reſiſtance to that Separation which muſt be made of BD, the Part of the Solid hanging out of the Wall, from that which is let into the Wall: And by what has been already ſhewn, the *Momentum* of the Power plac'd in C, to the *Momentum* of the Reſiſtance, which is in the Thickneſs of the Priſm, *i. e.* in the Connection of the Baſe BA with the contiguous Parts, hath the ſame Proportion as the Length CB has to half of BA; and therefore the abſolute



Reſiſtance againſt Fraction in the Priſm BD, (the abſolute Reſiſtance is that which is made if the Priſm be drawn directly; for then the Motion of the Mover is the ſame with that of the Body mov'd) to the Reſiſtance againſt Fraction

Fraction by Help of the Leaver BC, has the ſame Proportion as the Length BC has to half AB in the Priſm, which Half in the Cylinder is the Semidiameter of its Baſe. And this is the firſt Propoſition. But note that what I have here ſaid muſt be underſtood without the Conſideration of the Weight proper to the Solid BD, which Solid I took as depriv'd of all Weight. But in caſe we would conſider its Weight, together with the Weight E, we muſt add to the Weight E half the Weight of the Solid BD, ſo as the Weight of BD being, *e. g.* two Pounds, and the Weight E ten Pounds, we muſt take the Weight E as if it was eleven Pounds.

SIMP. And why not as if it was twelve, I pray?

SALV. The Weight E, *Simplicius*, hanging at the End C, gravitates in reſpect of the Leaver BC, with all its *Momentum* of ten Pounds, when if only BD hang there, it would gravitate with its whole *Momentum* of two Pounds; but, as you ſee, that Solid is uniformly diſtributed thro' its whole Length BC; wherefore its Parts near the End B, gravitate leſs than thoſe farther off: So that in the End, thoſe being compenſated with theſe, the Weight of the whole Priſm is brought to operate under the very Centre of Gravity which anſwers to the Middle of the Leaver BD: But a Weight hanging at the End C, has a *Momentum* double of that it would have hanging from the Middle: And therefore the half of the Weight of the Priſm muſt be added to the Weight E, when we uſe the *Momenta* of both, as if they were placed at the End C.

SIMP. I apprehend you very well; and farther, unleſs I'm deceiv'd, the Power of both Weights BD and E ſo placed, would have the ſame *Momentum* as if the whole Weight of BD, with twice the Weight E, were hung in the Middle of the Leaver BC.

SALV. You're very right; and pray remember this. Now we may eaſily underſtand,

PROP. II.

How, and with what Proportion, a Ruler or a Priſm broader than thick, reſiſts Fraction more if forc'd according to its Breadth, than according to its Thickneſs.

The better to underſtand this, imagine a Priſm or Ruler *ad*, whoſe Breadth is *ac*, and Thickneſs much leſs *bc*;

Fig. I.

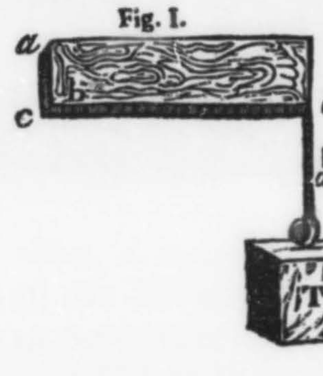


Fig. II.



Now the Thing here enquired after is this, *viz.* how it comes to paſs that it will reſiſt the great Weight T, if we attempt to break it edgeways, as in *Fig.* I. but plac'd flatways,

flatways, as in *Fig.* II. it will not reſiſt the Weight X, which is leſs than T: And the Thing is plain, ſince we ſuppoſe the *Fulcrum* in one caſe to be under the Line *ac*, and in the other, under *bc*, and the Diſtances of the Forces to be equal in both Caſes, *viz. cd*; But in the former Caſe the Diſtance of the Reſiſtance from the *Fulcrum*, which is half the Line *ca*, is greater than the Diſtance in the other Caſe, which is half the Line *cb*: Wherefore the Power of the Weight T muſt be neceſſarily greater than the Weight X, as much as the half of the Breadth *ca* is greater than half the Thickneſs *bc*, that ſerving for a Counter-Leaver to *ca*, and this to *cb*, to overcome the ſame Reſiſtance, *i. e.* the Quantity of Fibres of the whole Baſe *ab*.

Wherefore we conclude that the ſame Ruler or Priſm, which is broader than it is thick, reſiſts breaking more if plac'd edgeways than flatways; and that in Proportion of the Breadth to the Thickneſs.

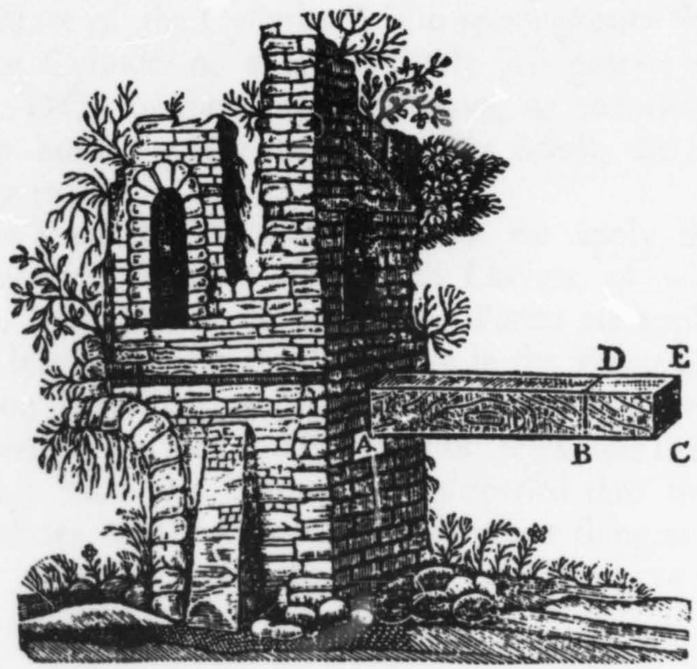
'Tis requiſite now that we begin,

PROP. III.

To find in what Proportion the *Momentum* of the proper Gravity increaſes in a Priſm or Cylinder, in relation to the proper Reſiſtance againſt Fraction, whilſt keeping its Paralleliſm to the Horizon it is made longer and longer: And this *Momentum* I find increaſes continually in a *duplicate* Proportion of that of the Prolongation.

For

For Demonſtration whereof, imagine a Priſm or Cylinder AB firmly faſten'd in the Wall at the End A, ſo as to be parallel to the Horizon, which afterwards imagine to be prolong'd to E, by adding thereto the Part BE. It is manifeſt that the Prolongation of the Leaver AB to C



by itſelf alone, *i. e.* taken abſolutely, increaſes the *Momentum* of the Force preſſing againſt the Reſiſtance to Fraction or Separation to be made in A, according to the Proportion of CA to BA; but moreover the *Weight* of the Solid BE adjoin'd to the *Weight* of the Solid AB, increaſeth the *Momentum* of the preſſing Gravity, according to the Proportion of the Priſm AE to the Priſm AB, which Proportion is the ſame with that of the Length AC to

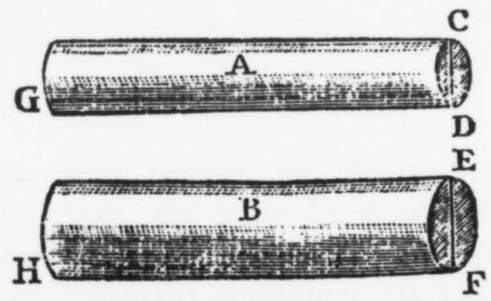
to the Length AB. Wherefore 'tis manifeſt that if the two Augmentations of both Lengths and Gravities be put together, the *Momentum* compounded of both is in a *duplicate Ratio* of either of them. We conclude, therefore, that the *Momenta* of the Forces of Priſms and Cylinders of equal Thickneſs, but unequal Lengths, are to one another in a *duplicate Ratio* of that of their Lengths; that is, are as the Squares of their Lengths.

We will ſhew in the ſecond Place, in what Proportion the Reſiſtance to Fraction in Priſms and Cylinders increaſes, ſuppoſing them of the ſame Length, but to increaſe in Thickneſs: And here I aſſert, That,

PROP. IV.

In Priſms and Cylinders of equal Length, but unequal Thickneſs, the Reſiſtance againſt Fraction increaſes in a *triplicate* Proportion of the Diameters of their Thickneſſes, that is, of their Baſes.

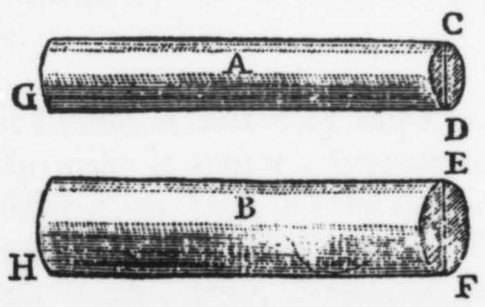
Let the two Cylinders be A and B, whoſe equal Lengths are DG and FH, and whoſe unequal Baſes are the Circles



whoſe Diameters are CD and EF. I ſay that the Reſiſtance of the Cylinder B againſt Fraction, to that of the Cylinder

Cylinder A, is in a *triplicate* Proportion of that of the Diameter FE, to the Diameter DC: For if we conſider the abſolute and ſimple Reſiſtance that reſides in the Baſes, *i. e.* in the Circles EF and DC to Fraction, by being forcibly pull'd directly or endways; without all Doubt the Reſiſtance of the Cylinder B is ſo much greater than that of the Cylinder A, as is the Circle EF greater than the Circle DC, for the Fibres, Filaments, or tenacious Parts, which hold together the Parts of the Solids, are ſo many the more.

But if we conſider that we, when we apply the Force tranſverſely, do make uſe of two Leavers, of which the Parts, or Diſtances at which the Forces are applied, are the Lines DG FH, the *Fulcra* are in the Points D and F, but the other Parts or Diſtances, at which the Reſiſtances are plac'd, are the Semidiameters of the Circle DC, EF; becauſe ſince the Filaments are deſperſed thro' the whole Superficies of the Circles, it is the ſame thing as if they were all reduced into the Centres: If, I ſay, we conſider



ſuch Leavers as theſe, we ſhall find the Reſiſtance in the Centre of the Baſe EF, oppoſed to the Force of H, to be ſo

ſo much greater than the Reſiſtance of the Baſe CD, againſt or oppoſed to the Force plac'd in G, (ſince the Forces in G and H are of equal Leavers DG and FH) as much as the Semidiameter FE is greater than the Semidiameter DC: the Reſiſtance againſt Fraction therefore in the Cylinder B, is more than the Reſiſtance of the Cylinder A, according to both the Proportions, *i. e.* of the Circles EF and DC, and of their Semidiameters, or, if you will, of their Diameters: but the Circles are in a *duplicate* Proportion of their Diameters; therefore the Proportion of their Reſiſtances, which is compos'd of them, is *triplicate* of that of their Diameters. *Q. E. D.*

But now becauſe Cubes are in a *triplicate* Proportion of their Sides, we may likewiſe conclude, *That the Reſiſtances of Cylinders of equal Length, are to one another as the Cubes of their Diameters.*

From what has been demonſtrated, we may likewiſe conclude, That,

COROLLARY.

The Reſiſtances of Priſms and Cylinders of equal Length are in *Seſquialter* Proportion of that of the Cylinders themſelves.

This is manifeſt, becauſe the Priſms and Cylinders of equal Height, have the ſame Proportion to each other as their Baſes have, that is, *duplicate* of the Sides or Diameters of their Baſes. But the Reſiſtance (as hath been demonſtrated) have a *triplicate Ratio* of the ſame Sides or Diameters: Therefore the Proportion of the Reſiſtances is *Seſquialter* to that of the ſaid Solids, and conſequently of the Weights of the ſaid Solids.

SIMP. Before we go any farther, I desire you'll answer me one Doubt, and that is this: I have not heard mentioned or consider'd another Sort of Resistance, that, in my Opinion, is diminished in Solids as they are longer and longer, and not only in taking them transversely, but also directly or longways; just as we see a very long one less fit to sustain a great Weight than a shorter is: Wherefore I believe a Ruler of Wood or Iron would sustain a much greater Weight, if it be short, than it would do if it were very long, provided it be always used lengthways, and not transversely, its own Weight comprehended, which in the longer is greater.

SALV. I fear, *Simplicius*, that you, with many others, are out in this Point, if I rightly apprehend your Meaning; You mean, I suppose, thus, that a Rope, *v. g.* forty Yards long, cannot sustain so great a Weight as one or two Yards of the same Rope will.

SIMP. I do mean so, and to me the Proposition seems very probable.

SALV. But I think it not only improbable, but false, as I shall easily make it appear: Suppose then this Rope AB to be fasten'd on high, by the End A; and at the other End let hang the Weight C, by the Force of which the Rope shall break; pray, *Simplicius*, assign me the particular Place where it will break.

SIMP. Let it be in D.

SALV. And

SALV. And why, pray, must it break in D?

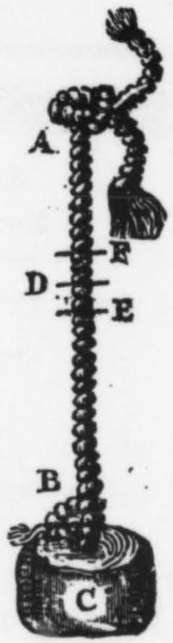
SIMP. Because the Rope is not strong enough in that Part to sustain, *e. g.* an hundred Pounds, which is the Weight of the Part of the Rope BD, together with that of the Stone C.



SALV. Whenever then we shall apply to this Rope the Weight of an hundred Pounds, it will break in D?

SIMP. I think so.

SALV. But tell me now, if one ſhould hang the ſame Weight not at the End of the Rope B, but to any Point as E, near the Point D; or elſe ſhould faſten the Rope not in A, but juſt above the ſame Point D, as at F: Tell me, I ſay, whether the Point D would feel the ſame Weight of an hundred Pounds?



SIMP. It would feel it, provided to the Stone C be added the Part EB of the Rope.

SALV. If then the Rope be drawn by the ſaid hundred Pounds of Weight, it will break in D, by your own Conceſſion; and yet FE is but a ſmall Part of the long Rope

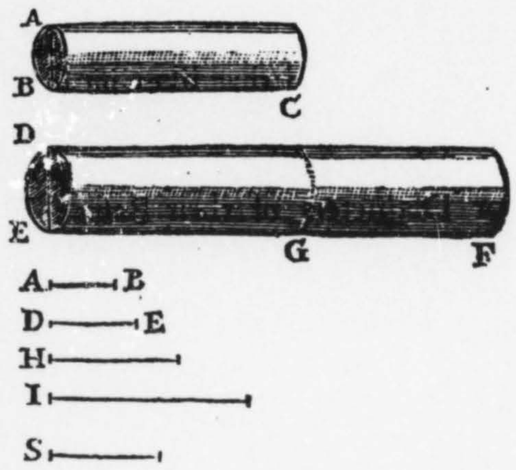
Rope AB: Why therefore do you continue in the Opinion that the long Rope is weaker than the ſhort one? I hope you're now ſatisfied, and that I have freed you from an Error, wherein you have had many Companions, and theſe otherwiſe very learned Men. Now let us proceed; and having now demonſtrated that the *Momentum* of Priſms and Cylinders increaſes above their Reſiſtances, according to the Squares of their Lengths (provided they are of the ſame Thickneſs); And that likewiſe if they are of the ſame Length, but different Thickneſs, they increaſe their Reſiſtances according to the Proportion of the Cubes of their Sides, or Diameters of their Baſes: Let us proceed, I ſay, and enquire what happens in ſuch Solids, which differ both in Length and Thickneſs: Where I obſerve, That,

PROP. V.

The Reſiſtances againſt Fraction of Priſms and Cylinders of different Length and Thickneſs, are in a Proportion compounded of the Proportion of the Cubes of the Diameters of their Baſes, and of the Proportion of their Lengths reciprocally taken.

Let

Let two ſuch Cylinders be ABC and DEF: I ſay, the Reſiſtance of the Cylinder AC, ſhall be to the Reſiſtance of the Cylinder DF, in a Proportion compounded of the Proportion of the Cube of the Diameter DE, and of the Proportion of the Length EF to the Length BC.



Suppoſe EG = BC, and let H be a third Proportional, and (I) a fourth, between the Lines AB and DE: And as EF is to BC, ſo let (I) be to S. Since now the Reſiſtance of the Cylinder AC is to the Reſiſtance of the Cylinder DG, as the Cube of AB to the Cube of DE; that is, as the Line AB to the Line (I): and the Reſiſtance of the Cylinder DG to the Reſiſtance of the Cylinder DF, as the Length FE to the Length EG; that is, as the Line (I) to the Line S: It will be, by Equality of Proportion, as the Reſiſtance of the Cylinder AC to the Reſiſtance of the Cylinder DF, as the Line AB to the Line S: But the Line AB has to the Line S a Proportion compounded

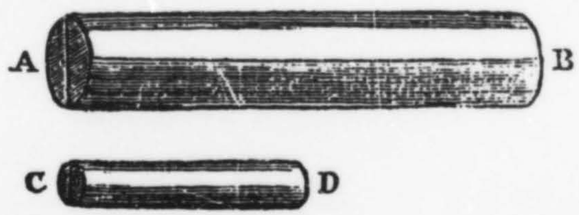
pounded of the Proportion of AB to (I), and of the Proportion of (I) to S: Therefore the Reſiſtance of the Cylinder AC, is to the Reſiſtance of the Cylinder DF, in a Proportion compounded of the Proportion of AB to (I), *i. e.* of the Cube of AB to the Cube of DE, and of the Proportion of the Line (I) to the Line S, *i. e.* of the Length EF to the Length BC. *Q. E. D.*

This Propoſition being demonſtrated, let us conſider what happens in ſimilar Cylinders and Priſms; concerning which we will demonſtrate, That,

PROP. VI.

Of like Cylinders and Priſms the *Momenta* compounded, *i. e.* thoſe reſulting from their Gravities, and from their Lengths, which are of the Nature of Leavers, have to one another a Proportion *Seſquialter* to that the Reſiſtances of their Baſes have.

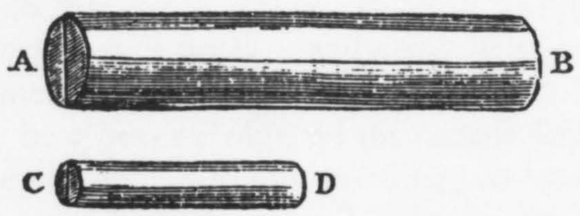
In order to demonſtrate which, deſcribe two ſimilar Cylinders AB and CD: Then I ſay that the *Momentum*



of the Cylinder AB, by which 'tis able to overcome the Reſiſtance of its Baſe B, hath a Proportion to the Reſiſtance of the Baſe D, *Seſquialter* to that which the ſame Reſiſtance of the Baſe B has to the Reſiſtance of the Baſe D:

And

And becauſe the *Momenta* of the Solids AB and CD, to overcome the Reſiſtances of their Baſes, are compounded of their Gravities, and the Forces of the Leavers; and the Force of the Leaver AB is = the Force of the Leaver CD, becauſe the Length AB has the ſame Proportion to the Semidiameters of the Baſe B, (the Cylinders being ſimilar) as the Length CD has to the Semidiameter of the Baſe D: It remains that the whole *Momentum* of the Cylinder AB, be to the whole *Momentum* of CD, as the ſole Gravity of the Cylinder AB to the ſole Gravity of the Cylinder CD, *i. e.* as the Cylinder AB to the Cylinder CD: But theſe Cylinders are in a *triplicate* Proportion of the Diameter of their Baſes B and D: and ſince the Reſiſtances of the ſame Baſes are to each other as the Baſes themſelves are, they are conſequently in a *duplicate* Proportion of the Diameters of their Baſes: Therefore the *Momenta* of Cylinders are in a *Seſquialter* Proportion of that which the Reſiſtances of their Baſes have.



SIMP. This Propoſition, indeed, is not only new to me, but unexpected, and at firſt Sight very remote from the Judgment I ſhould have made concerning it; for theſe Figures being in all other Reſpects alike, I ſhould have certainly

certainly concluded, That their *Momenta* alſo ſhould have obſerv'd the ſame Proportion to their proper Reſiſtances.

SAGR. This is the Demonſtration of that Propoſition, which, at the Beginning of our Diſcourſe, I thought I had a Sort of a Glimpſe of.

SALV. Some time ago I myſelf thought as you did; I believ'd that the Reſiſtances of like Solids were alike, until a certain Obſervation, tho' not altogether to be relied on, ſeem'd to repreſent to me that ſimilar Solids do not preſerve the ſame Tenor in their Toughneſs, but that the bigger are leſs apt to ſuſtain violent Accidents, or Shocks; as in falling, luſty Men are more liable to Hurt than little Children. And (as we ſaid at our Beginning) as we ſee a great Beam or Column, falling from on high, to break, whilſt a ſmall Stick does not, tho' falling from the ſame Height. This is the Obſervation which put me upon enquiring after that wonderful Property which I am now going to demonſtrate to you, *viz.* That of the infinite Number of ſimilar Solids, there are not two whoſe *Momenta* keep the ſame Proportion to their proper Reſiſtances.

SIMP. Now you bring into my Mind ſomething which *Ariſtotle* has among his Mechanical Queſtions, where he endeavours to give a Reaſon why Beams become weaker, and bend more, the longer they are, altho' the longer are thicker than the ſhorter: And, if I remember right, he reduces it to that of the ſimple Leaver.

SALV. 'Tis very true; And becauſe that Solution ſeem'd to leave ſome Room for Doubt, Monſignore *di Guevara*, who in his truly learned Commentaries hath greatly illuſtrated this Work, not to mention his other ſubtile Speculations, ſets himſelf to remove all theſe Difficulties: Yet he himſelf remain'd ſtill doubtful in this Point, *viz.* Whether the Lengths and Thickneſſes of ſuch Solids, increaſing in the ſame Proportion, ought to retain the ſame Proportion of Force and Reſiſtance againſt Fraction and Flexion.

After a long Conſideration of this Matter, I hit upon what I am going to tell you: And firſt I will demonſtrate, That,

PROP. VII.

In ſimilar and heavy Priſms and Cylinders, there is one and but one, that (charg'd with its own Weight) is reduced to the ultimate State betwixt breaking and not breaking; ſo that every greater, as being unable to reſiſt its own Weight, will break; and every leſſer will be able to reſiſt ſome additional Force apply'd to break it.

Let AB be a heavy Priſm, of the utmoſt Length it can poſſibly be to hold without breaking, ſo that if it be lengthen'd never ſo little, it will inſtantly break: I ſay, that this is the only one, amongſt all thoſe that are ſimilar to it, (which are infinite in Number) which is capable of being reduced into ſuch a doubtful and tickliſh State, that every greater will break by its own Weight, and every leſſer will not; but, on the contrary, will be able to reſiſt or bear ſome additional Weight.

Firſt;

Firſt; Let the Priſm CE be ſimilar and greater than AB: I ſay, this cannot conſiſt, but muſt break, being overcome by its own Gravity: Suppoſe the Part CD to be of the ſame Length with AB: Now becauſe the Reſiſtance of CD, is to that of AB, as the Cube of the Thickneſs of CD to the Cube of the Thickneſs of AB, *i. e.* as the Priſm CE to the Priſm AB, (ſince they are ſimilar) and therefore the Weight of CE is the utmoſt that can be ſuſtain'd at the Length of the Priſm CD; but the Length CE is greater; therefore the Priſm CE will break.



Secondly; Let FG be a ſimilar Priſm, and leſs than AB; we will likewiſe demonſtrate, (ſuppoſing FH = BA) that the Reſiſtance of FG is to the Reſiſtance of AB, as the Priſm FG to the Priſm AB, provided the Diſtance AB, *i. e.* FH be = FG; but it is greater: therefore the *Momentum* of the Priſm FG, plac'd in G, is not ſufficient to break the Priſm FG.

SAGR. The Demonſtration is ſhort and clear; and now the Truth and Neceſſity of the Propoſition plainly appears, which at firſt Sight ſeem'd very improbable: It would therefore be requiſite to alter ſufficiently the Proportion betwixt the Length and Thickneſs of the greater Priſm, by making it thicker or ſhorter, in order to reduce

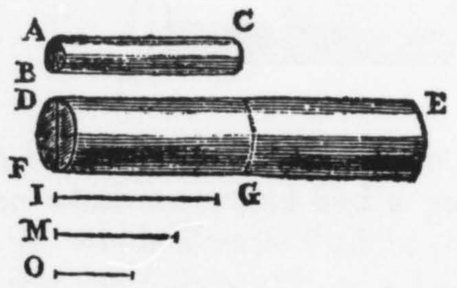
it to that nice and dubious State between breaking and not breaking: And now the Inveſtigation of this State would, methinks, be no leſs ſubtile and ingenious.

SALV. Nay, rather more, as it is more laborious: I'm ſure I ſpent no little time in finding it out; and I will now impart it to you.

PROP. VIII. PROB. I.

A Cylinder or Priſm of the greateſt Length, not to be broken by its own Weight, and alſo a greater Length being given, to find the Thickneſs of another Cylinder or Priſm, which is the only one and biggeſt of that given Length that can reſiſt its own Weight.

Let the Cylinder BC be the greateſt that can reſiſt its own Weight, and let DE be a Length greater than AC; it is required to find the Thickneſs of the Cylinder that is the greateſt of the Length DE that can reſiſt its own Weight. Let (I) be a third Proportional to the Lengths



DE and AC; and as DE is to (I), ſo let the Diameter FD be to the Diameter BA; and make the Cylinder FE: Then

Then I ſay, that this is the greateſt and the only one amongſt all that are ſimilar to it, that can ſuſtain its own Weight. To the Lines DE and (I), let M be a third Proportional, and O a fourth; and ſuppoſe FG to be = AC: Becauſe the Diameter FD is to the Diameter AB, as the Line DE to (I), and O is a fourth Proportional to DE and (I), the Cube of FD will be to the Cube of BA as the Line DE to O: But as the Cube of FD to the Cube of BA, ſo is the Reſiſtance of the Cylinder DG to the Reſiſtance of the Cylinder BC; therefore the Reſiſtance of the Cylinder DG is to the Reſiſtance of the Cylinder BC, as the Line DE to the Line O.

And becauſe the *Momentum* of the Cylinder BC is = its Reſiſtance, if we ſhew that the *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder BC, as the Reſiſtance DF is to the Reſiſtance BA, *i. e.* as the Cube of FD to the Cube of BA, *i. e.* as the Line DE to the Line O, we have our Deſire.

The *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder DG, as the Square of the Line DE to the Square of AC, *i. e.* as the Line DE to (I): But the *Momentum* of the Cylinder DG is to the *Momentum* of the Cylinder BC, as the Square of DF to the Square of BA, *i. e.* as the Square of DE to the Square of (I), *i. e.* as the Square of (I) to the Square of M, *i. e.* as the Line (I) to O; therefore by Equality of Reaſon: As the *Momentum* of the Cylinder FE to the *Momentum* of the Cylinder BC, as the Line DE to O, *i. e.* as the Cube of DF to the Cube of BA, *i. e.* the Reſiſtance of the Baſe DF to the Reſiſtance of the Baſe BA; which is what was ſought.

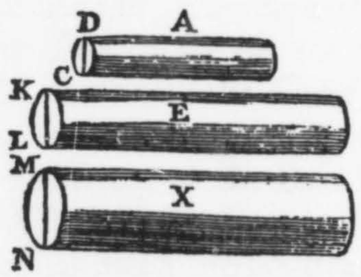
SAGR. This

SAGR. This is a long Demonſtration, and hard to be remember'd at once hearing; therefore pray, *Salviatus*, be pleaſed to repeat it.

SALV. With all my Heart——But may be I had better give you a more conciſe and ſhorter one; but then we muſt have a Figure ſomewhat different.

SAGR. This will be ſtill more obliging: But pray let me have this laſt Demonſtration in Writing, that I may conſider it at Leiſure.

SALV. You ſhall have it:——Now ſuppoſe a Cylinder A, the Diameter of whoſe Baſe let be the Line DC,



and let this A be the greateſt that can ſuſtain itſelf without breaking, than which we would find a greater ſimilar to the Cylinder A, which likewiſe ſhall be the greateſt, and the only one that can ſuſtain itſelf at a Length equal to a given Line; and let this Cylinder be E, the Diameter of whoſe Baſe let be KL; and let MN be a third Proportional to the Lines DC and KL, which MN let be the Diameter

Diameter of the Baſe of the Cylinder X, equal in Length to the Cylinder E: Then I ſay, that X is the Cylinder ſought.

Becauſe the Reſiſtance DC is to the Reſiſtance KL as the Square of DC to the Square of KL, *i. e.* as the Square of KL to the Square of MN, *i. e.* as the Cylinder E to the Cylinder X, *i. e.* as the *Momentum* E to the *Momentum* X: But the Reſiſtance KL to the Reſiſtance MN, is as the Cube of KL to the Cube of MN, *i. e.* as the Cube of DC to the Cube of KL, *i. e.* as the Cylinder A to the Cylinder E, *i. e.* as the *Momentum* A to the *Momentum* E: and thence by Perturbation of Proportion, as the Reſiſtance DC to the Reſiſtance MN, ſo the *Momentum* A to the *Momentum* X: and thence the Cylinder X is in the ſame Conſtitution of *Momentum* and Reſiſtance with the Cylinder A: But let's make this Problem more general; and let the Propoſition be this,

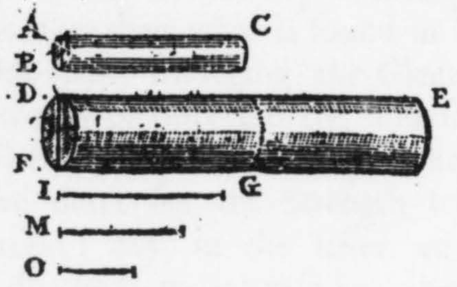
PROP.

The Cylinder AC being given with any Proportion at Pleaſure of its *Momentum* to its Reſiſtance; and any Length DE being alſo given; to find the Thickneſs of the Cylinder whoſe Length is DE, and whoſe *Momentum* has the ſame Proportion to its Reſiſtance, as the *Momentum* of the Cylinder AC has to its Reſiſtance.

Reſuming the foregoing Figure, *Page* 188. and nearly the ſame Progreſs, we ſay,

Becauſe

Becauſe the *Momentum* of the Cylinder FE is to the *Momentum* of the Part DG, as the Square of ED to the Square of FG, *i. e.* as the Line DE to (I) : and the *Momentum* of the Cylinder FG to the *Momentum* of the Cylinder AC, is as the Square of FD to the Square of AB,



i. e. as the Square of DE to the Square of (I), *i. e.* as the Square of (I) to the Square of M, *i. e.* as the Line (I) to O : Therefore, *ex Equali*, the *Momentum* of the Cylinder FE is to the *Momentum* of the Cylinder AC, as the Line DE to O, that is, as the Cube of DE to the Cube of (I), that is, as the Cube of FD to the Cube of AB, that is, as the Reſiſtance of the Baſe FD to the Reſiſtance of the Baſe AB. *Q. E. F.*

Now from what has been demonſtrated, you may plainly ſee how impoſſible it is, not only for Art, but alſo for Nature herſelf, to increaſe her Machines to an immenſe Bigneſs : That immenſely great Ships, Palaces, Temples, can't be built by Art, whoſe Oars, Yards, Beams, * Iron Bolts, and, in a Word, whoſe other Parts will conſiſt or hold together : Nor can Nature produce Trees

* Catene di Ferro.

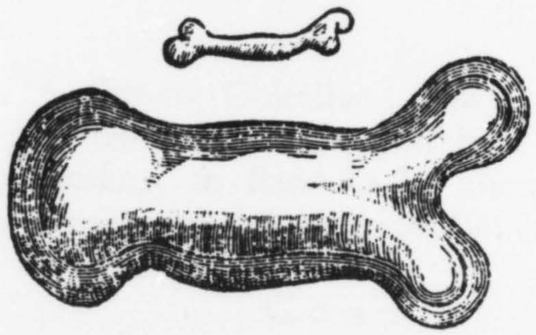
of

of a moſt immenſe Bigneſs, ſince their Arms or Boughs would at length break by their own Weight : As it is alſo impoſſible for her to give Bones for Men, Horſes, or other Animals, ſo made as to ſubſiſt, and proportionally to perform their Offices, when ſuch Animals ſhould be enlarged to immenſe Heights, unleſs ſhe uſes Matter much harder and more reſiſting than ſhe commonly does ; or elſe ſhould make ſuch Bones of a Thickneſs out of all Proportion ; whence the Figure and Look of the Animal muſt become monſtrous : Which, may be, my moſt witty Poet hinted at, where, deſcribing a huge Giant, he ſays,

Non ſi può compartir quanto ſià lungo
Si ſmiſuratamente è tutto groſſo.

Whatever Height we to the Giant give,
He cannot without equal Thickneſs live.

To explain this a little by an Example, I have here drawn the Figure of a Bone, three times longer than a common one, and increaſed in Thickneſs in ſuch Proportion, as that it may, in its great Animal, perform its Office proportionally to what the leſſer Bone does in the leſſer Animals : The Figures are theſe ;



Where you ſee what a Figure, out of all Proportion, that of the augmented Bone makes.

Whence 'tis manifeſt, that he that would, in an huge Giant, keep the ſame Proportion that the Members have in an ordinary ſiz'd Man, muſt either find Matter much harder, and more reſiſting, to make Bones of, or elſe muſt admit, that its Conſiſtency or Strength is in Proportion much weaker than what is found in Men of a middle Stature. Otherwiſe, nlairging the Giant to an immenſe Height, he would be oppreſs'd by, and fall under his own Weight: Whereas, on the contrary, when Bodies are diminiſhed, we don't ſee the Strength to diminiſh in the ſame Proportion: nay, in the leſſer we obſerve the Robuſtneſs or Strength to increaſe in a greater Proportion: So that I believe that a little Dog could carry two or three Dogs of equal Size with himſelf, on his Back; but I don't think an Horſe could carry ſo much as one ſingle Horſe of his own Size.

SIMP. I muſt own I don't throughly agree with you in this; and the immenſe Bulks that we ſee in Fiſhes have occaſion'd it: Thus, for Inſtance, a Whale, as I have heard, is as big as ten Elephants, yet ſuſtains itſelf.

SALV. The Objection you have ſtarted, *Simplicius*, hints to me another Condition I did not before think of, by Help whereof, even Giants and other vaſt Animals may conſiſt, and move themſelves as eaſily as ſmaller do: And this will happen, not only when greater Strength is added to the Bones and other Parts, whoſe Office it is to ſuſtain their own Weight, and the Weight that's upon them;

them; but the Structures of the Bones keeping the ſame Proportions, the ſame Fabricks would juſt in the ſame manner, nay, indeed, would more eaſily conſiſt, if the Gravity of the Matter of thoſe Bones, as alſo that of the Fleſh, or of whatever elſe is to reſt itſelf upon the Bones, be diminiſh'd in that Proportion: And indeed Nature has made uſe of this Artifice in the framing of Fiſhes, making their Bones and Muſcles, not only very light, but without any Gravity.

SIMP. I ſee, *Salviatus*, whither your Diſcourſe tends: You'll ſay, That ſince the Element of Water is the Habitation of Fiſhes, which by its Corpulency, or, as others will, its Gravity, diminiſhes the Weight of Bodies immerſed in it; for this Reaſon the Matter of Fiſhes, not weighing any thing, may, without any Detriment to the Bones, be ſuſtained. But this is not enough, becauſe, altho' the reſt of the Subſtance of the Fiſh weigh not, yet without doubt the Matter of the Bones muſt gravitate, or muſt have its Weight: Who will ſay that the Rib of a Whale, that's as big as a Beam, does not weigh a great deal, and when in Water does not ſink to the Bottom? It ought therefore to follow from what you have ſaid, that in ſo vaſt a Bulk, theſe ſhould not be able to ſubſiſt.

SALV. A ſhrewd Objection: But to anſwer your Doubt; pray tell me whether you have ever obſerv'd Fiſhes, at Pleaſure, to ſtand or lie immoveable under Water, ſo as neither to deſcend to the Bottom, nor riſe to the Top, without ſtirring their Fins.

SIMP. I have often obſerv'd it.

SALV. Fiſhes then being able to ſtay themſelves without any Motion, in the Midſt of the Water, 'tis an infallible Argument that the Compoſition of their corporeal Maſs, equalleth the ſpecifick Gravity of the Water; ſo that if there be found in them ſome Parts heavier than Water, it is neceſſarily required that there be others ſo much lighter, as to procure an *Equilibrium*.

Wherefore if the Bones be heavy, 'tis neceſſary that the Muſcles and other Matter be light, that theſe with their Lightneſs may compenſate the Weight of the Bones: Thus in Aquatick Animals, the contrary happens to what befalls the Terreſtrial Ones; namely, in the latter 'tis the Office of the Bones to ſuſtain their own Weight with that of the Fleſh; in the former the Fleſh ſuſtains its own Weight, with that of the Bones: Let's no longer wonder then that there may be vaſt Animals in the Water, but not on the Land, that is, in the Air.

SIMP. I am ſatisfied; and can't help obſerving that thoſe Animals which we call Terreſtrial, ought, with greater Reaſon, to be call'd Aerial, ſince they in reality live in the Air, and are ſurrounded by the Air, and breathe therein.

SAGR. *Simplicius*'s Doubt and its Solution pleaſes me mightily: And I eaſily comprehend farther, that one of theſe huge Fiſhes being hauled on Shore, could not long ſuſtain itſelf; but that the Joints or Connexions of the Bones

†

Bones being relaxed, its Maſs would be cruſh'd under its own Weight.

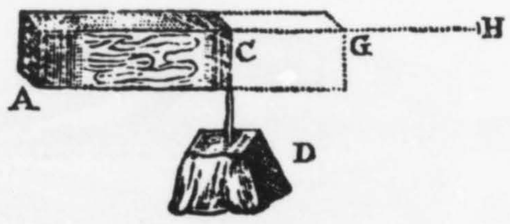
SALV. At preſent I'm of the ſame Opinion: and I almoſt think that the ſame thing would happen to that huge Ship, which, whilſt floating on the Sea, is not diſſolv'd by its own Weight, and the great Burden of its Lading and Artillery; but was it on dry Ground, and ſurrounded with Air, perhaps it would fall to Pieces.

But let us proceed and demonſtrate how,

PROP. IX. PROB. II.

A Priſm or Cylinder with its Weight, and the greateſt Weight it can ſuſtain being given, to find the greateſt Length, beyond which if it be prolong'd, it would break by only its own Weight.

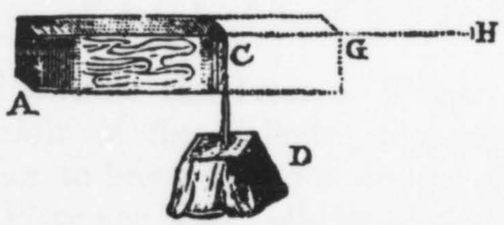
Let there be given the Priſm AC, with its Weight, and likewiſe let the greateſt Weight D be given that can be



ſuſtain'd at its Extremity C; 'tis required to find the greateſt Length to which the ſaid Priſm may be prolong'd without breaking: Make the Line AH in the ſame Proportion to the Length AC, as the Compound of the Weight

of

of AC, and the double Weight of D, is to the Weight of the Priſm AC; and let AG be a mean Proportional between AC and AH: Then I ſay that AG is the Length ſought. For the gravitating *Momentum* of the Weight D in C, is equal to the *Momentum* of the double Weight of D, if it be plac'd in the Middle of AC, where is alſo the Centre of the *Momentum* of the Priſm AC: The *Momentum* therefore of the Reſiſtance of the Priſm AC which is in A, is equivalent to the Gravitation of the Double of the Weight D, together with the Weight of AC, hang'd alſo in the Middle of AC: And ſince it has been made as the *Momentum* of the ſaid Weights ſo plac'd, *i. e.* of the Double of D with AC, to the *Momentum* of AC, ſo the Line HA to the Line AC, between which the Line AG is a mean Proportional; therefore the *Momentum* of the Double of D, with the *Momentum* of AC, will be to the *Momentum* of AC, as the Square of GA to the Square of AC: But the preſſing *Momentum* of the Priſm GA, is to the *Momentum* of AC, as the Square of GA to the Square



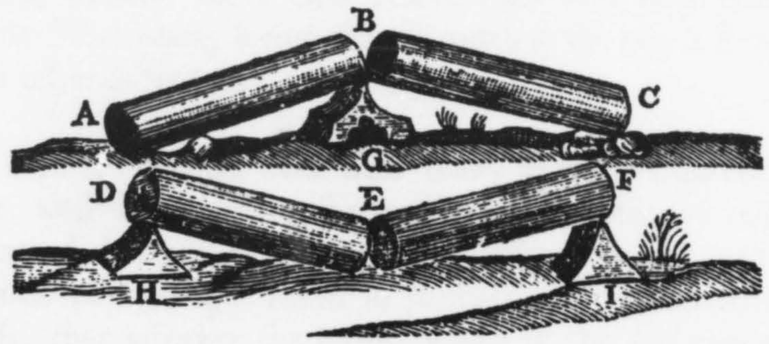
of AC; Therefore that Length AG is the greateſt ſought, *i. e.* that to which the Priſm AC being prolong'd, ſuſtains itſelf, but beyond which Length were it produc'd, it would break. Hitherto we have conſider'd the *Momenta* and Reſiſtances

Reſiſtances of Priſms and Cylinders, one End whereof has been ſuppoſed fix'd, and to the other only the Force of a preſſing Weight apply'd, conſidering either that Weight by itſelf, or join'd to that of the Solid: or elſe the ſole Gravity of the ſaid Solid. Now let us ſay ſomething of thoſe ſame Priſms and Cylinders, ſuppoſing them fix'd or ſuſtain'd at both Ends, or elſe to reſt upon one ſole Point taken between the Ends: And firſt, I ſay that,

PROP. X.

The Cylinder that charg'd with its own Weight is reduced to its greateſt Length, beyond which it could not ſuſtain itſelf, whether it be ſuſtain'd in the Midſt by one *Fulcrum*, or by two at its Extremities, may be in Length double of that faſten'd in a Wall as before, *i. e.* which is ſuſtain'd only at one End.

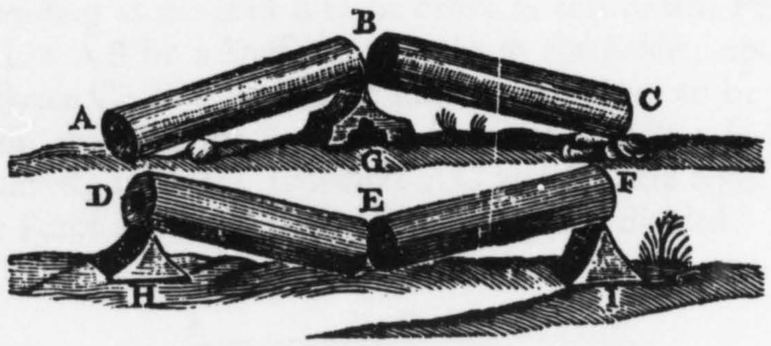
This is very manfeiſt; for if we ſuppoſe the half AB of the Cylinder ABC to be the utmoſt Length it can



ſuſtain itſelf at, whilſt fix'd at the End B, it will ſuſtain itſelf in the ſame manner, if, being laid on the *Fulcrum* G,

it

it be counterpoiſed by its other Half BC: And in like manner, if the Length of the Cylinder DEF be ſuch that only one half of it can be ſuſtain'd, whilſt 'tis fix'd at the End D, and conſequently the other EF fix'd in the Point F, 'tis manifeſt that if the *Fulcra* H and (I) be plac'd under



the Extremities D and F, any *Momentum* of Force or Weight added in E, will there make a Fracture.

That which requires a more ſubtile Speculation, is this; when abſtracting the proper Gravity of thoſe Solids, 'tis propoſed to us,

PROP. XI. PROB. III.

To find whether that Force or Weight, which apply'd to the Middle of the Cylinder, ſupported at its Ends, would ſuffice to break it, could do the ſame applied to any other Place not in the Middle of that Cylinder.

As for Example: If going to break a Staff, we hold it at each End in our Hand, and clap our Knee againſt the Middle of it; whether the ſame Force which ſuffices to break it after this manner, would alſo ſuffice in caſe the Knee

Knee were ſet not againſt the Middle of it, but nearer to one End.

SAGR. I think *Ariſtotle* has mention'd this Problem in his *Mechanical Queſtions*.

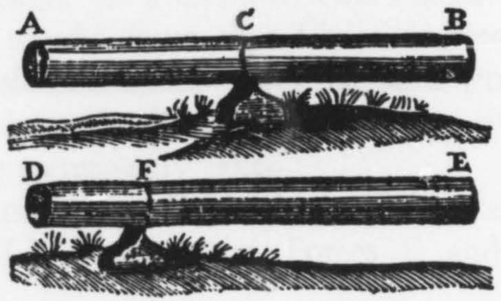
SALV. His is not quite the ſame; for he only ſeeks a Reaſon why leſs Strength is required to break the Staff with one's Knee, holding it in our Hands at the very Ends, that is, as far diſtant as poſſible from the Knee, than if we hold it nearer its Middle; and he gives a general Reaſon for the Thing, reducing it to the *Leaver*; which Leavers are longer when the Arms are extended, and the very Ends graſp'd: But we ſeek more than this, *viz.* Whether ſetting the Knee to the Midſt of the Staff, or to any other Part of it, always keeping the Hands at its Ends, the ſame Force be ſufficient in all Situations to break it.

SAGR. At firſt View, methinks, the ſame Force ſhould ſerve, becauſe thoſe two Leavers do as it were retain the ſame *Momentum*, ſeeing that ſo much as the one is ſhorten'd, the other is lengthen'd.

SALV. You ſee now how ready at Hand Equivocations are, and how cautiouſly and circumſpectly we ought to proceed, leſt we run into them: What you ſay, and what indeed at firſt ſight ſeems ſo probable, is, in Strictneſs, ſo falſe, that whether the Knee, which is the *Fulcrum* of the two Leavers, be plac'd or not plac'd againſt the Middle, makes ſuch an Alteration, that 'tis not enough to apply

four, nor ten, nor an hundred, no nor a thouſand times that Force which is ſufficient to break it in the Middle, to break it in ſome other Place: Firſt, we will conſider this in general, then come to the particular Determination of the Propoſition, *viz.* To find how the Forces are alter'd according as the Staff is to be broke in this or that Point.

Let AB be a Staff to be broke in the Midſt, upon the *Fulcrum* C; and let DE be juſt ſuch another, to be broke upon the *Fulcrum* F, diſtant from its Middle: Firſt, 'tis manifeſt, ſince the Diſtances AC and BC are equal, that the Force at the Ends B and A, is equally divided.



Secondly, becauſe the Diſtance DF is leſs than the Diſtance AC, the *Momentum* of the Force plac'd in D is leſs than the *Momentum* in A; that is plac'd at the Diſtance CA, and that according to the Proportion of the Line DF to the Line AC; and conſequently it muſt be increaſed to equal, or exceed the Reſiſtance of F: But now the Diſtance DF, in reſpect of the Diſtance AC, may be diminiſh'd *ad infinitum*: Therefore 'tis requiſite that it be poſſible to increaſe the Force which ſhall be applied in D alſo *ad infinitum*, to equal the Reſiſtance in F.

But,

But, on the contrary, as the Diſtance FE increaſes above CB, we muſt leſſen the Force in E, to equal or compenſate the Reſiſtance in F; but the Diſtance FE, in reſpect of CB, cannot increaſe *ad infinitum*, by removing the *Fulcrum* F towards the End D, nor indeed to its Double: Wherefore the Force in E, to equal or compenſate the Reſiſtance in F, will be always more than half the Force in B. Thus we eaſily ſee the Neceſſity of augmenting infinitely the *Momenta* of the collected Forces in E and D, to equal or exceed the Reſiſtance plac'd in F, according as the *Fulcrum* F continually approaches nearer and nearer to the End D.

SAGR. What ſhall we ſay, *Simplicius*? Muſt we not own *Geometry* to be a more powerful Inſtrument than all others to ſharpen the Wit, and to diſpoſe it to reaſon rightly? And that *Plato*, with great Reaſon, deſired his Scholars in the firſt Place to be grounded in the Mathematicks? For my Part, tho' I before well underſtood the Nature of the Leaver, how, its Length increaſing and decreaſing, the *Momentum* of the Force and of the Reſiſtance is increaſed or diminiſhed; yet in the Determination of the preſent Propoſition I was infinitely deceiv'd.

SIMP. The Truth is, I begin to ſee that *Logic*, how inſtrumental ſoever it be to direct and regulate our Diſcourſe, does by no means reach the Acuteneſs of *Geometry*, in ſtirring up our Minds to Invention.

SAGR. *Logic* ſeems to me to teach us how to diſcover whether Diſcourſes and Demonſtrations already made and

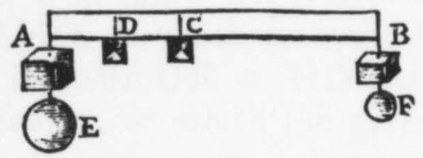
invented, be ſolidly concluſive; but that it teaches how to diſcourſe and reaſon concluſively, indeed I don't believe: But I wiſh *Salviatus* would demonſtrate to us in what Proportion the *Momenta* of Forces increaſe, to overcome the Reſiſtance to Fraction of the ſame Staff, in Places farther and farther from the Middle.

SALV. This Proportion proceeds in ſuch Manner, That,

PROP. XII.

If in the Length of a Cylinder two Places be mark'd, in which we would have that Cylinder break, the Reſiſtances at thoſe two Places have the ſame Proportion to each other, as have the Rectangles made by the Diſtances of thoſe Places reciprocally taken.

Let the Forces A and B be the leaſt that can break the Cylinder in C, and let the Forces E and F be the



leaſt that can break it in D: then, I ſay, that the Forces A and B have the ſame Proportion to the Forces E and F, as the Rectangle ADB has to the Rectangle ACB.

For the Forces A and B have to the Forces E and F a Proportion compounded of that of the Forces A and B, to the Force B; of that of the Force B to the Force F; and of that of the Force F to the Forces F and E: But as the Forces

Forces A and B are to the Force B, ſo is the Line BA to AC; and as the Force B is to the Force F, ſo is the Line DB to the Line BC; and as the Force F is to the Forces F and E, ſo is the Line DA to the Line AB; therefore the Forces A and B have to the Forces E and F a Proportion compounded of theſe three, *i. e.* of the right Line BA to AC; of the right Line DB to BC; and of the right Line DA to AB: But of the two Proportions DA to AB, and BA to AC, is compoſed the Proportion of DA to AC; therefore the Forces A and B have to the Forces E and F a Proportion compounded of the Proportions DA to AC, and of DB to BC: But the Rectangle ADB hath to the Rectangles ACB a Proportion compounded of the ſame Proportions of DA to AC, and of DB to BC; therefore the Forces A and B are to the Forces E and F, as the Rectangle ADB to the Rectangle ACB; which is as much as to ſay that the Reſiſtance againſt Fraction in C, has the ſame Proportion to the Reſiſtance againſt Fraction in D, as the Rectangle ADB to the Rectangle ACD. *Q. E. D.*

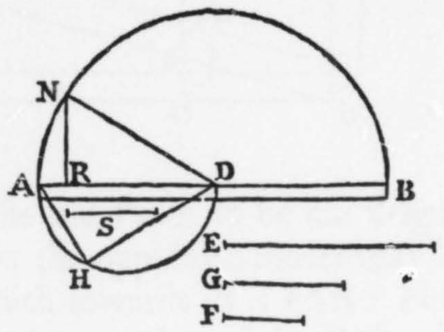
As a Conſectary of this Theorem we may eaſily ſolve the following curious Problem.

PROP. XIII. PROB. IV.

The greateſt Weight being given that can be ſuſtain'd at the Middle of a Cylinder or Priſm, where its Reſiſtance is leaſt; and a Weight being likewiſe given greater than that; to find in the ſaid Cylinder or Priſm, a Point at which the given greater Weight may be the greateſt can be there ſupported:

Let

Let the given Weight, that is greater than the greateſt Weight that can be ſupported at the Middle of the Cylinder AB, be to the ſaid greateſt Weight in Proportion as the Line E is to the Line F. Now 'tis required to find the Point in the Cylinder, at which the greater given Weight is the greateſt that can be ſuſtain'd.



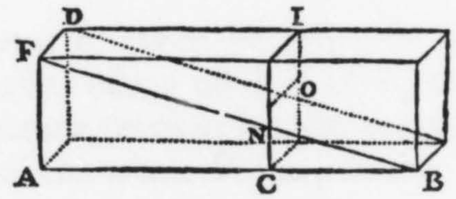
Between the two Lines E and F let G be a mean Proportional; and as E is to G, ſo let AD be to S; then S will be leſs than AD: Let AD be the Diameter of the Semicircle AHD, in which draw the Line AH = S, and join H and D, make DR = HD; then I ſay, that R is the Point ſought, at which the given Weight, which is greater than the given greateſt that can be ſuſtain'd at the Middle of the Cylinder D, would be the greateſt that could be there ſupported.

On the Line BA deſcribe the Semicircle ANB, and raiſe the Perpendicular RN, and join N and D: Now becauſe the two Squares of NR and RD are = to the Square of ND, *i. e.* to the Square of AD, *i. e.* to the two Squares AH and HD; and the Square of HD is equal to the Square of DR; therefore the Square of NR, *i. e.* the Rectangle

Rectangle ARB is = to the Square of AH, *i. e.* to the Square of S: But the Square of S is to the Square of AD, as the Line F to the Line E, *i. e.* as the greateſt ſupportable Weight at D to the given greater Weight: Therefore this greater ſhall be ſupported in R, and be the greateſt Weight that can there be ſupported; which is what was ſought.

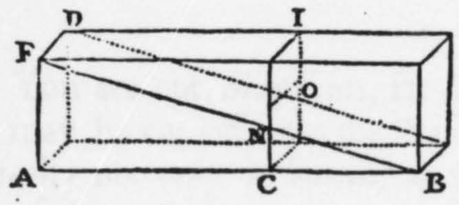
SAGR. I underſtand you very well; and am further thinking, that ſince the Priſm AB becomes ſo much the ſtronger, and reſiſts more againſt Preſſion in thoſe Parts that are more and more diſtant from its Middle, that, I ſay, in very great and heavy Beams we may cut or ſaw off a good deal from towards their Ends, which would much leſſen their Weight; a thing in Beams for very large Rooms very commodious, and of no ſmall Advantage. Now how curious it would be to find what Figure that Solid muſt be of, to have an equal Reſiſtance in all its Parts; ſo that it ſhould not be by any Weight more eaſily broke in the Middle than in any other Place?

SALV. I was juſt going to mention a notable Thing to you, and a very pleaſant one, to this Purpoſe: And the better to explain my Meaning, ſee the following Figure.



The

The Figure DB is a Priſm, whoſe Reſiſtance againſt Fraction in the Extremity AD, by a Force preſſing at the End B, is ſo much leſs than the Reſiſtance that would be found in the Place CI, as the Length CB is leſs than the Length CA, as has been juſt now demonſtrated.



Now ſuppoſe the ſaid Priſm to be cut diagonally along the Line FB, ſo as the oppoſite Surfaces may be two Triangles, one of which towards us is FAB : Now this Solid is of a Nature contrary to that of the Priſm, *i. e.* it reſiſts Fraction by the Force plac'd in B, ſo much leſs at the Term C than at the Term A, as the Length CB is leſs than the Length BA ; as we ſhall eaſily prove thus ;

The Section CNO being ſuppoſed parallel to the other AFD, the Line FA in the Triangle FAB has the ſame Proportion to the Line CN, as the Line AB to the Line CB ; and therefore if we ſuppoſe the *Fulcra* of the two Leavers to be plac'd in the Points A and C, whoſe Diſtances BA, AF, BC, and CN will be ſimilar ; and therefore ſuch *Momentum* as the Force plac'd in B, at the Diſtance BA, has above the Reſiſtance plac'd at the Diſtance AF, ſuch will the ſame Force in B, at the Diſtance BC, have above the ſame Reſiſtance, were it plac'd at the Diſtance CN : but the Reſiſtance to be overcome in the *Fulcrum* C, plac'd at the Diſtance CN, by the Force B, is ſo much leſs than the

the Reſiſtance in A, as the Rectangle CO is leſs than the Rectangle AD, *i. e.* as the Line CN is leſs than the Line AF, *i. e.* as the Line CB is leſs than the Line BA ; therefore the Reſiſtance of the Part OCB againſt Fraction in C, is ſo much leſs than the Reſiſtance of the Whole DAO againſt Fraction in A, as the Length CB is leſs than the Length AB.

Wherefore we have taken from the Beam or Priſm one half, by cutting it diagonally, and we have the Wedge or triangular Priſm FBA left ; which two Solids, that is, the whole Priſm, and that left, are of contrary Qualities, *i. e.* that, *viz.* the Priſm DB, the ſhorter it is, the more it reſiſts ; and this, *viz.* the Wedge FBA, the ſhorter you make it, the weaker it grows : Now this being ſo, it ſeems very reaſonable, nay neceſſary, that it may be ſo cut, that, taking off what is ſuperfluous, a Solid may remain of ſuch Figure as in all its Parts ſhall have the ſame Reſiſtance.

SIMP. It muſt unavoidably be ſo ; for where there's a Tranſition from a bigger to a leſs, we muſt needs meet with the Equal.

SAGR. But now the Buſineſs is to find the Curve along which the Priſm muſt be cut to have this Effect.

SIMP. This ſeems to me to be eaſy ; for if by cutting the Priſm diagonally, taking off half of it, the Body that remains be of a Nature (as to its Reſiſtance againſt Fraction) contrary to that of the whole Priſm ; ſo that in all Places wherein this acquires Strength, that in the ſame Proportion

 loſes

loſes it; methinks if we take a *Medium*, and take off only one half of that half, the remaining Figure will neither gain nor loſe any Strength, in all thoſe Places in which the two other Bodies loſt and gain'd an equal Portion of theirs.

SALV. You are out, *Simplicius*, for I will prove to you, that what may be cut off from the Priſm without weakening it, is really not only a fourth, as you ſuppoſe, but a third Part of it.

Now it remains (as *Sagredus* hinted)

PROP. XIV. PROB. V.

To find according to what Line the Section is to be made; and this I will demonſtrate to be *Parabolical*.

But firſt I muſt demonſtrate the following *Lemma*.

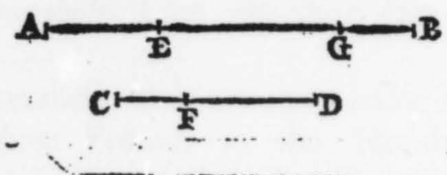
LEMMA I.

If there be two Ballances or Leavers divided by their *Fulcra*, in ſuch Sort that the two Diſtances, at which the Forces are to be plac'd, may have to each other a *duplicate* Proportion of the Diſtances, at which the Reſiſtances are; which Reſiſtances are to each other as their Diſtances: I ſay, the ſuſtaining Powers will be equal.

Let AB and CD be two Leavers, divided upon their *Fulcra* E and F, in ſuch Sort that the Diſtance EB has to the Diſtances ED a *duplicate* Proportion of that which the Diſtance EA has to FC: I ſay, the Powers which in B and

and D ſhall ſuſtain the Reſiſtances A and C, ſhall be equal to each other.

Let EG be ſuppoſed a mean Proportional between EB and FD: then it will be BE : EG :: EG : FD, and ſo is AE to CF; And ſo is ſuppoſed the Reſiſtance of A to the Reſiſtance of C: And becauſe it is EG : FD :: AE : CF, it will be by Permutation GE : EA :: DF : FC, and

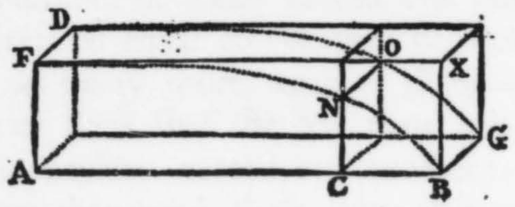


therefore (becauſe the two Leavers DC and GA are divided proportionally in the Points F and E) in caſe the Power which plac'd in D, countervails or equals the Reſiſtance of C, was plac'd in G, it would equal the ſame Reſiſtance of C plac'd in A: But by what has been granted, the Reſiſtance of A has to the Reſiſtance of C, the ſame Proportion as AE has to CF, *i. e.* as BE has to EG; therefore the Power G, or, if you will, D, plac'd at B, will ſuſtain the Reſiſtance C plac'd in A. *Q. E. D.*

This being underſtood; on the Surface FB of the Priſm DB, let the *parabolic* Line be deſcrib'd, whoſe Vertex is B; according to which let the Priſm be cut or ſaw'd, ſo that there may remain the Solid comprehended between the Baſe AD, the Rectangle AG, and the Surface DGBF, bent according to the Curvity of the *parabolic* Line FNB. Then I ſay, that this Solid is throughout or every where of equal Reſiſtance.

DEMONSTRATION.

Let this remaining Solid be cut by a Plane CO, parallel to AD; and imagine two Leavers divided and placed upon the *Fulcra* A and C: And let the Diſtances of one be BA and AF, and of the other BC and CN: Then



becauſe in the *Parabola* FBA it is AB : BC :: FA*q* : CN*q*; 'tis manifeſt that the Diſtance BA of one Leaver has to the Diſtance BC of the other a *duplicate* Proportion of that which the other Diſtance AF has to the other CN: And becauſe the Reſiſtance to be equal'd by the Leaver BA, has to the Reſiſtance to be equal'd by the Leaver BC, the ſame Proportion as the Rectangle DA has to the Rectangle OC; which is the ſame as the Line AF has to the Line NC, which are the two other Diſtances of the Leavers: It is manifeſt by the foregoing *Lemma*, that the ſame Force which apply'd to the Line BG equals the Reſiſtance DA, will alſo equal the Reſiſtance CO: And the ſame may be demonſtrated, if that Solid be cut in any other Place; therefore ſuch a Solid is every where of equal Reſiſtance.

Again,

Again, that the Priſm being cut according to the *parabolic* Line FNB, a third Part of it will be taken off, appears thus;

Becauſe the *Semiparabola* FNBA and the Rectangle FB are Baſes of two Solids comprehended between two parallel Planes, *i. e.* between the Rectangles FB and DG; therefore they are in the ſame Proportion to each other as their Baſes are: but the Rectangle FB is *Seſquialter* of the *Semiparabola* FNBA; therefore by cutting the Priſm according to the *parabolic* Line, the third Part of it is taken off.

Whence 'tis manifeſt, that we may leſſen the Weight of Beams Thirty-three Pounds in the Hundred, without weakening them at all, which in building large Ships may be of great Benefit: for in ſuch Kind of Fabricks, Lightneſs is of infinite Advantage.

SAGR. So many and ſo great are the Advantages, that it would be tedious, nay, 'tis impoſſible to mention them all. Let's paſs them therefore by, for I ſhould rather deſire to be convinc'd that that Diminution of Weight is made, according to the aſſign'd Proportion: That the diagonal Section takes away half the Weight, I well know, but that the other *parabolic* one takes away a third Part of the Priſm, (tho' I can believe it on the Word of *Salviatus*, who always ſpeaks Truth) yet I would rather chuſe to ſee it demonſtrated.

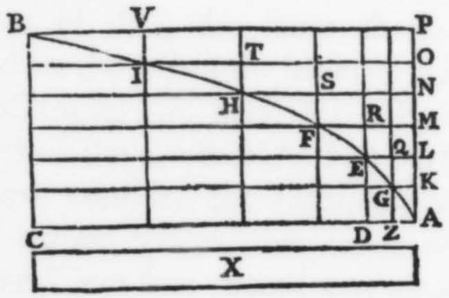
SALV. You'd have me demonſtrate to you, that the Exceſs of the Priſm above the *parabolic* Solid, is a third Part of the whole Priſm. This I am ſure I have formerly demon-

demonſtrated: I'll try whether I can put the Demonſtration together again; to which Purpoſe I remember I made uſe of a certain *Lemma* of *Archimedes*, which he has in his Book of *Spirals*; and 'tis this;

LEMMA II.

If any Number of Lines exceed one another equally, and the Exceſs be equal to the leaſt of thoſe Lines; and if there be as many more, each of them = the greateſt: the Squares of theſe ſhall be leſs than *triple* the Squares of thoſe Lines which exceed one another: but the ſame ſhall be more than *triple* thoſe other that remain after the Square of the longeſt is taken away.

This being granted: In this Rectangle ACBP, inſcribe the *parabolic* Line AB: Then we are to prove the mix'd Triangle BAP, whoſe Sides are BP and PA, and whoſe Baſe is the *parabolic* Line BA, to be the third Part of the whole Rectangle CP.



Now, if it be not the third Part, it muſt be more or leſs; If it be, ſuppoſe, leſs, then let the Space X be ſuppoſed = its Defect, or to what it wants of this third Part; then dividing

viding the Rectangle CP continually into equal Parts, with Lines parallel to the Sides BP and CA, we ſhall at length arrive at ſuch Parts, one of which ſhall be leſs than the Space X. Now let one of thoſe Parts be the Rectangle OB, and thro' the Points where the other Parallels cut the *parabolic* Line, draw Lines parallel to AP: here now I'll ſuppoſe, deſcribed about our Mixt-lin'd Triangle, a Figure compoſed of Rectangles; which are BO, IN, HM, FL, EK, GA, which Figure will certainly be leſs than the third Part of the Rectangle CP, ſince the Exceſs of this Figure above the Mixt-lin'd Triangle, is much leſs than the Rectangle BO, which BO is leſs than the Space X.

SAGR. Hold a little, pray: for I'm not ſatisfied that the Exceſs of this circumſcribed Figure above the Mixt-lin'd Triangle, is conſiderably leſs than the Rectangle BO.

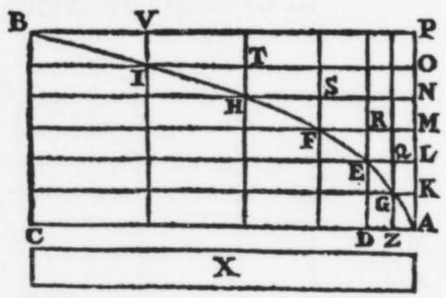
SALV. Is not the Rectangle BO = all thoſe Rectangles thro' which our *parabolic* Line paſſes, I mean theſe: BI, IH, HF, FE, EG, GA; of which one Part only lies without the Mixt Triangle? And was not this Rectangle BO given or ſuppoſed leſs than the Space X?

Therefore if the Triangle, together with the Space X, as our Adverſary ſuppoſes, be = the third Part of the Rectangle CP, the circumſcribed Figure which adds to the Triangle, ſo much leſs than the Space X, will be yet leſs than the third Part of the ſaid Rectangle CP: but this is impoſſible, becauſe it is more than a third Part; therefore it is not true that our Mixt-lin'd Triangle is leſs than the third Part of the Rectangle.

SAGR. You

SAGR. You have ſolved the Doubt I ſtarted; but you have ſtill to demonſtrate to us, that the circumſcribed Figure is more than a third Part of the Rectangle CP; which I believe will be pretty difficult.

SALV. Not at all: for in the *Parabola* the Square of the Line DE has the ſame Proportion to the Square of ZG, as the Line AD has to AZ, which is the ſame as the ▭ KE has to the ▭ AG, (their Heights AK and KL being equal) therefore the ſame Proportion as the Square of ED



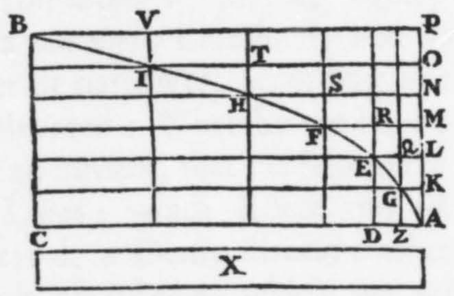
has to the Square of ZG, *i. e.* the Square of LA has to the Square of AK, the ſame the ▭ KE has to the ▭ KZ: And in the ſelf-ſame Manner may we prove that the other Rectangles LF, MH, NI, OB, are to each other as the Squares of the Lines MA, NA, OA, PA.

Let us now conſider the circumſcribed Figure to be compoſed of certain Spaces, that are to one another as the Squares of the Lines that exceed one another by an Exceſs = the leaſt Line, and the Rectangle CP to be compoſed of ſo many other Spaces, each of which are equal to the longeſt Line, which are all the Rectangles equal to OB.

Then,

Then, by the foregoing *Lemma*, the circumſcribed Figure is more than the third Part of the Rectangle CP; but it was ſuppoſed leſs, which you ſee is impoſſible: therefore the Mixt-lin'd Triangle is not leſs than the third Part of the ▭ CP. I ſay likewiſe, that it is not more, for if it be more than one third Part of the ▭ CP, ſuppoſe the Space X to be = the Exceſs of the Triangle above the third Part of the ▭ CP; then dividing and ſubdividing the Rectangle into Rectangles leſſer and leſſer, but always equal to each other, we ſhall meet at laſt with ſuch, one whereof ſhall be leſs than the Square X: Let this Diviſion be made, and let the Rectangle BO be leſs than the Space X; and having deſcrib'd the Figure as before, we ſhall have in the Mixt-lin'd Triangle an inſcrib'd Figure compoſed of the Rectangles VO, TN, SM, RL, QK, which alſo ſhall not be leſs than the third Part of the great Rectangle CP. For the Mixt-lin'd Triangle exceeds the inſcribed Figure by a far leſs Exceſs than it exceeds the third Part of the ▭ CP; becauſe the Exceſs of the Triangle, above the third Part of the Rectangle CP, is = to the Space X, which is leſs than the ▭ BO: and this ſtill conſiderably leſs than the Exceſs of the Triangle above the inſcribed Figure: For to the Rectangle BO, all the ſmall Rectangles AG, GE, EF, FH, HI, IB, are equal, much leſs yet than the Half of which is the Exceſs of the Triangle above the Figure inſcribed; Wherefore the Triangle exceeding the third Part of the Rectangle CP, by much more (it exceeding it by the Space X) than it exceedeth its inſcribed Figure, that ſame Figure ſhall alſo be more than the third Part of the Rectangle CP; but it is leſs by the foregoing *Lemma*: the Rectangle CP taken as the Aggregate of all the biggeſt Rectangles, having the

ſame Proportion to the Rectangles compoſing the inſcribed Figure, as the Aggregate of all the Squares of the Lines = the longeſt, to the Squares of the Lines which exceed one another equally, the Square of the longeſt being firſt taken away, and therefore (as it happens in Squares) the total Aggregate of the biggeſt (*i. e.* of the Rectangle CP) is more than the triple of the Aggregate of thoſe that exceed one another by an Exceſs = the leaſt of them, the Square of the greateſt being firſt taken away, which compoſe the inſcribed Figure: Therefore the Mixt-lin'd Triangle is neither greater nor leſs than the third Part of the Rectangle CP; it is therefore equal to it.



SAGR. A very curious Demonſtration; and ſo much the more ſo, as it affords us the Quadrature of the *Parabola*, proving it to be *Seſquitertial* of the Triangle inſcrib'd in it; the ſame indeed that *Archimedes* demonſtrated by two very different, but both very admirable Proceſſes of many Propoſitions: And the ſame that has lately been demonſtrated by *Lucas Valerius* the *Archimedes* of our Age; which Demonſtration is given us in a Treatiſe of his, concerning the Centre of Gravity of Bodies.

SALV. A

SALV. A Treatiſe as excellent as any that has been written by the famous Geometricians of the preſent, or any the paſt Ages; which, when our *Academic* read, he deſiſted to proſecute his Diſcoveries, which he was then writing about, concerning the ſame Subject; he there finding the whole Buſineſs ready done, and moſt happily demonſtrated to his Hands by the ſaid *Lucas Valerius*.

SIMP. This our *Academic* told me himſelf, and I often intreated him to let me ſee the Demonſtrations he had hit upon, before he met with *Valerius*'s Book; but I never ſaw them.

SALV. I have a Copy of them, and will ſhew them you, becauſe it's pleaſant to ſee the different Methods Authors take to inveſtigate the ſame Concluſions, and their Demonſtrations; where ſome of the Concluſions have different Explanations, altho' in Effect they are equally true.

SAGR. I ſhould be very glad to ſee them; pray be ſo kind as to bring them with you the next time you come. But now, *Salviatus*, ſince the Matter is thus as you have prov'd, in Relation to the Reſiſtance of a Solid made from a Priſm, by a *parabolic* Section, an Operation not only Elegant but Uſeful, in many Mechanical Operations; it would be of great Service to Artificers, had they an eaſy and ready Way to deſcribe this *parabolic* Line upon the Surface of the Priſm.

SALV. There are ſeveral Ways of deſcribing ſuch Lines, but two above all the reſt are very expeditious ones; and thoſe I'll now ſhew you: The former of which is really admirable; for by it, in leſs time than any one can deſcribe neatly with a Pair of Compaſſes on Paper, four or ſix Circles of different Magnitudes, I can deſcribe thirty or forty *parabolic* Lines, as accurately and exactly as the Circumferences of his Circles are. 'Tis thus: I take a Braſs Ball exquiſitely round, no bigger than a Nut, or a Boy's Marble; this thrown upon a Steel Mirror, plac'd not perpendicular, but ſomething inclined to the Horizon, ſo that the Ball may run along it, preſſing lightly upon it, it will then leave us a *parabolic* Line finely and exactly deſcrib'd; and this wider or narrower, according as the Projection is more or leſs elevated: Whereby we have prov'd, by a clear and ſenſible Experiment, that the Motion of *Projects* is made in *parabolic* Lines; which Effect our *Academic* was the firſt that obſerv'd, a Demonſtration whereof he has given us in his Book of *Motion*, which we will peruſe at our next Meeting. Now this Ball by which we ſay a *parabolic* Line is deſcribed, in the Manner aforeſaid, muſt be firſt a little warm'd and moiſten'd, by holding it in the Hand; for thus it will leave its Track more apparent upon the Mirror.

The other Way of deſcribing the Line ſought upon the Priſm is this;

Let two Nails be drove at a convenient Height into a Wall, both equidiſtant from the Horizon, and diſtant from each other by twice the Breadth of the Rectangle, upon which the *Semiparabola* is to be deſcrib'd; then upon theſe two

two Nails, hang a ſmall Line of ſuch Length, as that its Curvature might reach the whole Length of the Priſm: then that Line will hang in the Curve of a *Parabola*; and if this Curve be mark'd by Points upon the Wall, we ſhall have a whole *Parabola* there deſcribed, which, a Perpendicular hanging from a Point in the Midſt between theſe two Nails, will divide into two equal Parts; which Curve any Artificer that knows any thing, will be able, without any Difficulty, afterwards to transfer to the oppoſite Surfaces of the ſaid Priſm. The ſame Line might alſo be drawn upon the Side of the Priſm, by Points, or by Help of the Geometrical Lines, delineated upon the Compaſs of our *Friend*, without any Trouble.

Hitherto we have demonſtrated the various Concluſions belonging to the Contemplation of theſe Reſiſtances in Solids againſt Fraction; and have firſt open'd the Way to this *Science*, by ſuppoſing the direct Reſiſtance known, which may be proſecuted farther, to the gaining, by their Help, ſtill new Concluſions, with their Demonſtrations, which in Nature are infinite. But to put an End to this Day's Conference, I will only add the Contemplation of the Reſiſtances of hollow Solids, which are made Uſe of by *Art*, and more by Nature in a thouſand Operations; where, without increaſing the Weight, ſhe greatly augments the Strength; As is ſeen in the Bones of Birds, and in moſt Tubes, or hollow Trunks, which, tho' light, yet greatly reſiſt againſt bending and breaking.

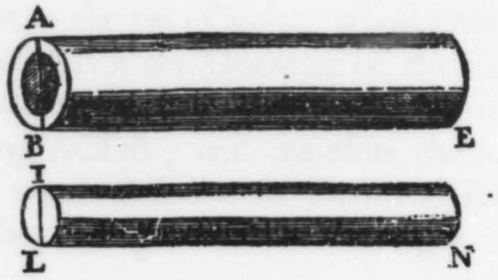
Thus, if a Wheat Straw, which ſupports an Ear that's heavier than the whole Stalk, was made of the ſame Quantity of Matter, but ſolid, it would much eaſier bend or break than it does: And with the ſame Reaſon *Art* has

obſerv'd, and Experience confirm'd, that an hollow Cane, or Trunk of Wood or Metal, is much ſtronger and more firm, than if, being of the ſame Weight and Length, it was ſolid, which then would conſequently be not ſo thick: And therefore *Art* has contrived a Way to make Canes hollow within, when they are required to be light and ſtrong. We will next then demonſtrate, That,

PROP. XV.

The Reſiſtances of two Cylinders of equal Weight and Length, one of which is hollow, and the other ſolid, have the ſame Proportion to each other, as their Diameters have.

Let the Tube or hollow Cylinder AE, and the ſolid Cylinder IN, be equal in Weight and Length: Then I ſay, that the Reſiſtance of the Tube AE, againſt Fraction, to the Reſiſtance of the ſolid Cylinder IN againſt Fraction,



has the ſame Proportion as the Diameter AB has to the Diameter IL. This is very manifeſt, for the Tube and Cylinder having the ſame Length, the Circle IL, that is, the

the Baſe of the Cylinder ſhall be equal to the Ring AB, of the Tube AE, (that Superficies which remains, when a leſſer Circle is taken out of a greater concentric with it, I call a Ring) and therefore their abſolute Reſiſtances ſhall be equal; but becauſe in breaking croſsways the Cylinder IN, we make uſe of the Length IN for a Leaver, and of the Point L for a *Fulcrum*, and of the Semidiameter or Diameter LI for a Counter-leaver; and becauſe in the Tube the Part of the Leaver, or the Line BE, is = the Line LN; but the Counter-leaver beyond the *Fulcrum*, is the Diameter or Semidiameter AB: It is manifeſt that the Reſiſtance of the Tube exceeds the Reſiſtance of the ſolid Cylinder, as much as the Diameter AB exceeds the Diameter IL, which is what was ſought: Therefore the Strength of the hollow Tube exceeds the Strength of the ſolid Cylinder, in Proportion of their Diameters; I mean when they are of the ſame Matter, Weight, and Length: It would not be amiſs, I think, in the next Place, to try to inveſtigate what happens, indifferently in other Caſes, between Tubes and ſolid Cylinders of equal Lengths, but unequal as to Weight, and more or leſs hollow'd out.

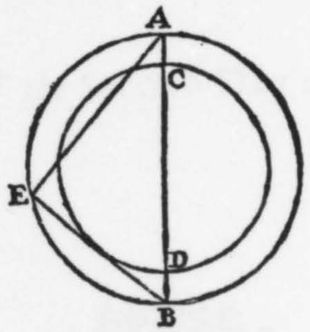
And firſt, we will demonſtrate how,

PROP. XVI.

PROP. XVI. PROB. VI.

A hollow Tube being given, to find a ſolid Cylinder equal to it.

This Operation is very eaſy: For let the Diameter of the Tube be AB, and the Diameter of its Concave be CD; in the greater Circle draw the Line AE equal to the Diameter CD, and draw EB: Now becauſe in the Semicircle AEB, the Angle E is right, the Circle whoſe Diameter is



AB, ſhall be = the two Circles, whoſe Diameters are AE and EB: But AE is the Diameter of the Concave of the Tube: therefore the Circle, whoſe Diameter is EB, will be = the Ring ACDB; and therefore the ſolid Cylinder, the Circle of whoſe Baſe has the Diameter EB, ſhall be = the Tube, they being of the ſame Length.

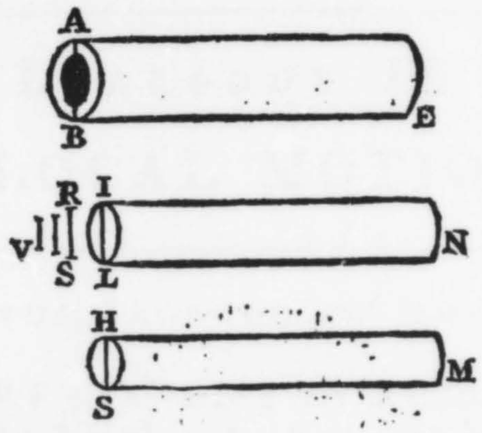
This being demonſtrated, we ſhall eaſily be able,

I PROP. XVII.

PROP. XVII. PROB. VII.

To find what Proportion the Reſiſtances of any Tubes and Cylinders whatever, provided their Lengths be equal, have to one another:

Let the Tube ABE, and the Cylinder RMS, be of equal Length, it is required to find what Proportion their Reſiſtances have to each other.



By the foregoing Propoſition, find a Cylinder as ILN, of an equal Length with, and equal to the Tube; and to the Lines IL and RS, (which are the Diameters of the Baſes of the Cylinders IN and RM) let the Line V be a fourth Proportional: Then I ſay, that the Reſiſtance of the Tube AE, to the Reſiſtance of the Cylinder RM, is as the Line AB to the Line V: For the Tube AE being = and of the ſame Length with the Cylinder IN, the Reſiſtance of the Tube ſhall be to the Reſiſtance of the

Cylinder, as the Line AB to the Line IL: But the Reſiſtance of the Cylinder IN, is to the Reſiſtance of the Cylinder RM, as the Cube of IL to the Cube of RS, *i. e.* as the Line IL to the Line V: Therefore, *ex Æquo*, the Reſiſtance of the Tube AE, to the Reſiſtance of the Cylinder RM, has the ſame Proportion as the Line AB has to the Line V: Which is what was ſought.

The END of the SECOND DAY'S CONFERENCE.

GALILÆUS

GALILÆUS

HIS

DIALOGUES.

DIALOGUE III.

Of LOCAL MOTION.

SALVIATUS, SAGREDO, *and* SIMPLICIUS.

HERE we advance a very new *Science* on a very old *Subject*. May be nothing in Nature was earlier than MOTION; and many and great Volumes we ſee have been written by Philoſophers concerning it: But yet I find many Properties of it worth our knowing, and which hitherto have not been obſerved, much leſs demonſtrated.

Some ſlight ones have been indeed noted, as for Example, That the natural Motion of falling Bodies is continually accelerated: But according to what Proportion that Acceleration is made, hath not hitherto been ſhewn; for no

one, that I know of, hath ever demonſtrated, that the Spaces run thro' by a Body falling from Reſt in equal Times, are in the ſame Proportion as the odd Numbers, which follow in order from the *Unit* or *One*.

Again, it has been obſerv'd, indeed, that *Projects* (or Things thrown with Violence) do deſcribe a Line, after ſome manner or other, curv'd; but that this is the Curve of a *Parabola* no body has told us.

Now that theſe are ſo, with many other Things no leſs worthy our Knowledge, I will here demonſtrate: And, what is ſtill more, I will open a Paſſage to a moſt ample and excellent *Science*, of which theſe our Labours are as Elements; into the Depths of which *Science* more piercing Wits than mine will be better able to penetrate.

This Tract we'll divide into three Parts.

In the firſt Part, we'll conſider thoſe things that reſpect *equable* or *uniform* Motion.

In the ſecond, we'll treat of Motion *naturally accelerated*: And,

In the third, of *violent* Motion, or of *Projects*.

Of EQUABLE MOTION.

Concerning equable or uniform Motion, we have need but of one Definition, which I give as follows;

DEFINITION.

An equable or uniform Motion, is that when a Moveable, in all equal Times whatſoever, paſſes thro' equal Spaces.

N. B. I

N. B. I thought it proper to add to the old Definition (which ſimply calls that an equable Motion, whereby equal Spaces are paſſed thro' in equal Times) theſe Words, *All whatſoever*; thus, In *all* equal Times *whatſoever*: For it may happen that a Moveable may paſs thro' equal Spaces in equal Times, and yet the Spaces may not be equal which it hath gone thro' in leſſer, tho' equal Parts of the ſame Time.

On this Definition depend four *Axioms*, *viz.*

AXIOM I.

In the ſame equable Motion, that Space is greater which is paſſed over in a longer Time than that Space paſſed over in a ſhorter.

AXIOM II.

In the ſame equable Motion, that Time is greater wherein a greater Space is paſſed thro', than that wherein the Moveable paſſes thro' a leſſer Space.

AXIOM III.

The Space paſſed thro' with a greater Velocity in any Time, is greater than that paſſed thro' with a leſſer Velocity in the ſame Time.

AXIOM

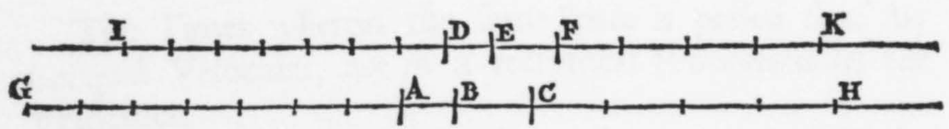
Axiom IV.

The Velocity wherewith a greater Space is paſſed over in any Time, is greater than the Velocity wherewith a leſſer Space is paſſed in the ſame Time.

Prop. I. Theor. I.

If a Moveable carried equably, and with the ſame Velocity, paſs thro' two Spaces, the Times of the Motion ſhall be to each other as the ſaid Spaces are.

Let the Moveable, by an equable Motion, paſs thro' with the ſame Velocity, the two Spaces AB and BC, and



let the Time of the Motion thro' AB be DE ; and the Time of the Motion thro' BC let be EF : then I ſay, that as the Space AB to the Space BC, ſo is the Time DE to the Time EF.

Produce on both Sides the Spaces and Times towards G, H, and I, K ; and in AG take any Number of Spaces = AB, and in DI the like Number of Times, each = DE : And again, in CH take any Number of Spaces = CB, and in FK as many Times = the Time EF : then will the Space BG, and the Time EI, be *Equimultiples* of the Space BA, and the Time ED, be the Number of the foremen-tioned Spaces and Times taken = AB and DE, what it will. And

And in like manner the Space HB and the Time KE, will be *Equimultiples* of the Space CB, and of the Time FE, at whatever Rate they be equimultiply'd : And be-cauſe DE is the Time of the Motion thro' AB, the whole Time EI ſhall be the Time of the Motion thro' BG, ſince the Motion is ſuppoſed Equable, and ſince in EI there are as many Times = DE, as in BG there are Spaces = BA ; In like manner we may conclude KE to be the Time of the Motion thro' HB. Now ſince the Motion is ſuppoſed Equable, if the Space GB was = BH, the Time IE would be alſo = the Time EK : and if GB be greater than BH, IE alſo will be greater than EK ; and if leſs, leſs : There are therefore four Magnitudes, AB the firſt, BC the ſe-cond, DE the third, and EF the fourth : And of the firſt and fourth, *viz.* of the Space AB, and of the Time DE, are taken *Equimultiples*, according to any Multiplication, *viz.* the Time IE, and the Space GB ; and it hath been demonſtrated, that theſe do at the ſame time, either equal, or want of, or exceed the Time EK, and the Space BH, which are *Equimultiples* of the ſecond and fourth ; there-fore the firſt hath to the ſecond, *i. e.* the Space AB hath to the Space BC, the ſame Proportion as the third has to the fourth, *i. e.* as the Time DE has to the Time EF. *Q. E. D.*

Prop. II. Theor. II.

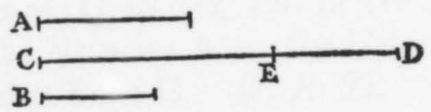
If a Moveable, in equal Times, paſs thro' two Spaces, thoſe Spaces will be to each other as the Velocities : And if the Spaces are as the Velocities, the Times will be equal. In the foregoing Figure, let the two Spaces AB and BC be

be paſſed over in equal Times; the Space AB with the Velocity DE, and the Space BC with the Velocity EF: then I ſay, that the Space AB is to the Space BC, as the Velocity DE to the Velocity EF; for taking, as before, *Equimultiples* both of the Spaces and the Velocities, according to any Multiplication whatever, *viz.* GB and IE of the Space AB, and of the Velocity DE, and likewiſe HB and KE of the Space BC, and of the Velocity EF, we may conclude, as above, that the *Multiples* GB and IE, both at the ſame time, fall ſhort of, are equal to, or exceed the *Equimultiples* BH and EK; therefore the Propoſition is manifeſt.

PROP. III. THEOR. III.

The Times wherein the ſame Space is paſſed thro' by unequal Velocities, are in a reciprocal Proportion to the Velocities.

Let the unequal Velocities be A the greater and B the



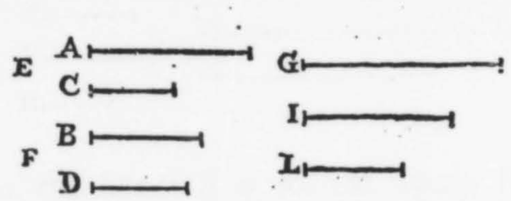
leſſer; and according to both theſe, let a Motion be made thro' the ſame Space CD:

Then I ſay, that the Time in which the Velocity A paſſes thro' the Space CD, ſhall be to the Time in which the Velocity B paſſes thro' the ſaid Space, as the Velocity B to the Velocity A. For make CD to DE, as A is to B, then, by the foregoing Propoſition, the Time wherein the Velocity A paſſes CD, ſhall be the ſame with the Time wherein the Velocity B paſſes CE. But the Time wherein the Velocity B paſſes CE, is to the Time in which it paſſes CD, as CE to CD; therefore the Time wherein the Velocity A paſſes CD, is to the Time in which the Velocity B paſſes the ſame CD, as CE to CD; that is, as the Velocity B to the Velocity A. *Q. E. D.*

PROP. IV. THEOR. IV.

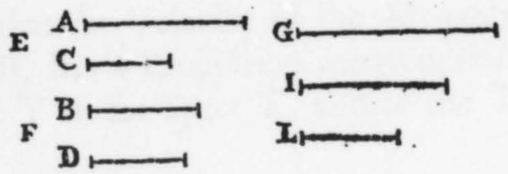
If two Moveables, move with an equable Motion, but with unequal Velocities, the Spaces which they paſs in unequal Times, are to each other in a Proportion compounded of the Proportion of the Velocities, and of the Proportion of the Times.

Let the two Moveables, moving with an equable Motion, be E and F; and let the Proportion of the Velocity



of the Moveable E, be to the Velocity of the Moveable F, as A is to B: And let the Time in which E is mov'd, be unto the Time wherein F is mov'd, as C is to D: Then I ſay, that the Space paſſed by E, with the Velocity A in the Time C, is to the Space paſſed thro' by F, with the Velocity B, in the Time D, in a Proportion compounded of the *Ratio* of the Velocity A to the Velocity B, and of the *Ratio* of the Time C to the Time D.

Let the Space paſſed by E, with the Velocity A, in the Time C, be G; and let G be to (I) as the Velocity A is to the Velocity B; and let (I) be to L as the Time C is to the Time D: then 'tis manifeſt, that (I) is the Space thro' which F is moved, in the ſame Time wherein E is moved thro' G; ſince the Spaces G and (I) are as the Velocities A and B, and ſince it is as the Time C to the Time D, ſo is (I) to L: And ſince (I) is the Space paſſed



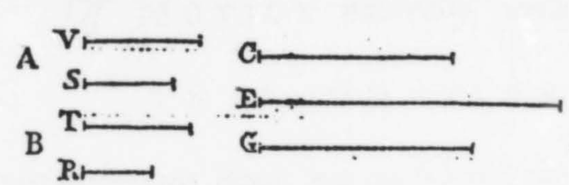
thro' by the Mover F, in the Time C, L will be the Space which is paſſed thro' by F in the Time D, with the Velocity B: But now the *Ratio* of G to L is compounded of the *Ratio* of G to (I), and of (I) to L, *i. e.* of the *Ratios* of the Velocity A to the Velocity B, and of the Time C to the Time D. *Q. E. D.*

Prop. V. Theor. V.

If two Moveables move with an equable Motion, but with unequal Velocities, and if the Spaces paſſed be alſo unequal, the Proportion of the Times will be compounded of the Proportion of the Spaces, and of the Proportion of the Velocities taken reciprocally.

Let

Let A and B be two Moveables, and let the Velocity of A be to the Velocity of B, as V to T, and let the Spaces paſſed be as S to R: Then, I ſay, the Proportion of the Time in which A is moved, to the Time in which B is moved, is compounded of the *Ratio* of the Velocity T, to the Velocity V, and of the *Ratio* of the Space S, to the Space R. Let C be the Time of the Motion A; and as the Velocity T is to the Velocity V, ſo let the Time C be to the Time E: And ſince C is the Time wherein A, with the Velocity V, paſſes the Space S, and ſince it is as the Velocity T, of the Moveable B, to the Velocity V, ſo the Time C to the Time E, E will be the Time wherein the Moveable B would paſs thro' the ſame Space S.

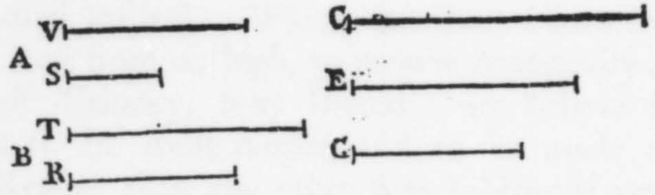


Again, as the Space S is to the Space R, ſo let the Time E be to the Time G: then 'tis manifeſt, that G is the Time wherein B would paſs thro' the Space R: And becauſe the Proportion of C to G is compounded of the *Ratios* of C to E, and of E to G; and ſince the Proportion of C to E, is the ſame with that of the Velocities of the Moveables A and B reciprocally taken, that is, with that of T and V: and ſince the Proportion of E G is the ſame with the Proportion of the Spaces S and R; therefore the Propoſition is manifeſt.

PROP. VI. THEOR. VI.

If two Moveables move with an equable Motion, the Proportion of their Velocities will be compounded of the *Ratio* of the Spaces run thro', and of the *Ratio* of the Times reciprocally taken.

Let the two Moveables A and B move with an equable Motion; and let the Spaces paſſed thro' by them be in Proportion of V to T, and let the Times be as S to R: then I ſay, that the Velocity of the Moveable A, to the Velocity of B, has a Proportion compounded of the *Ratio* of the Space V to the Space T, and of the Time R to the Time S.



Let C be the Velocity wherewith the Moveable A paſſes thro' the Space V, in the Time S; and the *Ratio* the Space V has to the Space T, the ſame let the Velocity C have to another E: then E will be the Velocity wherewith the Moveable B paſſes thro' the Space T in the ſame Time S: But if it be made as the Time R to the Time S, ſo the Velocity E to another G, G will be the Velocity wherewith the Moveable B paſſes thro' the Space T in the Time R: We have therefore the Velocity C, wherewith the Moveable A paſſes thro' the Space V in the Time S; and the Velocity G, wherewith the Moveable B paſſes the Space

Space T in the Time R; and the *Ratio* of C to G is compounded of the *Ratios* of C to E, and of E to G: But the *Ratio* of C to E, is ſuppoſed the ſame with the *Ratio* of the Space V to the Space T; and the *Ratio* of E to G, is the ſame with the *Ratio* of R to S; therefore the Propoſition is manifeſt.

SALV. Theſe Propoſitions are all our Author wrote concerning *Equable Motion*.

We will therefore paſs on to a more ſubtile and new Contemplation, *viz.* That which treats of Motion *naturally accelerated*; as is that which is generally obſerv'd by falling Bodies.

Of MOTION *naturally accelerated.*

INTRODUCTION.

IN the foregoing Book we conſidered the Accidents happening in the Equable Motion; now we ſhall treat of Accelerated Motion: And firſt, it is neceſſary to find out, and lay down a Definition moſt ſuitable to that which Nature makes uſe of. For altho' it be not inconvenient to feign a Motion at Pleaſure, and then contemplate its conſequent Paſſions; (as thoſe have done who have feign'd *Helixes* or *Conchords*, Lines ariſing from certain Motions, tho' Nature makes uſe of no ſuch; and upon that Suppoſition have laudably demonſtrated their Properties) yet ſeeing Nature makes uſe of a certain Kind of Acceleration in the Deſcent of heavy Bodies, we purpoſe to ſearch out and contemplate their Paſſions, if-ſo-be the Definition we intend

intend is but agreeable to the Eſſence of Motion naturally accelerated. And we believe, after a great deal of Trouble and Thought, we have found one that is ſo; and we believe ſo the rather, becauſe all the natural Experiments, which fall under the Obſervation of our Senſes, do anſwer, and are exactly agreeable to the Properties we ſhall hereafter demonſtrate of it. And in our Search concerning Motion naturally accelerated, we are, as it were, led by the Hand, by obſerving the uſual Method and Proceeding of Nature herſelf, in all her other Operations; wherein ſhe commonly makes uſe of the firſt, ſimpleſt, and eaſieſt Means: For I believe that no Man can think, that Swimming or Flying can be perform'd in a more ſimple or eaſy Manner than that very one which Fiſhes and Birds make uſe of by natural Inſtinct. When, therefore, I ſee a Stone falling from Reſt from on high, to acquire continually new Increments of Velocity, why ſhould I not believe ſuch Additaments to be moſt ſimple, and to be made thus with more Reaſon than any other Way? Now if we ſeriouſly conſider the Matter, we ſhall find no Additament, no Increment more ſimple than that which makes its Additions after the ſame Manner; as we may eaſily underſtand, by conſidering the great Affinity there is between Time and Motion. For as the Equability and Uniformity of Motion is defined and expreſſed by the Equability of the Times and Spaces, (for we call that Lation or Motion equable, by which equal Spaces are paſſed thro' in equal Times) ſo by the ſame Equality of the Parts of Time, we may denote or expreſs the Increments of Celerity ſimply made, conceiving in our Mind that ſuch Motion is continually accelerated uniformly, and at the ſame

ſame Rate, whilſt equal Additions of Celerity are given them in equal Times, be the Times what they will: ſo that taking any equal Particles of Time from the firſt Inſtant, in which the Moveable departs from Reſt, and begins its Deſcent, the Degree of Velocity in the firſt and ſecond Particles of Time, is double the Degree of Velocity that the Moveable acquires in the firſt Particle: and the Degree which it acquires in three Particles, is triple, and that in four, quadruple of the ſame Degree of the firſt Time: ſo as (for our better Underſtanding) if a Moveable ſhould continue its Motion according to the Degree or *Momentum* of Velocity acquired in the firſt Particle of Time, and ſhould continue falling equably with that ſame Degree, this Motion would be twice as ſlow as that which it would acquire, according to the Degree of Velocity acquired in two Particles of Time; ſo that we may reaſonably enough underſtand the Intention of Velocity to be made according to the Extenſion of the Time.

From whence we may frame the Definition of Motion, which we are going to treat of, as follows:

DEFINITION.

Motion equably or uniformly accelerated, is that which, departing from Reſt, ſuper-addeth to itſelf equal *Momenta* of Velocity in equal Times.

SAGR. Tho' I can't with Reaſon contradict this, or any other Definition given by any other Authors, Definitions being all Arbitrary: Yet I may, without Offence, queſtion whether this Definition, thus underſtood, and admitted in Abſtract,

Abſtract, doth ſuit, agree, and hold true in that Sort of accelerate Motion which heavy Bodies naturally deſcending do exerciſe. And becauſe the Author ſeems to promiſe us, that the natural Motion of heavy Bodies is ſuch as he has defined it, I could wiſh that ſome Scruples, which perplex me, were removed, that ſo I may apply myſelf with greater Attention to the following Propoſitions, and their Demonſtrations.

SALV. You and *Simplicius* are certainly in the Right to propoſe your Difficulties as they ariſe; which I imagine will be the very ſame which I met with when I firſt read this Tract, and which were partly reſolved and removed by weighing well the Author's Reaſoning, and partly by my own conſidering them.

SAGR. For my Part, when I conceive a heavy Body to depart from Reſt, *i. e.* from a Privation of all Velocity whatever, and to enter into Motion; and in that to acquire continually a greater Velocity, in ſuch Proportion wherein the Time increaſes from the firſt Inſtant of Motion; ſo as, *e. g.* Eight Pulſations to acquire eight Degrees of Velocity, four Degrees whereof it had acquired at the fourth Pulſation, two at the ſecond, one at the firſt: Since Time may be divided and ſubdivided infinitely, it ſeems to me to follow, ſince the antecedent Velocity always leſſens in ſuch Proportion, that there can be no Degree of Velocity ſo ſmall, or if you will, no Degree of Tardity ſo great, in which the ſaid Moveable is not found to be conſtituted after its Departure from infinite Tardity, *i. e.* from Reſt. So that if that Degree of Velocity which it had

had in four Pulſations of Time was ſuch, as, continuing equable, it would have fell two Miles in an Hour, and with that Degree of Velocity which it had in two Pulſations of Time, it would have paſſed one Mile in an Hour, it muſt follow, that in the Inſtants of Time nearer and nearer to the firſt Inſtant wherein it paſſed from Reſt to Motion, that it muſt be found ſo ſlow, that (continuing to move with ſo great Tardity) it would not have paſſed one Mile in an Hour, nor in a Day, nor in one, no nor in one thouſand Years: Nor, indeed, would it have run one Foot in a much greater Time, which is what we ſhall hardly get our Imagination to aſſent to, ſince our Senſe aſſures us, that a heavy Body is very quick in its Fall.

SALV. This is one of the Difficulties which I met with at my firſt thinking on this Subject, but I ſoon got rid of it: And that very Experiment which occaſions your Doubt, at length clear'd mine. You ſay, that Experience ſeems to ſhew, that the Body hath no ſooner departed from Reſt, but it has a very notable Velocity: But I ſay, that Experience clearly ſhews us, that the firſt Impetus's of the falling Body, how heavy ſoever it be, are moſt eaſy and ſlow. For imagine a heavy Body to be laid on ſome yielding Matter, and let it alone till it hath preſſed into it as much as by its ſimple Gravity it is able: 'tis manifeſt, that if you lift it up a Yard or two, and then let it fall upon the ſame yielding Matter, a new Preſſion will be made, greater than the former, which was made by its ſole Weight, which Effect is cauſed from the Velocity acquired by the falling Body conjoin'd with its Fall; and this Preſſion will be the greater, according as the Percuſſion

is made from the greater Height, *i. e.* according as the Velocity of the Percutient ſhall be greater. We may therefore, without Error, conjecture what the Velocity of a falling Body is, from the Quality and Quantity of the Percuſſion.

But tell me, Sirs, whether or no a Beetle, let fall upon a Stake, from a Height of four Yards, and drives it into the Ground, *e. g.* four Inches, doth not, when falling from an Height of but two Yards, drive it much leſs Way into the Ground, and leſs ſtill from an Height of one Yard, and leſs ſtill from one Foot high: And laſtly, if it be elevated but one Inch, what will it do more than if it were laid upon that Stake without any Blow? Certainly, but very little; and if it were lifted up but the Thickneſs of a Leaf, the Operation would be wholly imperceptible.

And becauſe the Effect of the Percuſſion depends on the Velocity of the Percutient, who will queſtion but that the Motion is very ſlow, and the Velocity extreme ſmall, where the Operation is altogether imperceptible?

Pray obſerve now the Force of Truth, whilſt the ſame Experiment, that, at firſt thinking, ſhews us one Thing, upon better Conſideration, aſcertains the contrary.

But not to have Recourſe ſolely to this Experiment, (which yet is undoubtedly demonſtrative) methinks we may come at this Truth by mere Diſcourſe. Suppoſe a heavy Stone to be ſuſtained at Reſt in the Air: this Stone let go, being heavier than Air, falls downwards, and not with an equable Motion, but at firſt by a ſlow one, and continually afterwards with an accelerated one, or faſter and faſter: And ſince the Velocity may be increaſed and diminiſhed

diminiſhed infinitely, what Reaſon have I to think that that Moveable, departing from infinite Tardity (for ſuch *Reſt* is) ſhould immediately enter into ten Degrees of Velocity, more than into four; or into four rather than two, one, or one half, or one hundredth, or, in a Word, of all ſtill leſs *ad infinitum*?

Pray hear me. You'll, I believe, eaſily grant, that a Stone departing from a State of Reſt, acquires Degrees of Velocity in the ſame Order wherein thoſe ſame Degrees are diminiſhed, and loſt, when by an impellent Power it is forced up again to the ſame Height it before fell from: and if this be ſo, 'tis plain, I think, that the aſcending Stone, its Velocity growing leſs and leſs till the Whole is ſpent, can't be reduced to a State of Reſt, till it has paſſed thro' all the Degrees of Tardity.

SIMP. But if the Degrees of Tardity be greater and greater *in infinitum*, they can never be all ſpent; and therefore ſuch an aſcending Body will never be brought to Reſt, but will be mov'd conſtantly ſlower and ſlower *in infinitum*, a thing we ſee does not happen.

SALV. This would happen, *Simplicius*, if the Mover ſhould ſtay any Time in each Degree; But it paſſes thro' them without ſtaying any longer than an Inſtant or Point of Time in any of them: And becauſe in every quantitative Time, tho' never ſo ſmall, there are infinite Inſtants, they aptly enough anſwer to the infinite Degrees of Velocity diminiſhed. Now that the aſcending Body does not ſtay for any quantitative Time, in any Degree of Velocity, appears thus: Becauſe if in any aſſign'd quantitative Time,

in the firſt Inſtant of Time, and likewiſe in the laſt, the Mover would be found to have the ſame Degree of Velocity; from the ſecond Degree, it might be likewiſe drawn upwards ſuch another Space as from the firſt it was carried to the ſecond; and in the ſame Manner it would paſs from the ſecond to the third, and ſo, in ſhort, its Motion would become uniform *ad infinitum*.

SAGR. From this Diſcourſe methinks may be gather'd a Solution of a Queſtion controverted amongſt Philoſophers, *viz.* What the Cauſe is of the Acceleration of the Natural Motion of heavy Bodies: For when I conſider that the *Impetus* impreſſed by the Projicient, on a Body driven upwards, continually diminiſhed, which *Impetus*, ſo long as it was greater than that other contrary one of Gravity, forc'd it upwards: but when both the *Impetus* and this Gravity come to an *Equilibrium*, the Mover ceaſes to riſe higher, and is in a State of Reſt, wherein the impreſſed *Impetus* is not annihilated, but only that Exceſs is ſpent, whereby it before exceeded the Gravity of the Mover, and therefore drove it upwards. Since now the external *Impetus* conſtantly leſſening, and conſequently the Gravity beginning to prevail, the Body begins to deſcend, tho' but ſlowly, on account of the Reſiſtance or Oppoſition of the impreſſed *Impetus*, which in ſome meaſure ſtill remains in the Mover: but becauſe that *Impetus*, being more and more overcome by the Gravity, continually diminiſhes; thence ariſes the continual Acceleration of the Motion.

SIMP. Your Reaſoning I own is pleaſant, but 'tis more ſubtile than ſolid: For altho' it were concluſive, yet it only

only ſalves thoſe natural Motions, which are preceded by a violent one, and wherein ſome Part of the external *Impetus* ſtill remains in Force: but where there is no ſuch remaining Impulſe, but the Moveable departs from a long Quieſcence, the Force of your whole Reaſoning vaniſhes.

SAGR. I believe you are out, Sir, and that this Diſtinction of Caſes, which you make, is ſuperfluous, or, to be plain with you, of no Moment at all: For, pray anſwer me, Can't ſometimes a greater, and ſometimes a leſs *Impetus*, be impreſſed by the Projicient upon the Project, ſo that at one time it may be forced one hundred Yards high, at another twenty, or four, or one?

SIMP. No doubt.

SAGR. Again; this impreſſed *Impetus* may ſo little exceed the Reſiſtance of the Gravity, as not to raiſe the Project above an Inch; and at length the Power of the Projicient may be only ſo much as exactly to ballance the Reſiſtance of the Gravity, and the Moveable is not driven upwards at all, but only ſuſtain'd.

When therefore you hold a Stone in your Hand, what elſe do you do but impreſs on it an *Impetus*, which forces it as much upwards, as its Faculty of Gravity draws it downwards? And do you not continue the ſame impreſs'd *Impetus* on it all the time you hold the Moveable in your Hand? It is not ſurely leſſen'd by holding it longer in your Hand? And this holding it, which hinders the Deſcent of the Stone, is ſurely the very ſame, whether it be perform'd by your

your Hand, or by a Table, or by a Rope, at which it is hung?

We may conclude therefore, *Simplicius*, that the Precedence of a long, a ſhort, or of a momentary Reſt, makes no Alteration at all, ſo as to hinder the Stone from always departing from Reſt, affected with ſuch an *Impetus*, contrary to its Gravity, as juſt ſuffices to keep it in Reſt.

SALV. I don't think this a ſeaſonable Time to enquire into the Cauſe of the Acceleration of Natural Motion; concerning which Philoſophers have greatly differed in Opinion: ſome reducing it to the approaching to the Centre; others to the fewer Parts of the *Medium* ſucceſſively remaining to be perforated: And others referring it to a certain Extruſion of the ambient *Medium*, which, upon conjoining upon the Back of, or behind the Moveable, preſſes upon, and continually thruſts it more; which Conceits, and others of the like Nature, 'tis hardly worth while to examine and anſwer. For as to our Author, it is enough at preſent that we underſtand, that his Buſineſs is to ſearch out and demonſtrate to us ſome Paſſions of an accelerated Motion, (let the Cauſe of that Acceleration be what it will) ſo that the *Momenta* of that Acceleration may continually increaſe after the Departure from Reſt, according to that moſt ſimple Proportion, wherewith the Continuation of the Time increaſes; which is as much as to ſay, that in equal Times there are made equal Additaments of Velocity. And if it ſhall be found that thoſe Accidents which ſhall hereafter be demonſtrated, are agreeable to Truth, in the Motion of heavy Bodies naturally deſcending and accelerated, we may conclude that our aſſumed

aſſumed Definition comprehends or takes in ſuch a Motion of heavy Bodies; and that it is true that their Acceleration continually increaſes, as the Time and Duration of Motion does.

SAGR. In the Mind I am in at preſent, it may more clearly, and without altering your Meaning, be defined thus,

> *That Motion uniformly accelerated, is that wherein the Velocity conſtantly increaſes, as the Space paſſed thro' is increaſed.*

So that, for Example, the Degrees of Velocity acquired by the Moveable, in a Deſcent of four Yards, ſhould be double that which it had when it had deſcended two Yards, and this double to that acquired at the firſt Yard: For it can't be doubted ſure, but that that Body which falls from an Height of ſix Yards, has, and ſtrikes with an *Impetus* double to that which it had when it had fell but three Yards, and triple of that which it had at two Yards, and ſix times that it had at the End of one Yard's fall.

SALV. I'm not a little pleaſed to have ſuch a Companion in Error as you are: Nay, I muſt tell you, there ſeems to be ſo much Likelihood and Probability in your Opinion, that our Author himſelf, when I propoſed this to him, could not deny but that he for ſome time continued in the ſame Miſtake. But that which I afterwards was greatly ſurprized at, was to ſee, in ſo few Words, two Propoſitions to be proved, not only to be falſe, but impoſſible; which carry with them ſuch ſeeming Truth, that, after having propoſed them to many Perſons, I never found one but what freely allowed them.

SIMP. For

SIMP. For my Part I ſhould certainly be of the Number of thoſe that allow, that a deſcending Body *vires acquirit eundo*, its Velocity increaſing as the Space does; and that the *Momentum* of the ſame Percutient is double, if it fall from a double Height: Theſe ſeem to me to be Propoſitions to be granted without the leaſt Heſitation or Doubt.

SALV. And yet they are as falſe and impoſſible, as 'tis for Motion to be made in an Inſtant; as I will now, in the cleareſt Manner, demonſtrate to you. If ſo be the Velocities have the ſame *Ratio* as the Spaces run thro', or to be run thro', thoſe Spaces are paſſed thro' in equal Times: If therefore the Velocities with which the falling Body deſcends thro' the Space of four Yards, were double the Velocities with which it paſſed thro' the two firſt Yards, (like as the Space is double the Space) then the Times of ſuch Tranſitions are equal: But for a Moveable to paſs thro' four Yards, and two Yards in the ſame Time, this can have place only in inſtantaneous Motion: But we ſee that the falling Body performs its Motion in Time, and to paſs thro' two Yards in leſs Time than it paſſes thro' four: Therefore its Velocity does not increaſe in the ſame *Ratio* with the Space run thro'.

And now, that the other Propoſition is alſo falſe, may with the ſame Clearneſs be demonſtrated: For ſince the Percutient is the ſame, the Difference and *Momentum* of the Percuſſions can be determin'd only by the Difference of the Velocities. If therefore a Percutient, falling from a double Height, makes a Percuſſion whoſe *Momentum* is double,

double, it muſt ſtrike with a double Velocity: But the double Velocity runs thro' the double Space in the ſame Time: And the Time wherein it falls from a greater Height, we ſee is longer.

SAGR. 'Tis very evident and wonderful eaſy now: But this extreme Eaſineſs wherewith you manifeſt the moſt abſtruſe Concluſions, will be apt to leſſen the Value they had whilſt they lay hid under contrary Appearances: Thus I dare ſay it is with the Generality of Mankind; they have a much leſs Eſteem for that Knowledge that's ſo very eaſily acquired, than they have for that about which more tedious and puzling Debates are bandy'd to and fro.

SALV. To thoſe who with great Brevity and Perſpicuity detect the Fallacies of ſuch Propoſitions as are received by the Generality for true, it would be tolerable, methinks, if, inſtead of Thanks, they met with only Contempt: But the Temper ſhewn by ſome Men, pretending to be, at leaſt, upon a *Par* in the ſame Studies with another, is next to intolerable; who, when they are ſenſible they've publiſh'd Concluſions as true ones, the Falſity whereof they afterwards ſee by a ſhort and eaſy Diſquiſition, have by that other been detected and laid open. This Temper I would not call *Envy*, which uſually in Time turns to Anger, if not to perfect Hatred of the Perſon that detects ſuch Fallacies: But I chuſe to call it an itching Deſire rather to defend their inveterate Errors, than to admit the Reception of new-diſcover'd Truths; which Humour ſometimes carries them ſo far as to write againſt thoſe Truths, which they are inwardly ſufficiently

convinced of, only to keep the Reputation of others under, in the Opinion of the numerous and ignorant Vulgar.

Of ſuch falſe Concluſions, received indeed for true, tho' eaſily to be refuted, I have heard no ſmall Number from our *Academic*: and I have kept Account of ſome of them.

SAGR. You muſt not deprive us of them, but impart them to us, when we ſhall appoint a particular Meeting for that Purpoſe. But to go on; I think by this Time we have eſtabliſh'd the Definition of Motion uniformly accelerate, to be treated of in the following Diſcourſes: and 'tis this;

DEFINITION.

Motion equably or uniformly accelerated (we ſay) is that which departing from Reſt, ſuper-adds to itſelf, in equal Times, equal Momenta *of Celerity.*

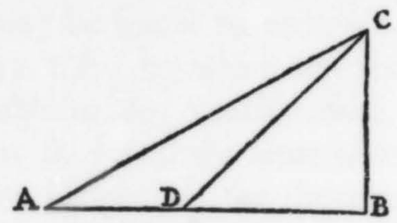
SALV. This Definition being allow'd, the Author asks and ſuppoſes but one Principle to be true; namely,

POSTULATUM.

He ſuppoſes that the Degrees of Velocity, acquired by the ſame Moveable, upon Planes of different Inclinations, are then equal, when the Elevations of the ſaid Planes are equal.

By the Elevation of an inclin'd Plane, he means a Perpendicular let fall from the Top of the ſaid Plane to its Baſe,

Baſe, or horizontal Line produced: But that you may better underſtand this, let the Line AB be parallel to the Horizon, upon which let the two Planes CA and CD be inclin'd; then the Perpendicular CB falling upon the horizontal Line AB, our Author calls the Elevation of the Planes CA and CD, and ſuppoſes that the Degrees of Velocity of the ſame Moveable, deſcending along the inclin'd



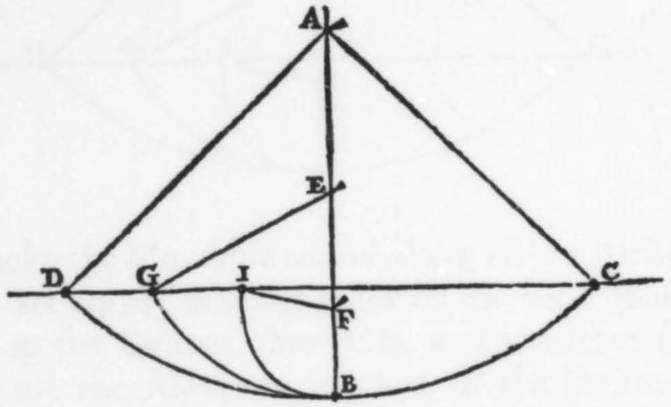
Planes CA and CD, which it hath acquired in the Terms A and D, are equal, their Elevations being the ſame, *viz.* CB. And ſuch alſo muſt the Degree of Velocity be underſtood to be, which the ſame Moveable, falling from the Point C, would have in the Point B.

SAGR. This Suppoſition ſeems to me ſo very probable, that it muſt be ſurely granted, without any Diſpute, ſuppoſing always that all accidental and external Impediments are removed, as alſo that the Planes be very hard and ſmooth, and that the Moveable be perfectly round, ſo that both Plane and Moveable may be free from all Roughneſs or Unevenneſs. Theſe Obſtacles and Impediments, I ſay, being removed, the Light of Nature dictates to me, that a Ball heavy and exactly round, deſcending by the Lines CA, CD, and CB, would neceſſarily come to the Terms A, D and B, with equal Impetus's.

 SALV. What

SALV. What you ſay carries great Probability with it; but I will now increaſe this Probability by an Experiment, ſo as it ſhall amount to very little leſs than a real Demonſtration.

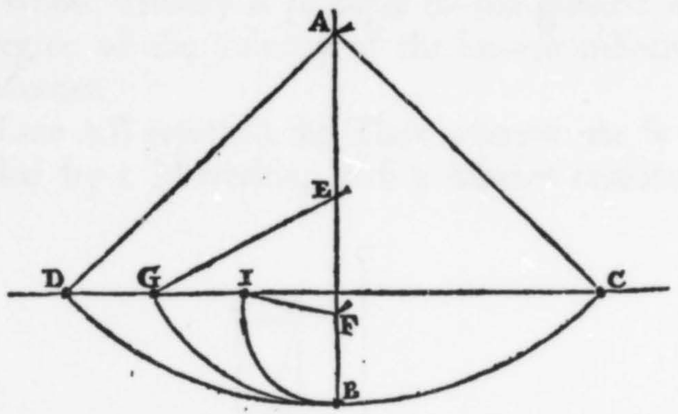
Conceive this Leaf of Paper to be a Wall, perpendicular to the Horizon, and to a Nail faſten'd in it, imagine a Ball of Lead of an Ounce or two Weight, to hang by a fine Thread as AB, two or three Yards long, and perpendicular to the Horizon: In this Wall draw alſo an horizontal Line as DC, which cuts the Perpendicular AB at right Angles, which AB let be hang'd two Inches or



thereabouts from the Wall. Then transferring the String AB, with the Ball, to AC, let go the Ball, and you'll ſee it deſcribe the Arch CBD, and to paſs ſo far beyond the Term B, that running along the Arch BD, it will riſe within a very little as high as the Parallel CD; it does not indeed quite reach it, being hinder'd by the Air and Thread: Whence we may ſafely conclude, that the *Impetus* of

of the Ball, acquired in the Point B, in its Deſcent thro' the Arch CB, is ſuch as ſuffices to force it upwards to the ſame Height, along ſuch another Arch BD: And now having often repeated this Experiment, drive into the Wall a Nail, juſt touching the Perpendicular AB, as in E or F, which let ſtand out five or ſix Inches, that the Thread AC returning as before, to carry back the Ball C, along the Arch CB; as ſoon as it is come to the Point B, ſtriking upon the Nail E, it may be forc'd to proceed along the Arch BG, whoſe Centre is E; by which we ſhall ſee what the ſame *Impetus* is able to do, which before being conceiv'd in the ſame Term B, forc'd the ſame Moveable thro' the Arch BD, to the Height of the horizontal Line CD. Now, Sirs, you'll with Pleaſure ſee the Ball to be carried to the Point G in the horizontal Line; and the ſame will happen if the Perpendicular ſtrikes the Nail plac'd lower, as in F, in which Caſe the Ball would deſcribe the Arch BI, always terminating its Aſcent exactly in the Line CD: And if the Nail be plac'd, by Way of Check, ſo low, that the Part of the Thread below it, in its Motion, can't reach to the Height of the Line CD, (which would happen if the Nail be nearer to the Point B, than to the Interſection of the Line AB, with the horizontal Line CD) then it would whirl quite round the Nail. This Experiment takes away all Doubt of the Truth of the Suppoſition; for ſince the two Arches CB and DB are equal, and ſimilarly ſituate, the *Momentum* acquired in the Deſcent thro' the Arch CB, is the ſame with that acquired in the Deſcent thro' the Arch DB: But the *Momentum* acquired in the Point B, in the Deſcent thro' the Arch CB, was ſufficient to force the ſame Moveable up thro' the Arch

Arch BD: Therefore alſo the *Momentum* acquired in the Deſcent thro' DB, is equal to that which forc'd the very ſame Moveable thro' the ſame Arch from B to D; ſo that univerſally, every *Momentum* acquired in the Deſcent thro' any Arch, is = that which hath Power to make that ſame Moveable aſcend the ſame Arch: But all the *Momenta*



which make the Moveable aſcend along all the Arches BD, BG, BI, are equal, as being made by the ſame *Momentum* acquired in the Deſcent thro' CB, as Experience ſhews: therefore all the *Momenta* acquired in the Deſcent thro' the Arches BD, BG and BI, are equal to one another.

SAGR. Your Reaſoning ſeems to me very concluſive, and the Experiment very applicable to demonſtrate the Truth of the *Poſtulatum*, that I think we may as ſafely admit it, as if it was demonſtrated.

SALV. No, *Sagredus*, I would not we ſhould aſſume more than we ought, eſpecially here, ſince we muſt in a particular

particular Manner make Uſe of this Aſſumption, in ſuch Motions as are made upon plain Superficies, and not upon curv'd ones; in which the Acceleration proceeds with Degrees very different from thoſe wherewith we ſuppoſe it to proceed on plain Surfaces: Inſomuch that altho' the Experiment I juſt mention'd, ſhews us that the Deſcent thro' the Arch CB, confers ſuch a *Momentum* on the Moveable, as can force it back to the ſame Height, along any Arch, as BC, BG, or BI; yet we can't ſhew, with like Evidence, that the ſame thing would happen in caſe a moſt perfect Ball were to deſcend along flat Planes, inclin'd according to the Inclinations of the *Chords* of theſe ſame Arches: But we may rather believe that Angles being form'd by thoſe flat Planes, at the Point B, the Ball which deſcends along the Plane inclin'd, as is the Chord CB, finding an Obſtacle in the Planes, aſcending according to the Chords BD, BG, BI, by ſtriking againſt them, will loſe ſome Part of its *Impetus*, and could not riſe again to the Height of the Line CD. But that Obſtacle being removed which prejudiceth the Experiment, we may certainly conceive in our Mind, that the *Impetus* (which has its Force from the Quantity of the Deſcent) would be able to remount the Moveable to the ſame Height. Let us therefore take this at preſent as a *Poſtulatum*, the Truth whereof we ſhall hereafter find eſtabliſh'd, when we ſhall ſee other Concluſions built upon this *Hypotheſis*, to anſwer and moſt exactly to agree with Experience.

This ſingle Principle being ſuppoſed, our Author goes on and demonſtrates the following Propoſitions.

PROP. I.

PROP. I. THEOR. I.

The Time wherein any Space is paſſed by a Moveable, with a Motion uniformly accelerate from Reſt, is = the Time in which the ſame Space would be paſſed by the ſame Moveable, carried with an equable Motion, the Degree of whoſe Velocity is *ſubduple* to the greateſt and ultimate Degree of the Velocity of the former uniformly accelerate Motion.

Let the Line AB repreſent the Time wherein the Space CD is paſſed by a Moveable, with a Motion uniformly



accelerate from Reſt in C; and let the greateſt and laſt Degree of Velocity acquired in the Inſtants of Time A, B, be repreſented by the Line EB, drawn at Pleaſure to AB; and join AE: Divide AB into any Number of equal Parts, and

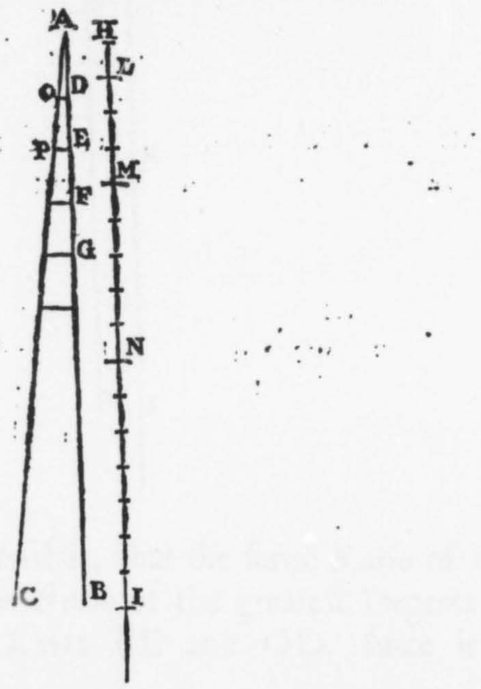
and thro' the Points of Diviſion draw as many Lines to the Line AE, ∥ EB; theſe will repreſent the increaſing Degrees of Velocity after the firſt Inſtant A. Then divide BE into two equal Parts in F, and draw FG and AG, ∥ BA and BF: then a Parallelogram AGFB will be conſtituted = the Triangle AEB, whoſe Side GF divides the Line AE into two equal Parts in (I). For if the Parallels of the Triangle AEB be extended unto GF, we ſhall have the Aggregate of all the Parallels contained in the quadrilateral Figure, equal to the Aggregate of thoſe comprehended in △ AEB; for thoſe in △ IEF are = thoſe contained in △ GIA, and thoſe which are in the Trapezium AIFB are common: Now ſince all and ſingular the Inſtants of the Time AB, do anſwer to all and ſingular the Points of the Line AB; and ſince the Parallels drawn from theſe Points comprehended in △ AEB, repreſent the increaſing Degrees of augmented Velocity, and the Parallels contained within the Parallelogram, do in like manner repreſent as many Degrees of Velocity not augmented, but equable; it appears that as many *Momenta* of Velocity were ſpent in the accelerate Motion, according to the increaſing Parallels of the △ AEB, as in the equable Motion, according to the Parallels of the Parallelogram CB: For what is wanting of *Momenta* in the firſt half of the accelerate Motion (for the *Momenta* are deficient, repreſented by the Parallels of △ AGI) is made up by the *Momenta* repreſented by the Parallels of △ IEF. 'Tis manifeſt, therefore, that thoſe Spaces will be equal, paſſed over in the ſame Time by two Moveables, one whereof is moved with a Motion uniformly accelerated from Reſt, but the other with an equable Motion, according to a

Momentum ſubduple of the greateſt *Momentum* of the Velocity of the accelerated Motion. *Q. E. D.*

PROP. II. THEOR. II.

If a Moveable deſcend from Reſt, with a Motion uniformly accelerate, the Spaces paſſed by it in any Times whatſoever, are to each other in a *duplicate* Proportion of the ſame Times, *i. e.* as the Squares of the ſame Times.

Let the Line AB repreſent a Length of Time from any firſt Inſtant A, wherein take any two Times AD and AE;



and let HI be a Line in which the Moveable from the Point H, as the firſt Beginning of Motion deſcends, accelerating

celerating uniformly; and let the Space HL be paſſed over in the firſt Time AD, and let HM be the Space it ſhall deſcend in the Time AE: I ſay the Space MH is to the Space HL, in a *duplicate* Proportion of that which the Time EA has to the Time AD; or, if you will, the Spaces MH and HL have the ſame Proportion to one another, as the Squares of EA and AD have to each other. Draw the Line AC, making any Angle with AB; and from the Points D and E draw the Parallels DO, EP, of which DO will repreſent the greateſt Degree of Velocity acquired in the Inſtant D, of the Time AD; and PE the greateſt Degree of Velocity acquired in the Inſtant E, of the Time AE: And ſince in the foregoing Propoſition it was demonſtrated, concerning Spaces paſſed over, that thoſe are equal to one another, one of which is paſſed by a Moveable out of Reſt, with a Motion uniformly accelerate; and the other which is paſſed in the ſame Time, by a Moveable carried with an equable Motion, whoſe Velocity is *ſubduple* of the greateſt acquired in the accelerate Motion; it is manifeſt, that the Spaces MH and LH are the ſame which with equal Motions, whoſe Velocities would be as the Halves of PE and OD, would be paſſed over in the Times EA and DA. If therefore it be prov'd that theſe Spaces MH and LH are in a *duplicate Ratio* of the Times EA and DA, we have gain'd what we wanted.

Now in the fourth Propoſition of the firſt Book it was demonſtrated, that of Moveables carried with an equable Motion, the Spaces paſſed over are in a Proportion to one

another, compounded of the *Ratio* of the Velocities, and of that of the Times : Now here the *Ratio* of the Velocities is the ſame with the *Ratio* of the Times (for the ſame Proportion as the Half of PE has to the Half of OD, or the Whole PE to the Whole OD, the ſame AE has to AD); therefore the Spaces are in a *duplicate* Proportion of the Times, or the Spaces paſſed over are to each other as the Squares of the Times. *Q. E. D.*



Hence likewiſe 'tis manifeſt, that the ſame *Ratio* of the Spaces is *duplicate* to the *Ratio* of the greateſt Degrees of Velocity, *viz.* of the Lines PE and OD, ſince it is PE : OD : : EA : DA.

COROL

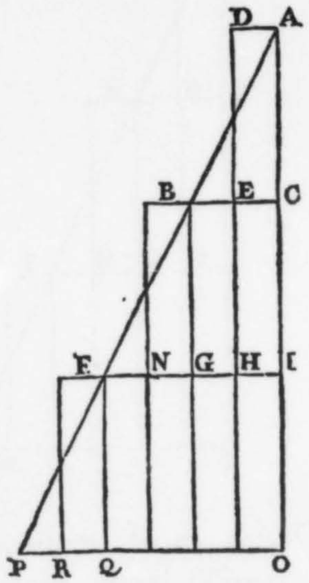
COROLLARY I.

Hence 'tis manifeſt, that if there were never ſo many equal Times, taken ſucceſſively from the firſt Inſtant, or Beginning of Motion ; as, for Inſtance, AD, DE, EF, FG, in which are paſſed over the Spaces HL, LM, MN, NI, thoſe Spaces will be one to another as the uneven Numbers are counted from the Unit, *viz.* as 1, 3, 5, 7 : For this is the *Ratio* of the Exceſſes of the Squares of the Lines exceeding one another equally, and the Exceſs of which Lines is equal to the leaſt of them ; or of the Squares conſequent, or that ſucceſſively follow one another from Unity : Whilſt, therefore, the Degrees of Velocity are increaſed, according to the ſimple *Series* of Numbers in equal Times, the Spaces run thro' in the ſame Times increaſe according to the *Series* of the uneven Numbers, counting from the Unit.

SAGR. Stay a little pray, whilſt I mention a Thing juſt come into my Mind ; to explain which the better, I'll deſcribe a Figure :

Let

Let then the Line AI repreſent the Continuation of Time, after the firſt Inſtant in A: Draw the right Line AF to the Point A, at any Angle with AI, and join the Points I and F; then biſect AI in C, and draw the Line CB ‖ IF: Then conſidering BC as the greateſt Degree of Velocity, which beginning from Reſt, in the firſt Inſtant A, goeth on augmenting according as the Parallels to BC do increaſe, which are in the △ ABC (which is really to in-

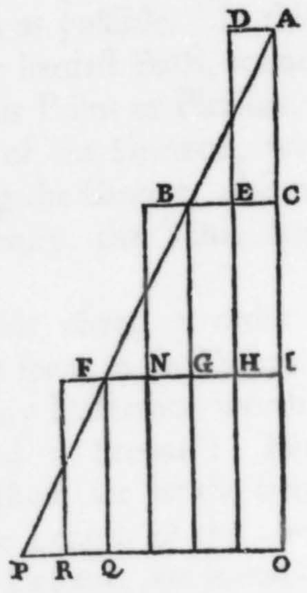


creaſe as the Time increaſes) by what has been ſaid, I admit, without Diſpute, that the Space run thro' by the falling Moveable, with the Velocity increaſed in the manner aforeſaid, would be = the Space that the ſame Moveable would paſs thro', in caſe it were in the ſame Time AC mov'd with an uniform Motion, whoſe Degree of Velocity ſhould be = EC, the Half of BC: I now proceed farther, and imagine the Moveable having deſcended with

with an accelerate Motion, to have in the Inſtant C the Degree of Velocity BC; 'tis manifeſt, that if it did continue to move with the ſame Degree of Velocity BC, without farther Acceleration, it will run thro' in the following Time CI, a Space double of that which it ran thro' in the equal Time AC, with the Degree of uniform Velocity EC, the Half of the Degree BC: But becauſe the Moveable deſcends with a Velocity increaſed always uniformly in all equal Times, it will add to the Degree BC, in the following Time, thoſe ſame *Momenta* of Velocity that increaſe according to the Parallels of the Triangle BFG, which is = △ ABC: Wherefore adding to the Velocity GI the Half of the Degree FG, which is the greateſt of all acquired by the accelerate Motion, and regulated by the Parallels of the Triangle BFG, we ſhall have the Degree of Velocity IN, with which, with an uniform Motion, it would have made in the Time CI; which Degree IN being *triple* the Degree CE, proves that the Space run thro' in the ſecond Time CI, ought to be *triple* of that it run thro' in the firſt Time CA.

And if we conceive another equal Part of Time as IO, to be added to AI, and the Triangle to be enlarg'd to APO, it is manifeſt, ſince thro' all that Time IO the Motion is continued with the Degree of Velocity IF, acquired by the accelerate Motion in the Time AI, ſince the Degree IF is *quadruple* of the Degree EC, the Space run thro' in the Time IO, 'twill be *quadruple* of that which was run thro' in the firſt equal Time AC: But now if the Increaſe of the uniform Acceleration be continued in the △ EPQ, like to that in △ ABC, which being reduced to equable Motion, adds the Degree = FC, QR being added = EC,

we ſhall have the whole equable Velocity exerciſed in the Time IO, *quintuple* of the equable Velocity of the firſt Time AC: Wherefore alſo the Space run thro' is *quintuple* that which was run thro' in the firſt Time AC.



'Tis manifeſt, therefore, from this Computation, that the Spaces run thro' in equal Times, by a Moveable, which, departing from Reſt, acquires a Velocity conformable to the Increaſe of the Time, are to one another as the uneven Numbers, counting from the Unit, as 1, 3, 5, are: And that the Spaces run thro' being taken together, the Space run thro' in the *double* Time, is *quadruple* to that ran thro' in a *ſubduple* Time; that run thro' in the *triple* Time is *noncuple*; and, in a Word, that the Spaces run thro' are in a *duplicate Ratio* of the Times, *i. e.* as the *Squares* of the Times.

SIMP. This

SIMP. This ſimple and clear Diſcourſe of *Sagredus*, pleaſes me beyond the Author's more obſcure Demonſtration; ſo that I am thoroughly convinc'd that the thing muſt ſucceed, as has been ſaid, the Definition of Motion uniformly accelerated being eſtabliſhed and granted: But I yet queſtion whether this be the Acceleration which Nature makes uſe of in the Motion of its deſcending heavy Bodies: Wherefore that I, and others like me, may the more clearly underſtand this, it would, methinks, be very ſeaſonable, before we go on, to produce ſome of thoſe Experiments, which are ſaid to be many, which in divers Caſes agree with the Concluſions demonſtrated.

SALV. You, like a Perſon of great Skill, make a very reaſonable Demand; for ſo it is uſual and convenient to do in Sciences that apply Mathematical Demonſtrations to Phyſical Concluſions, as is ſeen in thoſe that treat of *Perſpective*, *Aſtronomy*, *Mechanics*, *Muſic*, and others, who with ſenſible Experiments confirm thoſe their Principles, which are the Foundation of the following Structure: And therefore I would not have you think it ſuperfluous, to diſcourſe with ſome Prolixity concerning this firſt and grand Fundamental, whereon lies the Weight of the immenſe Machine of infinite Concluſions, of which we have but a very ſmall Part mentioned by our Author in this Book, who thought it enough to open the Door, hitherto ſhut, to ſpeculative Genius's. As to Experiments, the Author made ſeveral; and to aſſure us that the Acceleration of deſcending heavy Bodies obſerves the forementioned Proportion, I have often with him made the following Trial.

We took a Piece of Wood, a Priſm about twelve Yards long, half a Yard wide, and about three Inches thick, in which Thickneſs we made a Groove very ſtrait, or direct, a little more than an Inch wide; and to make it very ſmooth and ſleek, we glewed within it a Piece of Vellum poliſh'd as ſmooth as poſſible. In this Groove we let fall a Ball made of the hardeſt Braſs, round, and well poliſh'd. Thus elevating this Priſm at Pleaſure, one or two Yards above the Plane of the Horizon, we let the Ball, (as I ſaid) deſcend along the Groove, obſerving in the manner I ſhall tell you preſently, the Time ſpent in its Fall from Top to Bottom.

We repeated this often, in order to be certain of the Quantity of Time ſpent in the Deſcent, and in theſe Times we never found any Difference worth mentioning, no not the tenth Part of a Second: This being done and throughly eſtabliſhed, we let the ſame Ball deſcend but a fourth Part of the Length of the Groove, and meaſuring the Time of the Deſcent, we found it to be exactly the Half of the other: And then making Trial of the other Parts, by comparing the Time of its Deſcent thro' the whole Length, with that wherein it run thro', for Inſtance, $\frac{1}{2}$ the Length, or $\frac{2}{3}$, or $\frac{3}{4}$, or, in a Word, with the Time of its running thro' any other Part of the Length, by Experiments near an hundred times repeated, we always found the Spaces run thro' to be to one another, as the *Squares* of the Times: and this in all Inclinations of the Plane, *i. e.* of the Groove in which the Ball was made to deſcend: Where alſo we obſerv'd the Times of the Deſcents along different Inclinations, to retain the ſame Proportion to one another,

another, which anon we ſhall find aſſign'd them, and demonſtrated by the Author.

Now as to the meaſuring of Time: We hung on high a large Bucket of Water, which, by a very ſmall Hole thro' the Bottom, emitted a ſmall Thread of Water, which was received by a ſmall Cup, all the Time the Ball was falling thro' the Groove, or the ſeveral Parts of it: Then thoſe Parcels of Water thus collected, were every Time weigh'd in a very exact Pair of Scales, and the Differences and Proportions of the Weight exhibited to us the Differences and Proportions of the Times, and that ſo accurately, that, as I before ſaid, thoſe Trials often repeated, never differ'd any thing worth ſpeaking of.

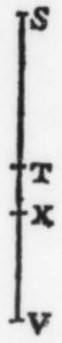
SIMP. I ſhould have been wonderfully pleaſed had I been preſent at thoſe Experiments: but ſince I may be aſſured of your Care in making them, and Fidelity in relating them, I am ſatisfied, and admit them as true, and ſafely to be relied on.

SALV. Then we may reſume our Reading, and go on:

COROLLARY II.

It is inferred, ſecondly, that if from the Beginning of the Motion, any two Spaces are taken, paſſed thro' in any Times, thoſe Times ſhall be to each other as either of thoſe Spaces is to a Space which is a mean Proportional between the two Spaces paſſed thro':

Thus, taking two Spaces ST and SV from the beginning of the Motion S, to which let SX be a mean Proportional; then the Time of the Fall thro' ST, ſhall be to the Time of the Fall thro' SV, as ST to SX; or again, the Time of the Fall thro' SV, to the Time thro' ST, is as VS to SX. For ſince it has been demonſtrated that the



Spaces run thro' are in a *duplicate Ratio* of the Times, or (which is the ſame) as the Squares of the Times: But the *Ratio* of the Space VS to the Space ST, is *duplicate* of the *Ratio* of VS to SX, or as the *Squares* of VS and SX are; 'tis manifeſt therefore that the Proportion of the Times of the Deſcents thro' SV, ST, is as that of the Spaces or Lines VS, SX.

SCHOLIUM.

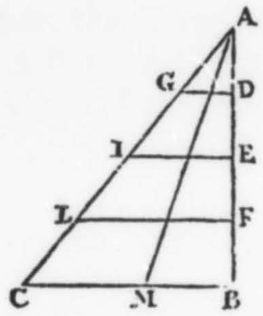
Now that which has been demonſtrated, in Motions perform'd perpendicularly, may be underſtood alſo to hold good in Motions made along Planes of any Inclination whatever; for it is ſuppoſed that in them the Degree of Acceleration

Acceleration increaſes in the ſame Proportion, *i. e.* according to the Increaſe of the Time, or, if you will, according to the ſimple and primary *Series* of Numbers.

PROP. III. THEOR. III.

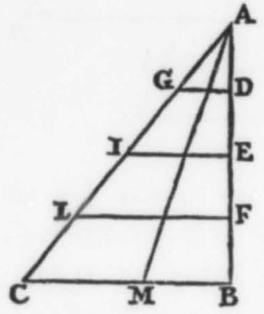
If the ſame Moveable, departing from Reſt, do move along an inclin'd Plane, and alſo along the Perpendicular, whoſe Heights are the ſame, the Times of their Motions will be to one another as the Lengths of that Plane, and that Perpendicular.

Let the inclined Plane be AC, and the Perpendicular AB, both which have the ſame Heights above the Horizon CB, *viz.* the very ſame Line BA; I ſay the Time of the Deſcent of the ſame Moveable along the Plane AC, has



the ſame Proportion to the Time of its Deſcent along the Perpendicular AB, as the Length of the Plane AC to the Length of the Perpendicular AB: For let any Number of Lines DG, EI, FL, be drawn ‖ the Horizon CB; 'tis manifeſt from the foregoing Aſſumption or Suppoſition, that

that the Degrees of the Velocity of the Moveable from A, the firſt Beginning of Motion acquired in the Points G and D, are equal, their Heights above the Horizon being equal; And in like manner the Degrees in the Points (I) and E, will be the ſame, as alſo the Degrees in L and F: And if not only theſe Parallels, but others, be imagin'd drawn from all the Points of the Line AB to the Line AC, the *Momenta* or Degrees of the Velocities, at the Terms or Extremes of each Parallel, will be always = one another: therefore the two Spaces AC and AB are paſſed over with the ſame Degrees of Velocity: But it has been demonſtrated, that if two Spaces are paſſed over by a Moveable, with the ſame Degrees of Velocity, the Times of the Motions have the ſame Proportion to each other as the Spaces have: therefore the Time of the Motion thro'



AC, to the Time thro' AB, is as the Length of the Plane AC to the Length of the Perpendicular AB. *Q. E. D.*

SAGR. Methinks this Trouble might have been ſpar'd, and we might have ſafely concluded the ſame thing, having before proved, that the Sum of the accelerated Motion of the

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the Tranſitions thro' AC and AB, is = the equable Motion, whoſe Degree of Velocity is *ſubduple* the greateſt Degree CB: Since therefore the two Spaces AC and AB are paſſed thro' with the ſame equable Motion, it hath already been ſhewn, by the firſt Propoſition of the firſt, that the Times of the Tranſitions will be to one another as the ſaid Spaces.

COROLLARY.

Hence is collected, that the Times of the Deſcents along Planes of different Inclinations, provided they have the ſame Elevation, are to one another as their Lengths.

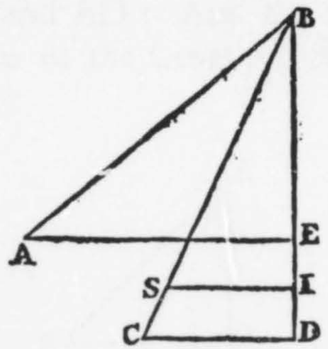
For if we ſuppoſe another Plane AM coming from A, and terminated by the ſame horizontal Line CB, 'tis demonſtrated by the laſt Propoſition, that the Time of the Deſcent thro' AM, to the Time thro' AB, is as the Line AM to AB; but as the Time AB to the Time thro' AC, ſo the Line AB to AC; therefore, *ex æquali*, as AM to AC, ſo is the Time along AM, to the Time thro' AC.

PROP. IV. THEOR. IV.

The Times of the Tranſitions along equal Planes, but unequally inclined, are to each other in a *ſubduple* Proportion of the Elevations of the ſame Planes, reciprocally taken.

Imagine

Imagine two equal Planes, but unequally inclin'd, as BA and BC, to proceed from the Point B; draw the horizontal Lines AE, CD, as far as the Perpendicular BD: Let BE be the Elevation of the Plane BA, and BD that of the Plane BC, and let BI be a mean Proportional between the Elevations DB and BE; then 'tis manifeſt, that the Proportion of DB to BI is *ſubduple* of the Proportion of DB to BE: Now I ſay, and muſt prove, that the Proportion of the Times of the Deſcents, or Tranſitions along the Planes BA and BC, is the ſame with the



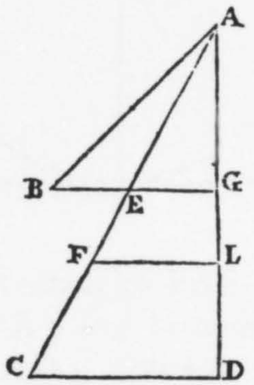
Proportion of DB to BI taken reciprocally: that is, I muſt prove that the Time along BA, is to the Time along BC, as DB is to BI: draw IS ‖ DC: Now ſince it has been demonſtrated, that the Time of the Deſcent thro' BA, to the Time of the Fall thro' the Perpendicular BE, is as BA to BE; and the Time thro' BE is to the Time thro' BD, as BE to BI; and the Time along BD to the Time along BC, as BD to BC, or as BI to BS: therefore, *ex æquali*, the Time along BA to the Time along BC, will be as BA to BS, or as CB to BS; but CB is to BS, as DB to BI, therefore the Propoſition is manifeſt.

PROP. V.

PROP. V. THEOR. V.

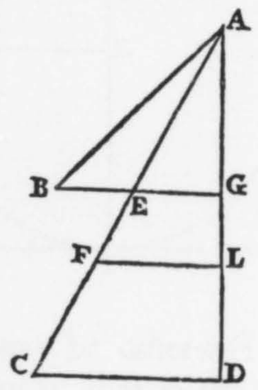
The Proportion of the Times of the Deſcent along Planes that have different Inclinations and Lengths, as alſo their Elevations unequal, is compounded of the Proportion of the Lengths of thoſe Planes, and of the *ſubduple* Proportion of their Elevations taken reciprocally.

Let AB and AC be Planes inclined after different Manners, whoſe Lengths are unequal, and whoſe Elevations are alſo unequal: I ſay, that the Proportion of the Time of the Deſcent through AC, to the Time through AB, is compoſed of the *Ratio* of AC to AB, and of the *Subduple* of their Elevations taken reciprocally: For,



draw the Perpendicular AD, which let the horizontal Lines BG CD meet; and between the Elevations DA and AG, let AL be a mean Proportional; and from the Point L draw a Parallel to the Horizon, meeting the Plane AC in F, then AF will be a mean Proportional between CA and AE: And becauſe the Time through

AC, to the Time thro' AE, is as the Line FA to AE; and the Time along AE, to the Time along AB, as the ſame AE to the ſame AB; 'tis manifeſt that the Time along AC, to the Time along AB, is as AF to AB: It remains, therefore, to be demonſtrated, that the Proportion of AF to AB, is compoſed of the *Ratio* of CA to AB, and of the *Ratio* of GA to AL, which is a *Ratio ſubduple* of the Elevations DA and AG reciprocally taken. But that is manifeſt putting CA between FA and AB; for the *Ratio* of FA to AC, is the ſame with that of LA to AD, or of GA to AL, which is *ſubduple* of the *Ratio* of the Elevations GA and AD: And the *Ratio* of CA to AB is the very *Ratio* of the Lengths; therefore the Propoſition is manifeſt.



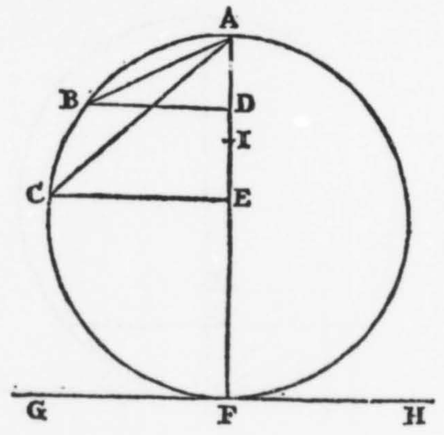
PROP. VI. THEOR. VI.

If from the uppermoſt and lowermoſt Points of a Circle, placed perpendicularly to the Horizon, certain inclined Planes be drawn to the Circumference, the Times of the Deſcents along theſe Planes, will be equal to each other.

Let

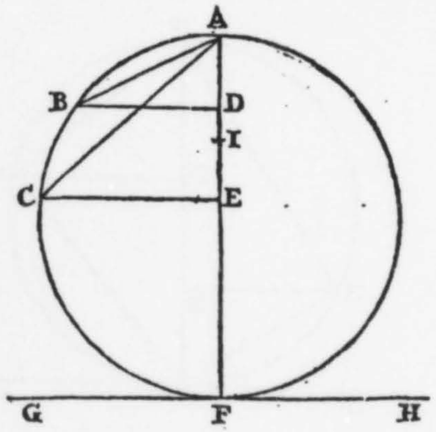
†

Let GH be the Horizon, and ACF a Circle erected upon it, whoſe Diameter FA, paſſing thro' its loweſt Part, or Point of Contact, let be erected perpendicular to the horizontal Line GH; and from its uppermoſt Point A let any Number of inclin'd Planes, as AB and AC be drawn to the Circumference; I ſay, that the Times of the Deſcents along thoſe Planes are equal to one another. Draw BD and CE perpendicular to the Diameter AF, and let AI be a mean Proportional between the Heights EA and AD of the Planes.



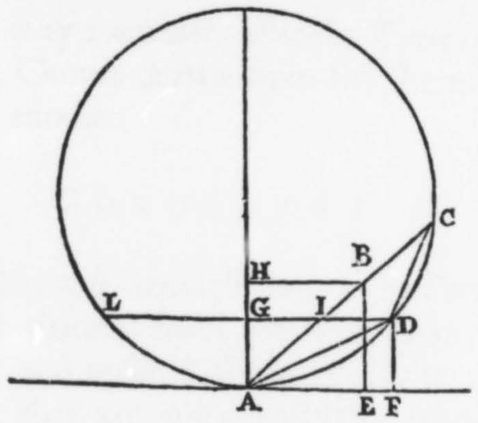
Now becauſe the Rectangles FAE and FAD are = the Squares of AC and AB; and becauſe it is as ▭ FAE to ▭ FAD, ſo AE to AD; therefore it is as the Square of CA to the Square of AB, ſo the Line EA to the Line AD. But as the Line EA to DA, ſo the Square of IA to the Square of AD; therefore the Squares of the Lines CA and AB are to one another as the Squares of the Lines IA and AD; and therefore as the Line CA to AB, ſo is IA to AD: But in the foregoing

Propoſition it was demonſtrated, that the Proportion of the Time of Deſcent through A C, to the Time of Deſcent thro' AB, is compounded of the *Ratios* of CA to AB, and of DA to AI, which is the ſame with the *Ratio* of BA to AC; therefore the Proportion of the Time of Deſcent through AC, to the Time of Deſcent through AB, is compounded of the *Ratios* of CA to AB, and of BA to AC; therefore the Proportion of thoſe Times is a Proportion of *Equality*; therefore the Propoſition is manifeſt.



The ſame Thing may be otherwiſe demonſtrated from the *Mechanicks*; namely, that in the following Figure, a Moveable will paſs thro' CA and DA in equal Times; For, let BA be = DA, and draw the Perpendiculars BE and DF, then 'tis manifeſt, from the Elements of *Mechanicks*, that the *Momentum* of the Weight elevated upon the Plane, according to the Line ABC to its whole *Momentum*, is as BE to BA; and that the *Momentum* of the ſame Weight upon the Elevation AD, to its whole *Momentum*,

mentum, is as DF to DA or BA; therefore the *Momentum* of the ſame Weight upon the Plane, inclin'd according to DA, to the *Momentum* upon the Inclination, according to ABC, is as the Line DF to the Line BE: Wherefore the Spaces which the ſame Weight ſhall paſs thro' in equal Times, upon the Inclinations CA and DA, will be to each other as the Lines BE and DF, by the ſecond Propoſition of the firſt Book: But as BE to DF, ſo it may be demonſtrated, that AC is to DA; therefore the ſame Moveable will paſs thro' the Lines CA and DA in equal Times.



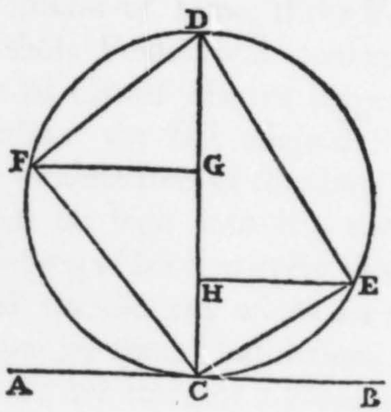
Now that BE is to DF, as CA to DA, is thus demonſtrated. Join CD, and thro' D draw DGL ‖ to AF, cutting CA in I, and thro' B draw BH alſo parallel to AF; then ∠ ADI will be = ∠ DCA; they ſtanding upon the equal Arches LA and AD, and ∠ DAC is common: Therefore the Sides of the ſimilar Triangles CAD and DAI, which are about the equal Angles, will be proportional; and thence it will be, CA : AD :: AD : AI, *i.e.* as BA to AI, or as HA to AG, *i.e.* as BE to DF. *Q. E. D.*

But

But the ſame Thing may be more expeditiouſly demonſtrated thus;

Let a Circle, as DFCE, be ſo erected upon the Horizon, as that its Diameter DC may be perpendicular thereto; and from the uppermoſt Point D, let there be an inclin'd Plane, as DF, drawn to the Circumference: I ſay, the Deſcent thro' the Plane DF, and that thro' the Diameter DC of the ſame Moveable, are perform'd in equal Times: For,

Draw FG parallel to the Horizon AB, which will be perpendicular to the Diameter DC, and draw FC; now



becauſe the Time of the Fall along DC is to the Time of the Fall along DG, as a mean Proportional between CD and DG, is to DG; and the mean Proportional between CD and DG, being DF; ∠ DFC in the Semicircle being a Right one, and FG perpendicular to DC; therefore the Time of the Fall through DC, is to the Time of the Fall through DG, as the Line FD to DG. But it has been demonſtrated, that the Time of the Deſcent through DF, is to the Time of the Deſcent through DG, as the ſame

ſame Line DF to DG; therefore the Times of the Deſcent thro' DF, and thro' DC, are to the Time of the Fall thro' DG, in the ſame Proportion; therefore they are equal. It may likewiſe be demonſtrated, that if from the loweſt Term C, be drawn the Chord CE, and EH being drawn ‖ to the Horizon, and ED drawn; that the Time of the Deſcent thro' EC will be equal to the Time of the Fall thro' the Diameter DC.

COROLLARY I.

Hence we may conclude, that the Times of the Deſcents along all the Chords drawn from the Terms C or D, are equal to one another.

COROLLARY II.

Hence 'tis likewiſe manifeſt, that if the Perpendicular and inclin'd Plane deſcend from the ſame Point, along which Perpendicular and inclin'd Plane the Deſcents are made in equal Times, they are in a Semicircle, whoſe Diameter is that very Perpendicular.

COROLLARY III.

Hence again, 'tis manifeſt, that the Times of the Motions along inclin'd Planes, are then equal, when the Elevations of equal Parts of the ſame Planes ſhall be to one another, as the Lengths of thoſe Planes.

For it has been ſhewn, that the Times thro' CA and DA, in the laſt Figure but one, are equal, the Elevation of

of the Part AB being equal to AD, *i. e.* that BE ſhall be to the Elevation DF, as CA to DA.

SAGR. Pray, Sir, ſtop a little, 'till I am ſatisfied concerning ſomething juſt come into my Head; which if it prove true, will be very pleaſant, as are all ſuch Concluſions as are natural or neceſſary.

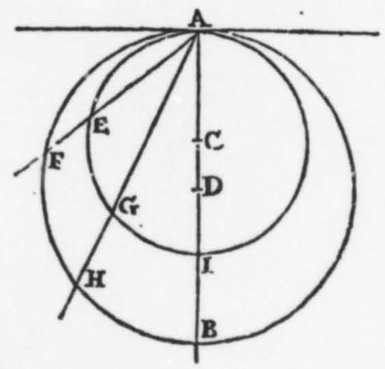
It is manifeſt, that if from a Point aſſigned in an horizontal Plane, be drawn an infinite Number of Lines every Way, upon each of which imagine a Point to move with an equable Motion; all theſe Points beginning to move in the ſame Inſtant of Time, if the Velocities of all of them be equal, thoſe Points will conſequently deſcribe Circumferences of Circles always bigger and bigger, and all concentric about the firſt aſſign'd Point; juſt as we ſee done in the Undulations of ſtanding Water, when a Stone is dropt from on high into it; the Percuſſion of which gives a Beginning of Motion every Way, and remains as the Centre of all the Circles; which are made ſucceſſively bigger and bigger by thoſe Undulations.

But now, if we imagine a Plane erected perpendicularly to the Horizon, and a Point in it aſſign'd or taken on high, from which imagine infinite Lines to proceed, inclin'd according to all Inclinations; upon, or along which, let heavy Moveables be imagined to deſcend, each with a Motion naturally accelerate, with ſuch Velocities that agree with the ſeveral Inclinations: If now we ſuppoſe thoſe Moveables to be continually viſible, in what Kind of Lines pray ſhould we ſee them conſtantly diſpoſed?

For my Part, I am amazed, when I conſider, that the preceding Demonſtrations prove them all to be conſtantly ſeen

ſeen in the ſame Circumference of Circles ſucceſſively increaſing, according as the Moveables in deſcending do more and more ſucceſſively recede from the higheſt Point, from whence they begun their Fall.

And that I may explain myſelf the better, let A be the Point on high, from which, according to any Inclinations, let fall the Lines AF, AH, as alſo the Perpendicular AB, in which taking the Points C and D, deſcribe Circles about them paſſing thro' the Point A and interſecting the inclin'd Lines in the Points F, H, B, and E, G, I. Then



by the foregoing Demonſtrations, it is manifeſt, that if the Moveables falling thro' thoſe Lines do depart from the Term A at the very ſame Time, that I ſay, when one is in E, another ſhall be in G, and the other in I: And thus continuing their Deſcent, in the very ſame Inſtant they will arrive in the Points F, H and B: And if theſe and infinite others continue ſtill to move along infinitely different Inclinations, they ſhall always ſucceſſively arrive at, or be in the very ſame Circumferences, which grow bigger and bigger *ad infinitum*.

From the two Kinds of Motion therefore which Nature uſes, is produced, or ariſes with admirable harmonious Variety, the Generation of infinite Circles.

The former is placed as in its Seat and Original Beginning, in the Centre of infinite concentric Circles; this latter reſides in the loſty Contact of infinite Circumferences of Circles, all excentric to one another. Thoſe ariſe from Motions which are all equal and equable; theſe from Motions which are all always inequable to one another, and all unequal to each other, as being made at infinitely different Inclinations.

But I add farther, that if from the two Points aſſigned, we conceive Lines to flow, not only along two Superficies, *viz.* the horizontal one, and the vertical or upright, but alſo every Way poſſible; as from thoſe beginning at one ſole Point, Circles from the leaſt to the greateſt were ſucceſſively produced; ſo here beginning from one ſole Point, infinite Spheres will be ſucceſſively produced; or, if you will, *one* Sphere continually increaſing to an infinite Magnitude; and this two Ways, *i.e.* let the Riſe of it be ſuppoſed either in the Centre, or in the Circumference of thoſe Spheres.

SALV. The Contemplation is really very ingenious, and ſuitable to *Sagredus*'s Genius.

SIMP. Tho' I conceive theſe two Ways of producing Circles and Spheres, by two different natural Motions; yet I don't throughly underſtand the Production, which depends on accelerate Motion: But yet becauſe for the Place of Emanation may one while be aſſign'd the loweſt Center, and

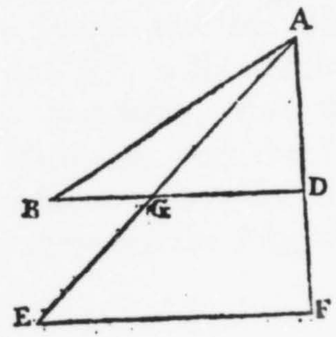
and another the higheſt Spherical Superficies; I can't but believe, that in theſe true and admirable Concluſions, there may be ſome great Myſtery conceal'd; ſome Myſtery, I ſay, in relation to the Creation of the Univerſe, whoſe Form is ſuppoſed Spherical, and concerning the Place of Reſidence of the firſt Cauſe.

SALV. I am pretty much of your Opinion: But ſuch profound Speculations we'll leave to wiſer Heads; and content ourſelves to be ſuch inferior Artificers, as diſcover and draw from the Quarry the Marbles, out of which the skilful Statuaries afterwards produce wonderful Images, which before lay hid under rude and misſhapen Cruſts. But now, if you pleaſe, we'll go on.

PROP. VII. THEOR. VII.

If the Elevations of two Planes ſhall have a Proportion *duplicate* of that of the Lengths of thoſe Planes, two Bodies ſhall move from the ſame Point of Reſt, the Length of thoſe Planes in equal Times.

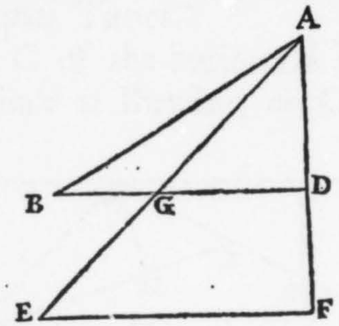
Let AE and AB be two unequal Planes, and unequally



inclined, and let the Elevation FA have a Proportion to

DA *duplicate* of AE to AB: Then, I ſay, that the Times of their being carried along the Planes AE and AB, from Reſt in A, are equal.

For draw to the Line of Elevations AF, the horizontal Parallels EF and BD, which Line BD let cut AE in G: Now becauſe the *Ratio* of FA to AD is *duplicate* of the *Ratio* of EA to AB, and becauſe it is as FA to AD, ſo EA to AG; therefore the *Ratio* of EA to AG is *duplicate*



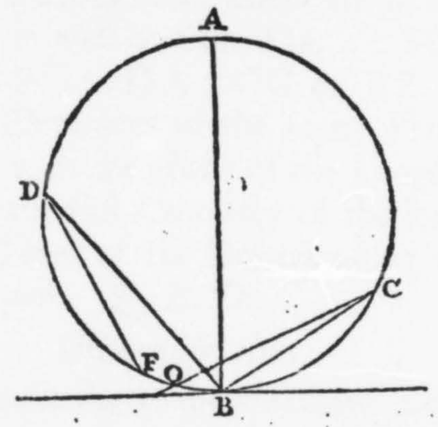
of that of EA to AB; and therefore AB is a mean Proportional between EA and AG; and becauſe the Time of the Deſcent thro' AB, to the Time thro' AG, is as AB to AG; and the Time of the Deſcent thro' AG, to the Time thro' AE, is as AG to a mean Proportional between AG and AE, which is AB; therefore *ex æquali*, the Time thro' AB, to the Time thro' AE, is as AB to itſelf; therefore the Times are equal. *Q. E. D.*

PROP. VIII. THEOR. VIII.

In Planes cut by the ſame Circle perpendicular to the Horizon in thoſe that meet with the End of the erect Diameter, whether at Bottom or at Top of it, the Times of Bodies being carry'd thro' thoſe Planes, are equal to the Time

Time of the Fall along, and the Length of that Diameter; and in thoſe that fall ſhort of the Diameter, the Times are ſhorter; but in thoſe that cut the Diameter, they are longer.

Let AB be the Diameter perpendicular to the Horizon of the Circle at Right Angles to the Horizon: Then that the Times of Bodies carried along Planes produc'd from the Terms A and B unto the Circumference of that Circle are equal, hath been already demonſtrated: And that the Time of the Deſcent along the Plane DF falling ſhort of



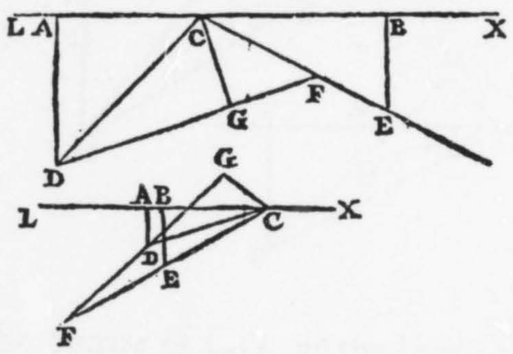
the Diameter, is ſhorter, is proved by drawing the Plane DB, which will be longer and leſs declining than DF; therefore the Time thro' DF, is ſhorter than the Time thro' DB, *i. e.* thro' AB. But laſtly, that the Time of the Deſcent along the Plane that cuts the Diameter, as CO, is longer, is in like Manner plain; for it is longer and leſs declining than CB; therefore the Propoſition is manifeſt.

PROP. IX.

PROP. IX. THEOR. IX.

If two Planes be inclin'd at Pleaſure from a Point in a Line parallel to the Horizon, and be cut by a Line which may make Angles alternately equal to the Angles contained between the ſaid Planes and horizontal Parallel, then the Motions thro' the Parts cut off by the ſaid Line, ſhall be performed in equal Times.

From the Point C of the horizontal Line LX, let any two Planes be inclin'd at Pleaſure, as CD and CE; and



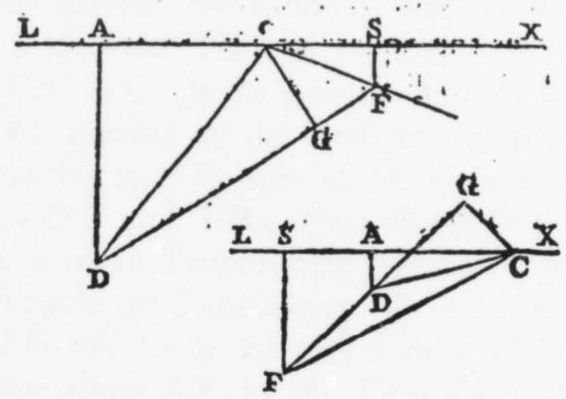
in any Point of the Line CD, make ∠ CDF = ∠ XCE: Then will the Line DF cut the Plane CE in F, in ſuch Manner that ∠'s CDF and CFD will be = ∠'s LCD and XCE taken alternately, *i.e.* ∠ CDF will be = XCE and CFD = LCD. For the common Angle DCF being taken from the three Angles of the △ CDF = two Right Angles, to which all the Angles made at the Point C, of the Line LX are equal, there remains in △ CDF two Angles CDF and CFD = the two Angles XCE and LCD: But by the Hypotheſis ∠ CDF is = ∠ XCE; therefore the remaining Angle CFD is = the remaining Angle DCL.

This

This being done, I affirm, that the Deſcents thro' CD and CF are equal; for let the Plane CE be ſuppos'd = the Plane CD, and from the Points D and E erect the Perpendiculars DA and EB unto the horizontal Parallel LX; and from C unto DF, let fall the Perpendicular CG: Now becauſe ∠ CDG is = ∠ ECB; and ∠'s DGC and CBE are Right, the Triangles CDG and CBE will be equiangular, and it will be DC : CG : : CE : EB; but DC is = CE; therefore CG will be = BE. And ſince of △'s DAC and CGF ∠'s ACD and CAD are = ∠'s CFG and FGC; therefore it will be CD : DA : : FC : CG, and alternately CD : FC : : DA : CG or BE. The *Ratio* therefore of the Elevations of the equal Planes CD and CE, is the ſame with the *Ratio* of the Lengths DC and CF, therefore by the firſt Corollary of the foregoing ſixth Propoſition, the Times of the Deſcent along and thro' the ſame, ſhall be equal. *Q. E. D.*

Otherwiſe thus,

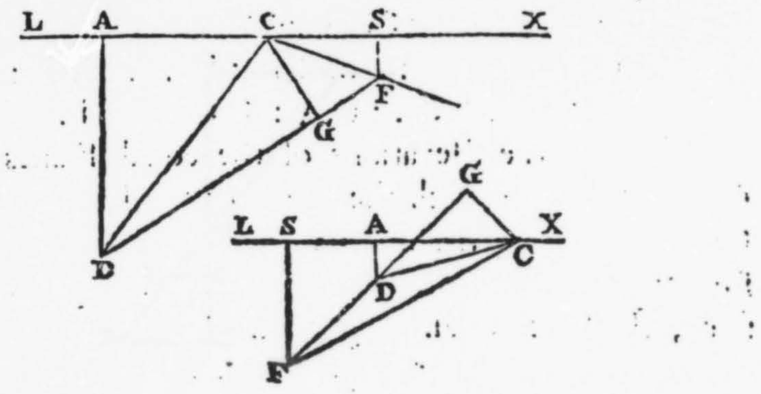
Draw FS perpendicular to the horizontal Line AS; then



becauſe △ CSF is ſimilar to △ DGC, it ſhall be SF : FC : : GC : CD. And becauſe △ CFG is ſimilar to △ DCA,

it

it will be FC : CG : : CD : DA; therefore *ex æquali*, SF : CG : : CG : DA; therefore CG is a mean Proportional between SF and DA: And as DA to SF, ſo the Square of DA to the Square of CG. Again, ſince △ ACD is ſimilar to △ CGF, it will be DA : DC : : CG : CF, and alternately, DA : CG : : DC : CF; and as the Square of DA to the Square of CG, ſo is the Square of DC to the Square of CF. But it has been proved that the Square of



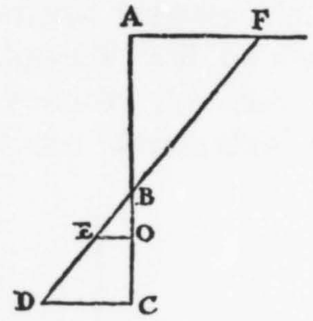
DA is to the Square of CG, as the Line DA to the Line FS; therefore, as the Square of DC to the Square of CF, ſo the Line DA to FS; therefore from the foregoing Seventh, ſince the Elevations of the Planes CD and CF have a *duplicate* Proportion of the ſame Planes, the Times of their Deſcents along the ſame will be equal.

PROP. X.

PROP. X. THEOR. X.

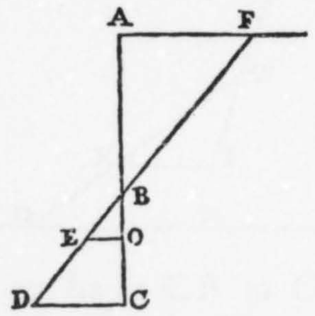
The Times of the Motions along different Inclinations of Planes whoſe Elevations are equal, are one to another as the Lengths of thoſe Planes, whether the Motions be made from Reſt, or a Motion from the ſame Height hath preceded them.

Let the Motions be made along ABC, and along ABD, to the Horizon DC; in ſuch manner that the Lation thro'



AB may precede the Motions thro' BD and thro' BC. Then, I ſay, the Time of the Lation thro' BD is to the Time thro' BC, as the Length BD to the Length BC. Draw AF parallel to the Horizon, to which continue out DB, meeting it in F, and let FE be a mean Proportional between DF and FB; draw alſo EO ‖ DC, then will AO be a mean Proportional between CA and AB; but if we ſuppoſe the Time along AB to be as AB, the Time along FB will be as FB (by Corol. of Theor. 3.); and the Time along AB, to the Time along all AC, ſhall be as AB to AO, a mean Proportional between AB and AC;

(by Cor. 2. of Theor. 2.) and the Time along FB, to the Time along all FD, ſhall be as FB to FE, a mean Proportional between FB and FD; wherefore the Time along the Remainder BC will be BO, and along the Remainder BD will be BE: But as BE is to BO, ſo is BD to BC; therefore the Times along BD and BC, after the Deſcent along AB and FB, or which is the ſame thing, along the common Part AB, ſhall be to one another as the Lengths BD and BC. But it has been already demonſtrated, that



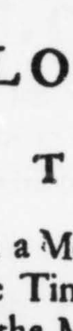
the Time along BD, is to the Time along BC, out of Reſt in B, as the Length BD to BC; therefore the Times of the Motions along different Planes, whoſe Elevations are equal, are to one another as the Lengths of the ſaid Planes, whether the Motion be made along the ſame, out of Reſt, or whether another Motion from the ſame Height do precede thoſe Motions. *Q. E. D.*

PROP. XI.

PROP. XI. THEOR. XI.

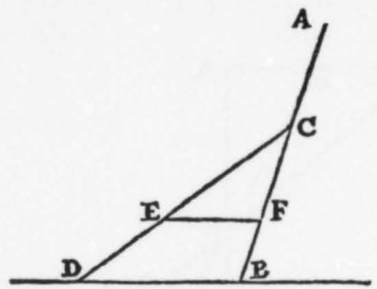
If a Plane, along which a Motion is made, out of Reſt, be divided at Pleaſure, the Time of the Motion along the firſt Part, to the Time of the Motion along the ſecond, is as that firſt Part to the Exceſs by which the ſame Part is exceeded by a mean Proportional between the whole Plane, and the ſame firſt Part.

Let the Motion be along the whole Plane AB, from Reſt in A, which let be divided at Pleaſure in C; and let AF be a mean Proportional between the whole BA and the firſt Part AC; then CF will be the Exceſs of the mean Proportional AF above the Part AC. Then, I ſay, that the Time of the Motion thro' AC to the Time



of the following Motion thro' CB, is as AC to CF. This is manifeſt; for the Time along AC, to the Time along the whole AB, is as AC to the mean AF; therefore by Diviſion, the Time along AC to the Time thro' the remaining Part CB will be as AC to CF. If therefore the Time thro' AC be ſuppoſed to be the very AC, the Time along CB will be CF. *Q. E. D.*

But if the Motion be not made along the continu'd Plane ACB, but by the inflected ones AC and CD, 'till it reaches the horizontal Line BD, to which from F, the Parallel FE is drawn. It may, in like manner, be demonſtrated, that the Time along AC, to the Time along CD, is as AC to CE: For the Time along AC to the Time along CB, is as AC to CF; but the Time along CB, after the Deſcent through AC to the Time through CD, after the ſame Deſcent through AC, has



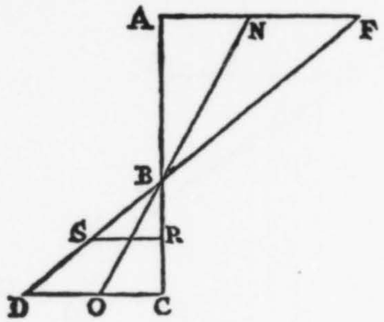
been demonſtrated to be as CB to CD, *i.e.* as CF to CE; therefore, *ex æquali*, the Time along AC, to the Time along CD, will be as the Line AC to CE.

PROP. XII. THEOR. XII.

If the Perpendicular and Plane, at Pleaſure inclin'd, be cut between the ſame horizontal Lines, and be taken mean Proportionals between them and the Parts of them contained between the common Section and upper horizontal Line; the Time of the Motion along the Perpendicular, to the Time of the Motion along the upper Part of the Perpendicular, and then along the lower Part of the cutting Plane, will have the ſame Proportion, as the Length of

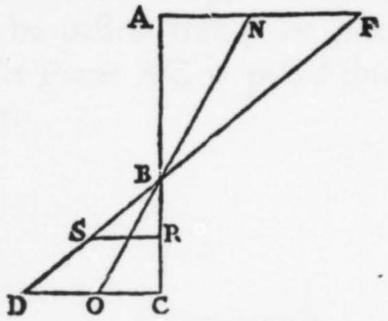
of the whole Perpendicular has to a Line compounded of the mean Proportional given in the Perpendicular, and of the Exceſs, by which the whole inclined Plane exceeds its mean Proportional.

Let the horizontal Lines be AF the upper, and DC the lower; between which let the Perpendicular AC, and the inclin'd Plane DF be cut in B; and let AR be a mean Proportional between the whole Perpendicular CA, and the upper Part AB; and let FS be a mean Proportional



between the whole inclin'd Plane DF, and its upper Part BF. Then, I ſay, that the Time of the Fall along the whole Perpendicular AC, to the Time along its upper Part AB, together with the lower Part of the Plane DF, *viz.* with BD, has the ſame Proportion as AC has to the mean Proportional of the Perpendicular, *i. e.* AR, together with SD, which is the Exceſs of the whole Plane DF above its mean Proportional FS: For let a Line be drawn from R to S, which will be ‖ to the two horizontal Lines: and becauſe the Time of the Fall along all AC, to the Time along the Part AB, is as CA to the mean Proportional AR, if we ſuppoſe AC to be the Time of

the Fall thro' AC, AR will be the Time of the Fall thro' AB, and RC that thro' the Remainder BC. For if the Time along AC be ſuppoſed, as was done to be AC itſelf, the Time along FD ſhall be FD; and in like manner DS may be concluded to be the Time along BD, after the Deſcent thro' FB, or thro' AB: The Time therefore along the whole AC, is AR with RC; and the Time along the inflected Planes AB and BD, will be AR with SD. *Q. E. D.*



The ſame happens if inſtead of the Perpendicular another Plane be ſuppoſed or taken, as *v. g.* NO; and the Demonſtration is the ſame.

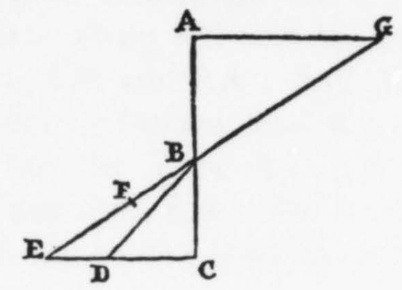
PROP XIII. PROB. I.

A Perpendicular being given to inflect a Plane to it, along which, ſuppoſing it to have the ſame Elevation with the given Perpendicular, it may move after its Fall along the Perpendicular, in the ſame Time as it is falling from Reſt along that Perpendicular.

Let the given Perpendicular be AB, to which extended to C, let the Part BC be equal, and draw the horizontal Lines

†

Lines CE and AG; 'tis required from B to inflect a Plane reaching to the Horizon CE, along which a Motion may be made after a Deſcent from A, in the ſame Time as along AB from Reſt in A. Make CD = CB, and drawing BD, let BE be drawn = BD and DC taken together. Then, I ſay, that BE is the Plane ſought. Produce EB meeting the horizontal Line AG in G, and let GF be a mean Proportional between EG and GB: Then EF will be to FB as EG to GF; and the Square of EF to the



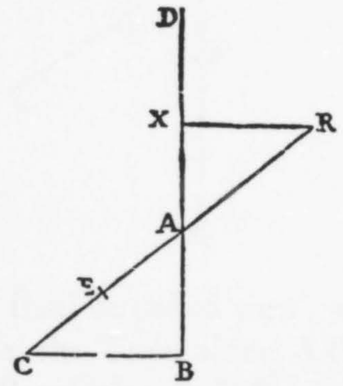
Square of FB, as the Square EG to the Square of GF, *i. e.* as the Line EG to GB; but EG is the Double of GB, therefore the Square of EF is the Double of the Square of FB; but the Square alſo of DB is double the Square of BC; therefore as the Line EF to FB, ſo is DB to BC; and by Compoſition and Permutation, as EB to DB × BC, ſo is BF to BC; but BE is = DB × BC; therefore BF is = BC or to BA. If, therefore, AB be ſuppoſed to be the Time of the Fall along AB, GB will be the Time of the Fall along GB, and GF the Time along the whole Plane GE; therefore BF will be the Time of the Fall thro' the Remainder BE, after the Fall from G, or from A. *Q. E. D.*

PROP. XIV.

Prop XIV. Prob. II.

A Perpendicular and a Plane inclin'd to it, being given, to find a Part in the upper Perpendicular, which ſhall be paſſed thro' from Reſt, in a Time = that wherein the inclin'd Plane is paſs'd thro', after the Fall along the Part found in the Perpendicular.

Let the Perpendicular be DB, and the Plane inclined to it AC; it is required in the Perpendicular AD to find a Part, which ſhall be paſſed thro', *ex quiete*, in a Time = that in which the Plane AC is paſs'd thro' after the Fall along the ſaid Part.



Draw the horizontal Line CB; and as BA × twice AC to AC, ſo make CA to AE; and as BA is to AC, ſo let EA be made to AR; and from R let fall the Perpendicular RX, to the Line DB; then, I ſay, that X is the Point ſought: And becauſe as BA × twice AC is to AC, ſo CA is to AE, by Diviſion it will be as BA × AC to AC, ſo CE to EA; and becauſe as BA to AC, ſo EA

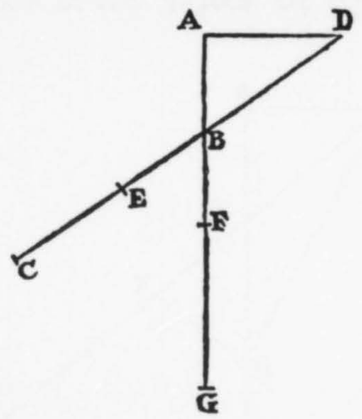
EA to AR; it will be by Compoſition as BA × AC to AC, ſo ER to RA: But as BA × AC to AC, ſo is CE to EA; therefore as CE to EA, ſo ER to RA, and both the Antecedents to both the Conſequents, *viz.* CR to RE; therefore CR, RE and RA are Proportionals: Furthermore, becauſe as BA to AC, ſo EA is ſuppoſed to be to AR, and becauſe of the Similarneſs of Triangles, as BA to AC, ſo XA to AR; therefore as EA to AR, ſo XA to AR, therefore EA and XA are equal.

And now if we underſtand the Time along RA to be RA, the Time along RC will be RE, a mean Proportional between CR and RA: And AE will be the Time along AC, after its Deſcent thro' RA, or after its Deſcent thro' XA; but the Time thro' XA is XA, ſo long as RA is the Time thro' RA: But it has been ſhewn, that XA and AE are equal; therefore the Propoſition is manifeſt.

Prop. XV. Prob. III.

A Perpendicular, and a Plane inflected to it, being given, to find a Part in the Perpendicular extended downwards, which ſhall be paſs'd thro' in the ſame Time as the inflected Plane is, after a Deſcent along the given Perpendicular.

Let the Perpendicular be AB, and the Plane inflected to it BC: It is required in the Perpendicular extended downwards, to find a Part, which from the Fall from A ſhall be paſſed thro' in the ſame Time as BC is paſſed thro' from the ſame Fall from A: Draw the horizontal Line AD, which let CB meet, extended to D, and let DE be a mean Proportional between CD and DB, and let BF be = BE; and laſtly, let AG be a third Proportional to BA and AF; then, I ſay, BG is the Space that, after the Fall AB, ſhall be paſſed thro' in the ſame Time



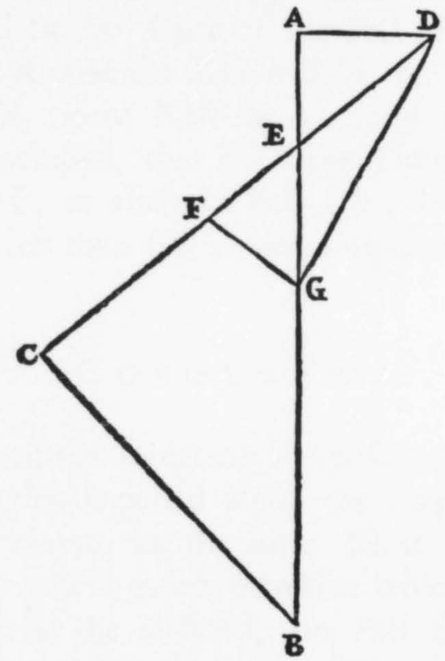
as the Plane BC ſhall be paſſed thro', after the ſame Fall: For if we ſuppoſe the Time along AB to be as AB, the Time along DB will be as DB; and becauſe DE is a mean Proportional between BD and DC, the ſame DE ſhall be the Time along the Whole DC, and BE the Time along the Remainder BC, from Reſt in D, or after the Fall thro' AB; and in like manner it may be concluded, that BF is the Time of the Fall thro' BG, after the ſame Fall thro' AB; but BF is = BE; therefore the Propoſition is manifeſt.

PROP. XVI.

PROP. XVI. THEOR. XIII.

If the Parts of an inclin'd Plane and Perpendicular, the Times of whoſe Motions from Reſt are equal, meet at the ſame Point; a Moveable coming from any greater Height, will ſooner run thro' the ſaid Parts of the inclined Plane, than that Part of the Perpendicular.

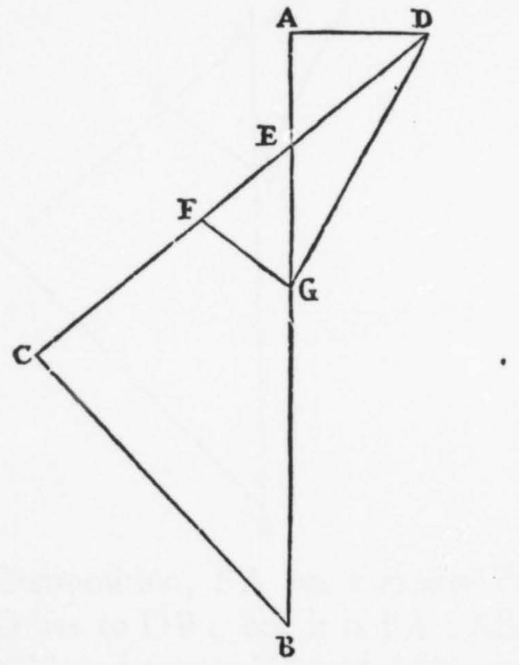
Let the Perpendicular be EB, and the inclin'd Plane CE meeting each other in the Point E, the Times of whoſe



Motions from Reſt in E are equal, and in the Perpendicular extended, take any higher Point at Pleaſure, as A; from which let the Moveable fall; I ſay, that the inclined Plane

EC will be ſooner paſſed over than the Perpendicular EB will after the Fall AE.

Join C and B, and having drawn the horizontal Line AD, produce CE 'till it meets it in D; and make DF a mean Proportional between CD and DE; and make AG a mean Proportional between BA and AE, and draw FG and DG: Now becauſe the Times of the Moveables being carried thro' EC and EB, from Reſt at E, are equal, ∠ C will be a Right one, by *Cor.* 2. of *Prop.* 6. and ∠ A is Right, and ∠'s at the Vertex are equal, therefore △'s



AED and CEB are Equiangular, and the Sides about the equal Angles proportional, therefore BE : EC : : DE : EA, ▭ BEA is = ▭ CED: And becauſe ▭ CDE exceeds ▭ CED by

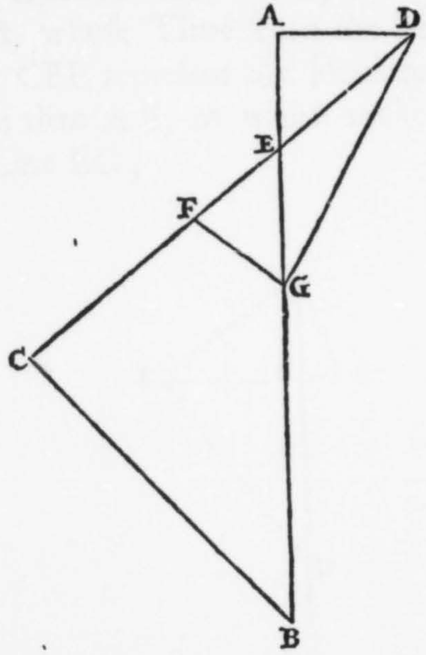
by the □ of ED, and ▭ BAE exceeds ▭ BEA, by □ of EA, the Exceſs of ▭ CDE above ▭ BAE, *i. e.* of □ FD above □ AG, will be the ſame with the Exceſs of □ of DE above the Square of AE, which Exceſs is □ of DA; therefore the □ of FD is = the two Squares of GA and AD, to which □ of GD is alſo equal; therefore the Line DF is = DG, and ∠ DGF = ∠ DFG, and ∠ EGF leſs than ∠ EFG, and the oppoſite Side EF leſs than the Side EG: Now if we ſuppoſe the Time of the Fall thro' AE to be as AE, the Time thro' DE will be as DE; and ſince AG is a mean Proportional between BA and AE, AG will be the Time of the Fall thro' the whole AB, and the Remainder EG will be the Time thro' the Remainder EB, from Reſt in A; and it may in like manner be concluded, that EF is the Time thro' EC, after the Deſcent DE, or after the Fall AE; but EF has been proved to be leſs than EG; the Propoſition therefore is manifeſt.

COROLLARY.

From this and the foregoing Propoſition, it is manifeſt, that the Space that is paſſed along the Perpendicular, after the Fall from above, in the ſame Time wherein the inclin'd Plane is paſs'd, is leſs than that which is paſs'd in the ſame Time as in the inclined, no Fall from above preceding, yet greater than the inclined Plane.

For it having but juſt now been proved, that of the Moveables coming from the ſublime Term A, the Time of its paſſing thro' EC is ſhorter than that of its running along EB; it is manifeſt, that the Space that is paſſed along

along EB, in a Time = the Time along EC, is leſs than the whole Space EB; but that the ſame Space along the Perpendicular is greater than EC, is manifeſt, by reaſſuming the Figure of the foregoing Propoſition, wherein the Part of the Perpendicular BG hath been demonſtrated to be moved over in the ſame Time as BC is, after the Fall AB: but now that this BG is greater than BC, is thus collected; ſince BE and FB are equal, and BA leſs than BD, FB has a greater Proportion to BA, than EB has to



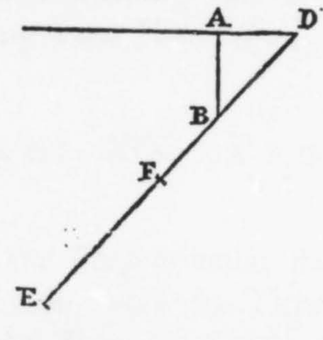
BD, and, by Compoſition, FA has a greater Proportion to AB than ED has to DB; but it is FA : AB : : GF : FB (for AF is a Mean between BA and AG); and in like manner ED : BD : : CE : EB; therefore by Compoſition, GB has a greater *Ratio* to BF than CB has to BE; therefore GB is greater than BC.

PROP. XVII.

PROP XVII. PROB. IV.

A Perpendicular and a Plane inclin'd to it, being given, to aſſign a Part in the given Plane, in which, after the Fall along the Perpendicular, the Motion may be made in a Time = that wherein the Moveable from Reſt paſſed thro' the given Perpendicular.

Let the Perpendicular be AB, and let the Plane inclined to it be BE: It is required in BE to aſſign a Space along which the Moveable, after the Fall thro' AB, may move in a Time = that wherein it paſſed from Reſt thro' the Perpendicular AB.



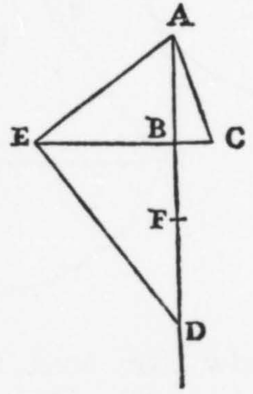
Let AD be an horizontal Line, which let the Plane produced meet in D, and make FB = BA, and make BD : DF : : FD : DE: Then, I ſay, the Time thro' BE, after the Fall along AB, is = the Time thro' AB, from Reſt in A: For if we ſuppoſe AB to be the Time along AB, DB will be the Time thro' DB: And becauſe it is BD : DF : : FD : DE, DF will be the Time thro' the whole Plane DE, and BF will be that thro' the Part BE from D; but the Time thro' BE after DB, is the ſame as after AB; therefore the Time thro' BE, after AB, will be BF; *i. e.* = the Time AB, from Reſt in A. *Q. E. F.*

PROP. XVIII.

PROP. XVIII. PROB. V.

Any Space in the Perpendicular being given, from the aſſigned Beginning of Motion, that is paſſed in a given Time, and any other leſſer Time being alſo given; to find another Length in the ſaid Perpendicular, that may be paſſed thro' in the given leſſer Time:

Let the Perpendicular be AD, in which let the Space given be AB, whoſe Time from the Beginning A, let be AB, and let CBE repreſent the Horizon, and let a Time be given leſs than AB, to which make BC equal in the horizontal Line EC;



'Tis required in the ſame Perpendicular to find a Space = the ſame AB, that may be paſſed in the Time BC.

Draw a Line from A to C; then becauſe BC is leſs than BA, ∠ BAC will be leſs than BCA. Make CAE = to it, and let the Line AE meet the Horizon in the Point E, to which ſuppoſe ED perpendicular, cutting the Perpendicular

pendicular in D; and make DF = BA; then, I ſay, that FD is that Part of the Perpendicular, along which the Lation from the Beginning of Motion in A, is performed in the given Time BC.

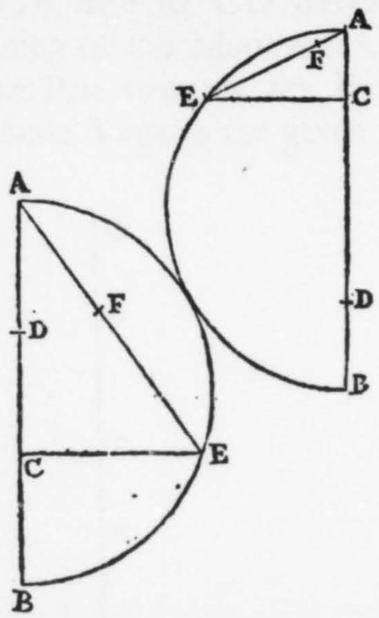
For ſince in the right-angled Triangle AED, from the right Angle E, a Perpendicular EB is drawn to the oppoſite Side AD, AE will be a mean Proportional between DA and AB, and BE a mean between DB and BA, or between FA and AB (for FA is = DB) and ſince AB is ſuppoſed to be the Time thro' AB, AE or EC will be the Time thro' the whole AD, and EB the Time thro' AF; therefore the remaining Part BC will be the Time thro' the remaining Line FD. *Q. E. F.*

PROP. XIX. PROB. VI.

Any Space in the Perpendicular paſſed over, from the Beginning of Motion, with the Time of the Fall, being given, to find the Time in which another equal Space, taken at Pleaſure in the ſame Perpendicular, ſhall be afterwards paſſed thro' by the ſame Moveable.

In the Perpendicular AB, let AC be any Space taken from the Beginning of the Fall in A, to which let another Space, as DB, taken at Pleaſure, be equal, and let the Time of the Fall thro', or Motion along AC be given; and let it be AC. Now it is required to find the Time of the Fall thro', or Motion along DB, after its Fall from A.

About the whole Line AB, as a Diameter, deſcribe the Semicircle AEB, and from C let fall CE perpendicular to



AB, and draw the Line AE, which will be longer than EC. Make EF = EC: Then, I ſay, that the Remainder FA is the Time of the Fall thro', or Motion along DB, after the Fall from A; For,

Demonstration.

Becauſe AE is a mean Proportional between BA and AC, and becauſe AC is the Time of the Fall thro' AC, AE

AE will be the Time of the Fall thro' the whole Line AB. And ſince CE is a Mean between DA and AC, (for DA is = BC) CE, *i.e.* EF, will be the Time of the Fall thro' AD; therefore what remains AF, is the Time of the Fall thro' the remaining Space DB. *Q. E. D.*

Corollary.

Hence is gathered, that if the Time of any Space, *ex quiete*, be as the ſaid Space; the Time thereof, after another Space is added, will be the Exceſs of the mean Proportional betwixt the Space added, and former Space taken together, and the ſaid Space above the mean Proportional, betwixt the firſt Space and the Space added.

As for Example.

Suppoſing that the Time thro' AB, from Reſt in A, be AB; AS being another Space added, the Time along AB,

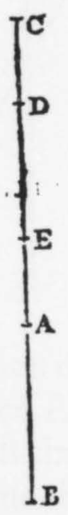


after the Fall thro' SA, will be the Exceſs of the mean Proportional between SB and BA above the mean Proportional between BA and AS.

PROP XX. PROB. VII.

Any Space, and a Part therein, after the Beginning of Motion, being given, to find another Part towards the End, that ſhall be paſſed thro' in the ſame Time as the firſt Part given is paſſed thro':

Let the Space be CB, and let CD be the Part in it given, after the Beginning of the Motion in C; now 'tis required to find another Part towards the End B, which may be paſſed in the ſame Time as the given Part CD is.



Take a mean Proportional between BC and CD, to which ſuppoſe BA equal: and let CE be a third Proportional between BC and CA; I ſay, EB is the Space that after the Fall out of C ſhall be paſſed over in the ſame Time as CD is.

For

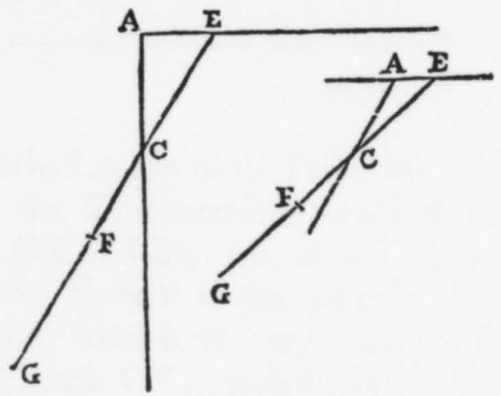
For if we ſuppoſe the Time along the whole CB, to be as CB; BA (the mean Proportional between BC and CD) ſhall be the Time along CD: And ſince CA is a Mean betwixt BC and CE, CA will be the Time thro' CE; but the whole BC is the Time thro' the whole CB, therefore the remaining Part BA, will be the Time thro' the remaining Part EB, after the Fall thro' C; but the ſame BA was the Time thro' CD, therefore CD and EB will be paſſed over in equal Times from Reſt in A. *Q. E. F.*

PROP. XXI. THEOR. XIV.

If from Reſt a Fall be made perpendicularly, in which from the Beginning of the Motion, a Part be taken at Pleaſure, paſſed over in any Time, after which let an inflex Motion follow, along any Plane inclin'd at Pleaſure; the Space, which in ſuch a Plane is paſſed in a Time = the Time of the Fall already made in the Perpendicular, will be more than double, and leſs than triple the Space already paſſed in the Perpendicular.

Under

Under the Horizon AE, let AB be a Perpendicular to it; along which, from the Beginning A, let a Fall be made, of which take a Part at Pleaſure as AC; then from C let a Plane CG be inclin'd at Pleaſure, along which, after the Fall from A to C, let the Motion be continued; then, I ſay, that the Space paſſed by ſuch Motion along CG, in a Time equal to the Time of the Fall thro' AC, is more than double, and leſs than triple that ſame Space AC: For make CF = AC, and extend the Plane GC to the Horizon in E, and make as CE to EF, ſo FE to EG;



i.e. let EF be continu'd to G, ſo that EF may be a mean Proportional between EC and EG: If then we ſuppoſe the Time of the Fall thro' AC, to be as the Line AC, CE ſhall be the Time thro' EC, and CF or CA the Time of the Motion thro' CG: Then it is to be proved, that the Space CG is more than double, and leſs than triple the ſaid CA. Thus then ſince it is as CE to EF, ſo is EF to EG, it will alſo be, ſo is CF to FG: but EC is leſs than EF, wherefore alſo CF will be leſſer than FG, and conſequently GC more than double FC or AC: and again, ſince

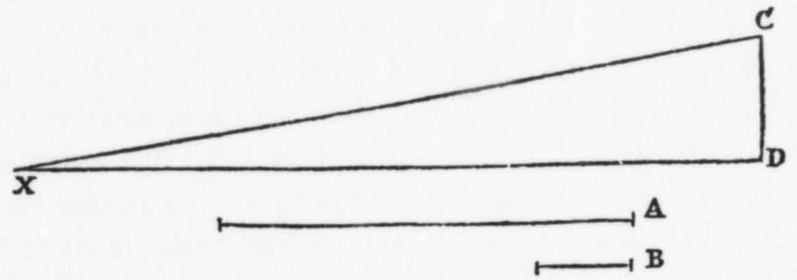
ſince FE is leſs than double EC (for EC is greater than CA or CF) GF alſo will be leſs than double FC, and conſequently GC leſs than triple CF or CA. *Q. E. D.*

But the ſame thing may be more univerſally propoſed; for that which has been proved as to the Perpendicular and inclined Plane, holds true alſo, if after the Motion along any inclin'd Plane, it moves along a Plane more inclined, as you may ſee by Fig. 2. and the Demonſtration is the ſame.

PROP. XXII. PROB. VIII.

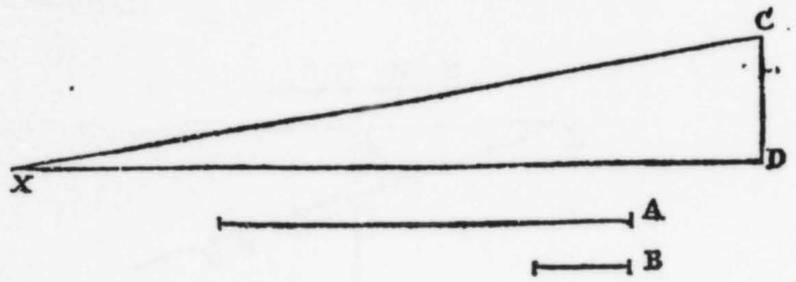
Two unequal Times being given, and the Space that from Reſt is paſſed over along the Perpendicular in the ſhorter of the given Times; to inflect a Plane from the higheſt Point of the Perpendicular unto the Horizon, thro' which the Moveable may deſcend, in a Time = the longer of the given Times:

Let the greater of the unequal Times be A, the leſſer B; and let the Space that from Reſt is paſſed along the



Perpendicular, in the Time B, be CD; then it is requir'd from the Term C to inflect a Plane 'till it reach the Horizon, that ſhall be paſſed over in the Time A. As B is

to A, ſo let CD be to a fourth Line, to which let CX, drawn from C to the Horizon, be equal; then 'tis manifeſt, that CX is the Plane thro' which the Moveable will paſs in the given Time A: For it has been demonſtrated, that the Time along, or thro' the inclined Plane has the ſame Proportion to the Time along its Elevation (or the



Perpendicular) as the Length of the Plane has to the Length of its Elevation; the Time therefore thro' CX, to the Time thro' CD, is as CX to CD, *i. e.* as the Time A to the Time B; but the Time B is that wherein the Perpendicular is paſſed thro' from Reſt; therefore the Time A is that wherein the Plane CX is paſſed thro'. *Q. E. F.*

Prop. XXIII. Prob. IX.

A Space paſſed from Reſt, along the Perpendicular, in any Time being given, to inflect a Plane from the loweſt Term of that Space, along which, after the Fall along the Perpendicular, a Space may be paſſed in the ſame Time = any given Space, that is more than double, and leſs than triple the Space paſſed along the Perpendicular:

Along

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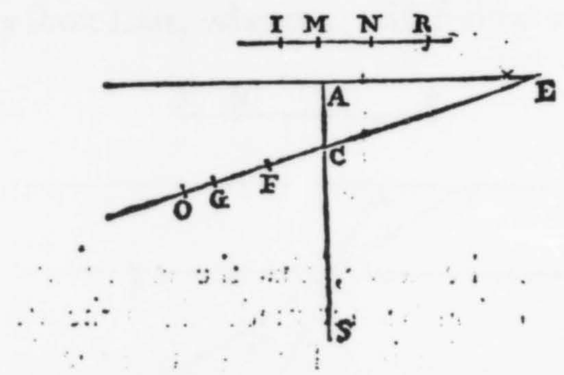
Along the Perpendicular AS, in the Time AC, let the Space AC be paſſed, *ex quiete* in A; to which let IR be more than double and leſs than triple: It is required from the Term C to inflect a Plane, along which a Moveable, after the Fall along AC, may in the ſame Time AC paſs a Space = the ſaid IR.

Let RN and NM be each equal to AC, and what Proportion the Remainder IM has to MN, the ſame let the Line AC have to a fourth Line, equal to which draw

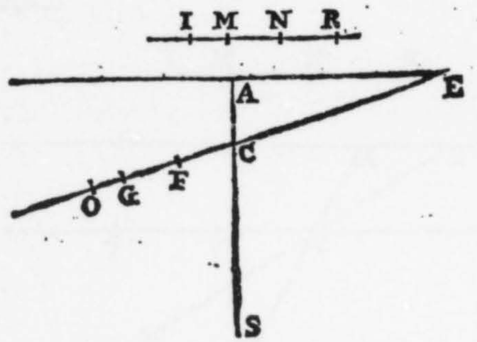


CE from C to the Horizon AE; continue CE towards O, and take CF, FG, and GO = the ſaid RN, NM, and MI; then, I ſay, that the Time along the inflected Plane CO, after the Fall AC, is = the Time AC, out of Reſt in A: For ſince it is OG : GF : : FC : CE; it will be by Compoſition OF : FG or FC : : FE : EC; and as one of the Antecedents to one of the Conſequents, ſo all the Antecedents to all the Conſequents, *i. e.* the whole OE : EF : : FC : EC; therefore OE, EF, and EC are continual Proportionals: And ſince it was ſuppoſed that the Time along AC was as AC, CE will be the Time thro' EC; and EF the Time along the whole EO; and the Remain-

S ſ

der

der CF that along the Remainder CO; but CF is = the ſaid CA; therefore what was required is done: For the Time CA is the Time of the Fall along AC, *ex quiete*, in A; and CF (which is = CA) is the Time along CO, after the Deſcent thro' AC, or after the Fall thro' AC. *Q.E.F.*



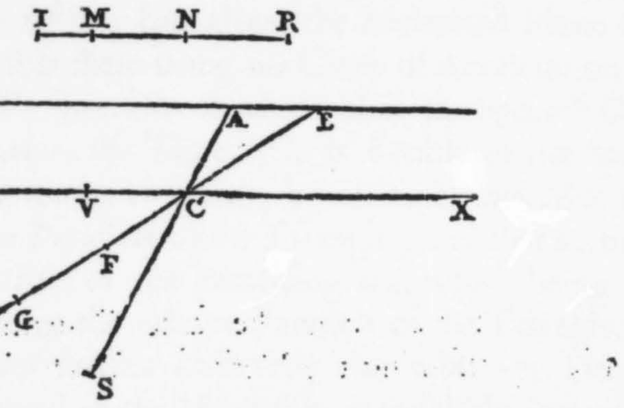
But here it muſt be noted, that the ſame will happen, if the preceding Motion be not made along the Perpendicular, but along an inclined Plane; as in the following Figure, wherein let the preceding Motion be made along the inclined Plane AS, below the Horizon AE; and the Demonſtration is the very ſame.

SCHOLIUM.

Upon due Conſideration, it will be manifeſt, that the leſs the given Line IR wants of being triple AC, the nearer the inflected Plane, along which the ſecond Motion is to be made, which ſuppoſe to be CO, approaches to the Perpendicular, along which, in a Time = AC, a Space ſhall be paſſed triple to AC.

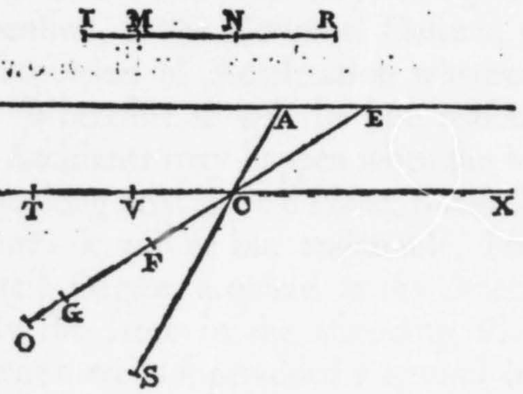
For

For in caſe IR were very near the triple of AC, IM would be very nearly = MN; and ſince CE has by Conſtruction the ſame Proportion to AC, as MN has to IM, 'tis manifeſt, that the ſaid CE will be found but very little bigger than CA, and conſequently the Point E will be very near the Point A, and the Lines CO and CS will make but a very acute Angle, they almoſt concurring in one Line. But, on the contrary, if the given Line IR were but a very little more than double the ſaid AC, IM would be but a very ſhort Line, whence it will follow alſo, that AC

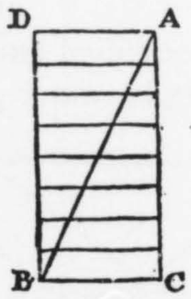


will be very ſhort in reſpect of CE, which will be very long, it approaching, in ſuch Caſe, very near to the horizontal Parallel drawn thro' C; Whence we may gather, that in the preſent Figure, if after the Deſcent thro' the inclined Plane AC, a Reflection be made along the horizontal Line, as along CT; the Space thro' which the Moveable wou'd of Conſequence be carried in a Time = the Time of the Deſcent thro' AC, would be exactly double the Space AC. And the ſame Way of Reaſoning ſeems to be applicable here: For it appears from what was but now ſaid,

ſince it is OE : EF : ; FE : EC, that FC determines the Time thro' CO. And if a Part of the horizontal Line TC, double to CA, be divided equally in V, and extended towards X, it will be infinitely produced, whilſt it ſeeks to meet the prolonged Line AE. And the *Ratio* of the infinite Line TX to the infinite Line VX, will be no other than the *Ratio* of the infinite Line VX, to the infinite Line XC.



We may come at the ſame Concluſion another Way, *viz.* by reſuming the Method of Reaſoning, us'd in the De-



monſtration of the firſt Propoſition, thus; the Triangle ABC repreſents, by the Parallels to its Baſe BC, the Degrees of Velocity continually increaſing, according to the Increments of

of the Times; from which, ſince they are infinite as the Points are infinite in the Line AC, and the Inſtants in any Time will reſult the Area of the Triangle: And if we conceive the Motion to be continued for juſt ſuch another Time, but no longer accelerately but equably, according to the greateſt Degree of Velocity, by the former Motion acquired, which Degree of Velocity is repreſented by BC; then of ſuch Degrees will be made the Aggregate or Parallelogram ADBC, which is double △ ABC. Wherefore the Space which with ſuch Degrees ſhall in the ſame Time be paſſed over, will be double the Space paſſed with the Degrees of Velocity repreſented by △ABC; but along the horizontal Plane the Motion, is equable there being no Cauſe of Acceleration or Retardation: We therefore conclude, that the Space * CT, paſſed in a Time = the Time AC, is double to the Space AC; for this is made, *ex quiete*, by an accelerate Motion, according to the Parallels of the Triangle; and that according to the Parallels of the Parallelogram, which being infinite, are double to the infinite Parallels of the Triangle.

It may be yet farther obſerved, that whatever Degree of Velocity is found in the Moveable, is indelibly impreſſed on it of its own Nature; ſince all external Cauſes of Acceleration or Retardation are removed, which happens in the horizontal Plane only: For in declining Planes, there is a Cauſe of greater Acceleration; and in the riſing ones of Retardation.

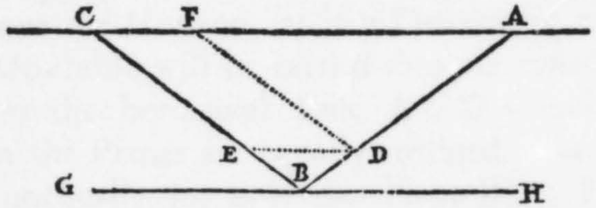
From whence in like Manner it follows, that the Motion along the horizontal Plane is perpetual: For if it be equable, it is neither weaken'd or retarded, much leſs deſtroy'd. Farther the Degree of Celerity acquired by the Moveable, by a

* Vid. Fig. ad Schol. præced.

Natural

Natural Deſcent, being of its own Nature indelible and eternal; what offers itſelf to be conſidered, is, that if after the Deſcent along a declining Plane, a Reflexion be made along another Plane that is riſing; in this latter there is an immediate Cauſe of Retardation; for in ſuch a Plane, the ſame Moveable naturally deſcends; whereupon there ariſes a certain Mixture of contrary Affections, *viz.* that Degree of Celerity acquired in the precedent Deſcent, which would of itſelf carry the Moveable uniformly, *in infinitum*, and of natural Propenſion, to the Motion of Deſcent, according to that ſame Proportion of Acceleration wherewith it always moves. Wherefore it will be but reaſonable, if, enquiring what Accidents may happen when the Moveable, after the Deſcent along any inclin'd Plane, is reflected along ſome riſing Plane; it will be but reaſonable, I ſay, if we take that greateſt Degree acquired in the Deſcent, to be kept perpetually the ſame in the aſcending Plane; but that in the Aſcent there is ſuperadded a natural Inclination downwards, *i. e.* the Motion from Reſt, accelerated according to the received Proportion. But leſt this ſhould not, without Difficulty, be underſtood, we will endeavour to explain it by a Scheme.

Let the Deſcent therefore be ſuppoſed to be made along the declining Plane AB; from which along another riſing



Plane BC, let be continued a reflex Motion; and, firſt, let the Planes be equal, and elevated at equal Angles above the

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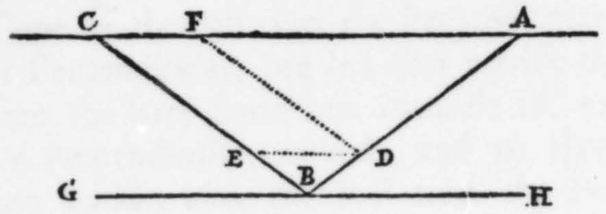
the Horizon GH. Now 'tis manifeſt, that the Moveable from Reſt in A, deſcending along AB, acquires Degrees of Velocity according to the Increaſe of its Time; and that the Degree in B is the greateſt of thoſe acquir'd, and by Nature immutably impreſſed; I mean, the Cauſes of new Acceleration or Retardation being removed: Of Acceleration, I ſay, which it would get if it ſhou'd move any farther along the Plane extended; and of Retardation, whilſt the Reflection is making along the riſing Plane BC; but along the horizontal Plane GH, the equable Motion, according to the Degree of Velocity acquired in its Paſſage from A to B, would be continual and extend *in infinitum*: And it would be ſuch a Velocity, as in a Time = the Time of Deſcent thro' AB, wou'd in the Horizon paſs over a Space double the Space AB: Now let us imagine the ſame Moveable, with the ſame Degree of Swiftneſs, to be equally moved thro' the Plane BC, ſo that alſo in this Time = the Time of the Deſcent thro' AB, it may paſs over along BC extended a Space the double of AB. But let us underſtand, that as ſoon as it begins to aſcend there naturally happens the ſame Thing to it, as happens to it in falling from A along AB, *viz.* a certain Deſcent *ex quiete* according to thoſe Degrees of Acceleration, by Virtue whereof, as it happens in AB, it may deſcend as much in the ſame Time along the reflected Plane, as it does along AB: Now it's manifeſt, that by this Mixture of an equable Motion of Aſcent, and of an accelerated one of Deſcent, the Moveable will be carried along the Plane BC (not a Space double the Space AB, as with only an equable Motion of Aſcent we ſuppoſed it to do; but the accelerated Motion of Deſcent leſſening the

Moveable's

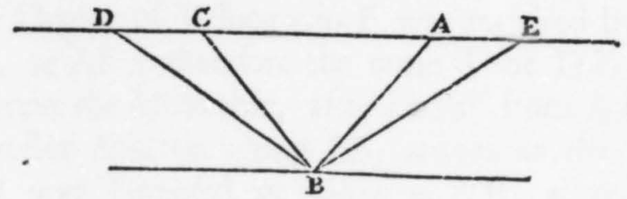
Moveable's Aſcent by one half; it will be carried, I ſay) by the equable Motion of Aſcent, or according to thoſe Degrees of Velocity which ſhall be equal, but to the Term C: And that two Points D and E being taken at Pleaſure, but equidiſtant from the Angle B, the *Tranſit* thro' DB is made in a Time = the Time of the Reflexion thro' BE, we may collect from hence. Draw DF ‖ BC; then it's



manifeſt, that the Deſcent along AD is reflected thro' DF. And if when the Moveable be got to D, it paſs along the horizontal Plane DE, the *Impetus* in E will be the ſame with the *Impetus* in D; therefore it will aſcend from E to C; and therefore the Degree of Velocity in D is = the Degree in E. *And therefore the Tranſit thro'* DB *is made in a Time = the Time of the Reflection thro'* BE.

From hence we may rationally aſſert, that if a Deſcent be made along any inclin'd Plane, after which a Reflexion follows along an elevated Plane, the Moveable, by the conceived *Impetus* will aſcend to the ſame Height or Elevation above the Horizon, as if a Deſcent be made thro' AB, the Moveable will be carried thro' the reflected Plane BC, up to the horizontal Line ACD; and this not only when the Planes are equally inclined, but alſo when they are unequally ſo, as is the Plane BD: For by the *Poſtulatum*

Poſtulatum, preceding, *the Degrees of Velocity are equal, which are acquired upon Planes unequally inclined, provided the Elevation of thoſe Planes above the Horizon be the ſame.* But if there being the ſame Inclination of the Planes EB and BD, the Deſcent thro' EB be ſufficient to force the Moveable along the Plane BD to D; ſeeing this Impulſe is made by the *Impetus* of Velocity conceived in the Point B: And if the *Impetus* be the ſame in B, whether the Moveable deſcend along AB, or along EB, 'tis manifeſt, that the Moveable ſhall be in the ſame Manner

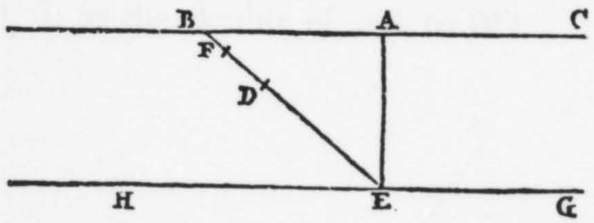


driven along BD, after its Deſcent along AB, and after that along EB; but it will happen that the Time of the Aſcent thro' BD will be longer than that thro' BC, as the Deſcent thro' EB is made in a longer Time than that thro' AB; but the Proportion of thoſe Times has been before demonſtrated to be the ſame with that of the Lengths of the Planes. Now it follows, that we ſeek the Proportion of the Spaces paſſed over in equal Times, along Planes whoſe Inclinations are different, but whoſe Elevations are ſtill the ſame, *i. e.* which are comprehended between the ſame horizontal Parallels, and this will be found by what follows.

Prop. XXIV. Theor. XV.

There being given between the ſame horizontal Parallels, a Perpendicular, and a Plane elevated from its loweſt Term, the Space that a Moveable, after the Fall along the Perpendicular paſſeth along the elevated Plane in a Time = the Time of the Fall thro' the Perpendicular, is greater than that Perpendicular, but leſs than double the ſame.

Between the ſame horizontal Parallels BC and HG, let there be a Perpendicular, as AE, and an elevated Plane EB, along which, after the Fall thro' the Perpendicular AE; from the Term E, let there be a Reflexion made



towards B: I ſay, the Space thro' which the Moveable aſcends, in a Time = the Time of the Deſcent AE, is greater than AE, but leſs than double the ſame AE.

Make ED = AE, and as EB is to BD, ſo let DB be to BF.

Firſt it ſhall be ſhewn, that the Point F is the Place to which the Moveable, with a reflex Motion along EB, ſhall arrive in a Time = the Time AE; and then that EF is greater than AE, but leſs than double the ſame AE.

Firſt,

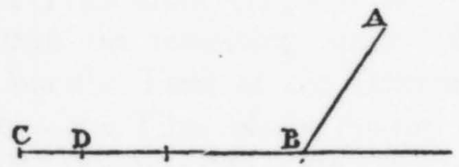
Firſt, then, If we ſuppoſe the Time of the Deſcent thro' AE to be as AE; the Time of the Deſcent thro' BE, or of the Aſcent thro' EB, will be as the Line BE; and ſince DB is a mean Proportional between EB and BF; and ſince BE is the Time of the Deſcent thro' the whole BE, BD will be the Time of the Deſcent thro' BF, and the remaining Part DE will be the Time of the Deſcent thro' the remaining Part FE; but the Time thro' FE *ex quiete* in B, and the Time of the Aſcent thro' EF is the ſame, ſince the Degree of Velocity in E was acquired by the Deſcent BE or AE; therefore the ſame Time DE ſhall be that wherein the Moveable, after its Fall from A thro' AE, with a reflex Motion along EB, arrives at the Point F: But ED was ſuppoſed or taken = AE, therefore, &c. Which was firſt to be proved.

Secondly, Becauſe as the whole Line EB is to the whole Line BD, ſo is the Part taken away DB to the Part taken away BF; it will be as the whole EB to the whole BD, ſo the Remainder ED to DF; but EB is greater than BD, therefore alſo ED is greater than DF, and EF leſs than double DE or AE. *Q. E. D.* And the ſame Thing will happen, if the precedent Motion be not made along the Perpendicular, but along an inclin'd Plane; and the Demonſtration is the ſame, provided that the reflex Plane be leſs riſing, *i. e.* longer than the declining one.

PROP. XXV. THEOR. XVI.

If after the Deſcent along any inclin'd Plane, a Motion follow along the Plane of the Horizon, the Time of the Deſcent along the inclin'd Plane will be to the Time of the Motion along any horizontal Line, as the double Length of the inclin'd Plane to the horizontal Line taken.

Let the horizontal Line be CB, the inclin'd Plane AB, and after the Fall thro' AB, let follow a Motion along the Horizon, in which take any Space BD; I ſay, the Time of the Fall thro' AB, to the Time of the Motion thro' BD, is as the double of AB to BD.



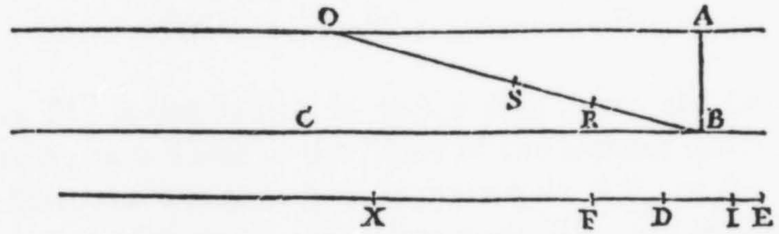
For, make BC double AB, then 'tis plain from what has been before demonſtrated, that the Time of the Fall thro' AB is = the Time of the Motion thro' BC; but the Time of the Motion thro' BC, to the Time of the Motion thro' BD, is as the Line BC to the Line BD; therefore the Time of the Motion thro' AB, to the Time thro' BD, is as the double of AB to BD. *Q. E. D.*

PROP. XXVI.

PROP. XXVI. PROB. X.

A Perpendicular between two horizontal Lines being given, as alſo a Space greater than the Perpendicular, but leſs than double the ſame, to elevate a Plane between the ſaid Parallels, from the loweſt Term of the Perpendicular, along which the Moveable may with a reflex Motion, after its Fall thro' the Perpendicular, paſs a Space = the Space given, and in a Time = the Time of the Deſcent along the Perpendicular.

Let AB be a Perpendicular between the horizontal Parallels AO and BC; and let FE be greater than BA, but

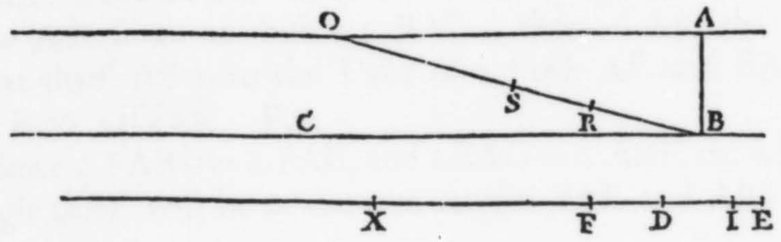


leſs than double the ſame. It is required from the Point B to elevate a Plane between the ſaid Parallels, along which the Moveable, after its Fall from A to B, may, with a reflex Motion, in a Time = the Time of the Fall thro' AB, aſcend a Space = the ſaid FE.

Suppoſe ED = AB, then the remaining Part DF is leſs, ſince the whole Line EF is leſs than the double of AB: Let DI be = DF, and as EI to ID, ſo make DF to a fourth FX; and from B draw the reflex Line BO = EX; the , I ſay, that a Plane along BO, is that along which, after

after the Fall AB, a Moveable in a Time = the Time of the Fall thro' AB paſſes, by aſcending a Space = the given Space EF. For,

Suppoſe BR and RS to be = ED and DF; then becauſe it is EI : ID : : DF : FX, it will be by Compoſition ED : DI : : DX : XF; *i.e.* ED : DF : : DX : XF, and : : EX : XD; *i.e.* BO : OR : : RO : OS; but now if we ſuppoſe the Time along AB to be AB, the Time along OB will be



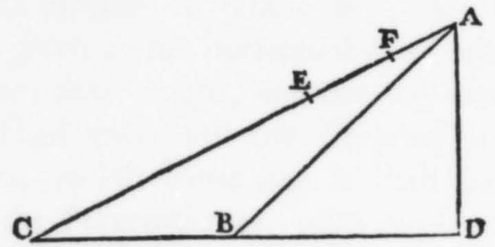
OB, and RO the Time along OS; and the remaining Part BR the Time thro' the remaining Space SB, deſcending from O to B; but the Time of the Deſcent thro' SB, *ex quiete* in O, is = the Time of the Aſcent from B to S, after the Fall AB; therefore BO is the Plane raiſed from B, along which, after the Fall through AB in the Time BR or BA, is paſſed the Space BS = the given Space EF. *Q. E. F.*

PROP. XXVII. THEOR. XVII.

If a Moveable deſcend along unequal Planes whoſe Elevation is the ſame, the Space that's paſſed over in the loweſt Part of the longer, in a Time = that wherein the whole ſhorter Plane is paſſed, is = the Space that is compoſed of the

the ſaid ſhorter Plane, and of the Part to which the ſaid ſhorter Plane hath the ſame Proportion that the longer Plane hath to the Exceſs, by which the longer Plane exceeds the ſhorter.

Let AC be the longer Plane, and AB the ſhorter, whoſe Elevations are the ſame; from the loweſt Part of AC (viz. from C) ſet off CE = AB; then make EF of the ſame Proportion to CE, as AE (*i.e.* as the Exceſs of the Plane CA above AB) has to the whole Line AC: Then, I ſay, the



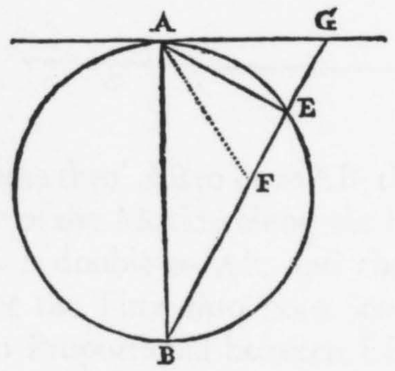
Space FC is that which is paſſed thro' after the Deſcent from A, in a Time = the Time of the Deſcent thro' AB: For ſince the whole CA is to the whole AE, as the Part taken away CE to the Part taken away EF; the remaining Part EA will be to the remaining Part AF, as the whole CA to the whole AE; therefore theſe three CA, AE, and AF, are continual Proportionals: but if the Time thro' AB be ſuppoſed to be as AB, the Time thro' AC ſhall be as AC; and the Time thro' AF ſhall be as AE, and that thro' the remaining Part FC ſhall be as EC: but EC is = the ſaid AB, therefore, &c. *Q. E. D.*

PROP. XXVIII.

Prop. XXVIII. Theor. XVIII.

Let the horizontal Line AG be a Tangent to a Circle, and from the Point of Contact draw the Diameter AB; draw alſo the two Chords AE and EB at Pleaſure, and let the Proportion of the Time of the Fall thro' AB, to the Time of the Deſcent through both AE and EB be required: Produce BE 'till it meets the Tangent Line in G, draw AF ſo as to biſect ∠ BAE; then, I ſay, that the Time thro' AB is to the Time thro' both AE and EB, as AE is to $\overline{AE \times EF}$. For,

Since ∠ FAB is = ∠ FAE, and ∠EAG = ∠ ABF, the whole Angle GAF will be = the two Angles FAB and ABF; to



which Angles, ∠ GFA is alſo equal; therefore the Line GF is = GA; and becauſe ▭ BGE is = □ of GA, it will be alſo = □ of GF, and the three Lines BG, GF, GE Proportionals: but now if we ſuppoſe AE to be the Time thro' AE, GE will be the Time thro' GE, and GF the Time thro' the whole GB, and EF the Time thro' EB, after the Deſcent from

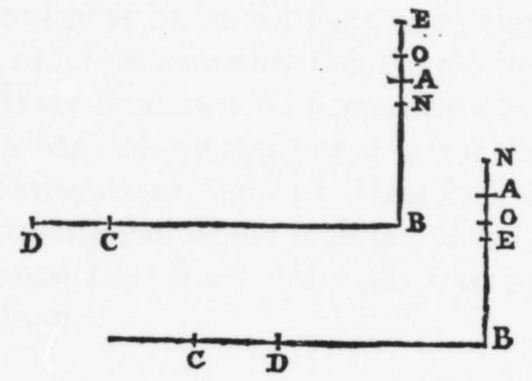
G, or from A thro' AE; therefore the Time thro' AE, or through AB, to the Time through $\overline{AE \times EB}$, is as AE to $\overline{AE \times EF}$. *Q. E. D.*

Shorter thus; Make GF = GA; then 'tis manifeſt that GF is a mean Proportional between BG and GE; the reſt as above.

Prop. XXIX. Prob. XI.

Any horizontal Space being given, from whoſe End a Perpendicular is erected, in which let a Part be taken = half the Space given in the horizontal one; then a Moveable falling from that Height, and turning along the horizontal Space, ſhall paſs thro' the Perpendicular and the horizontal Space, in leſs Time than it ſhall paſs thro' any other Part of the Perpendicular, with the ſame horizontal Space.

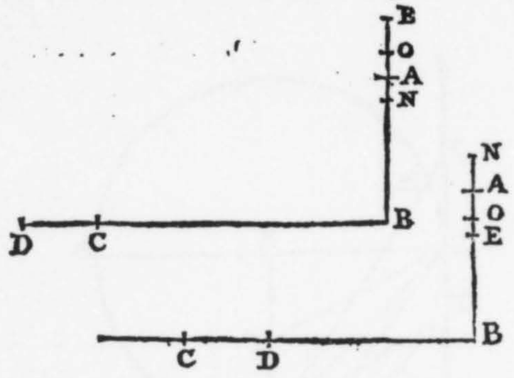
Let there be an horizontal Plane, in which let any



Space as BC be given; from the Term B let there be a Perpendicular, wherein let BA be half BC; then, I ſay,

 that

that the Time in which a Moveable, let fall from A, paſſes both the Spaces AB and BC, is the ſhorteſt of all the Times wherein the ſame Space BC, with a Part of the Perpendicular, whether greater or leſs than the Part AB can be paſſed. Let a greater be taken, as EB, in Fig. 1. or a leſſer as EB in Fig. 2; then we muſt prove, that the Time in which the Spaces EB and BC are paſſed, is longer than the Time in which AB and BC are paſſed.



Suppoſe the Time thro' AB to be as AB, then the ſame AB ſhall be the Time of the Motion along the horizontal Space BC; becauſe BC is double to AB, and therefore the double of BA ſhall be the Time thro' both Spaces AB and BC. Make BO a mean Proportional between EB and BA; then will BO be the Time of the Fall thro' EB. Again, let the horizontal Space BD be double the ſaid EB; then it's manifeſt, that the Time of it after the Fall EB is the ſame BO; for as DB is to BC, or as EB to BA, ſo make OB to BN; and ſince the Motion along the horizontal Plane is equable, and ſince OB is the Time thro' BD, after the Fall from E, therefore NB will be the Time thro' BC, after

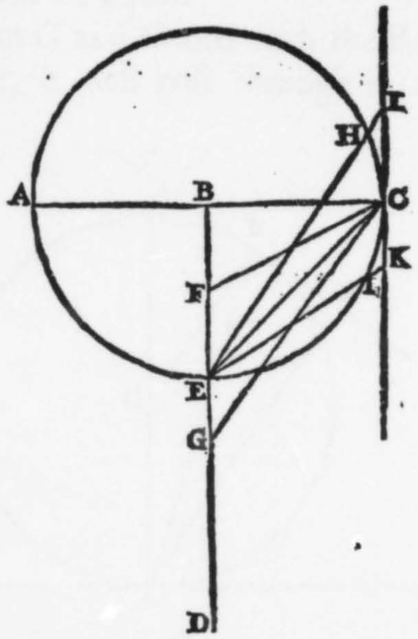
after the Fall from the ſame Height E: Whence 'tis manifeſt, that OA, together with BN, are the Time along $\overline{EB \times BC}$. And ſince the double of BA is alſo the Time thro' $\overline{AB \times BC}$; it remains to be proved, that OB, together with BN, are greater than the double of BA. And now becauſe OB is a mean Proportional between EB and BA, the *Ratio* of EB to BA is double that of OB to BA: And ſince EB is to BA, as OB is to BN, therefore the *Ratio* of OB to BN, ſhall alſo be double that of OB to BA. But now the *Ratio* of OB to BN is compounded of the *Ratios* of OB to BA, and of AB to BN; therefore the *Ratio* of AB to BN is the ſame with that of OB to BA; therefore BO, BA, and BN, are three continual Proportionals, and OB, together with BN, greater than the double of BA. Whence the Propoſition is manifeſt.

PROP. XXX. THEOR. XIX.

If a Perpendicular be let fall from any Point of the horizontal Line, and if from another Point in the ſame horizontal Line, a Plane be drawn 'till it meet the Perpendicular; thro' which a Moveable may deſcend in the ſhorteſt Time to the ſaid Perpendicular; ſuch a Plane ſhall cut off from the Perpendicular a Part = the Diſtance of the Point taken in the horizontal Line from the Point the Perpendicular was let fall from.

Let the Perpendicular BD be let fall from the Point B of the horizontal Line AC, in which let there be any Point C, taken at Pleaſure; and ſuppoſe the Diſtance BE, in the Perpendicular, to be = the Diſtance BC, and draw CE; then, I ſay, that of all Planes that can be drawn from the Point C to the Perpendicular CE, is that along which, in the ſhorteſt of all Times, a Deſcent is made to the Perpendicular. For,

Let the Planes CF and CG be inclined, the one above, the other below the Plane CE, and draw IK touching the



Circle, whoſe Radius is BC, in the Point C; this Tangent IK will be ‖ to the Perpendicular BD; draw EK ‖ FC, 'till it meet the Tangent in K, and cutting the Circumference of the Circle in L; then 'tis manifeſt, that the Time

Time of the Fall thro' LE is = the Time of the Fall thro' CE; but the Time thro' KE is longer than thro' LE, therefore the Time thro' KE is longer than thro' CE, but the Time thro' KE is = the Time thro' CF (KE and CF being equal, and equally inclined): In like manner, ſince CG and IE are equal and equally inclined, the Times of the Motions through them will be equal; but the Time thro' HE ſhorter than IE, is ſhorter than the Time thro' IE; therefore alſo the Time thro' CE (which is = the Time thro' HE) will be ſhorter than the Time thro' IE; therefore the Propoſition is manifeſt.

PROP. XXXI. THEOR. XX.

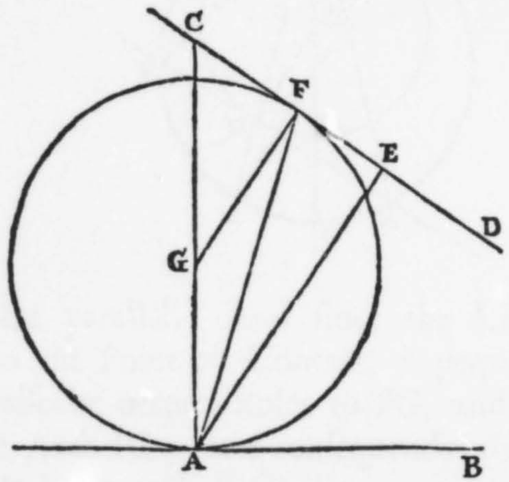
If a right Line be at Pleaſure inclin'd upon the horizontal Line; the Plane produced from a given Point in the horizontal Line, 'till it meet the inclin'd Plane along which the Deſcent is made in the ſhorteſt of all Times; ſuch a Plane, I ſay, is that which biſects ∠ made by two Lines drawn from the given Point, the one perpendicular to the horizontal Line, the other to the inclin'd Line.

Let

Let CD be a Line inclin'd at Pleaſure to the horizontal Line AB; and in the Line AB, let any Point as A be given, from which erect AC perpendicular to AB, and AE perpendicular to CD, and biſect the Angle CAE by the Line FA; then, I ſay, that of all Planes inclined to the Plane CD, out of any Point of CD, to the Point A, the Plane extended thro' FA, is that along which the Deſcent is made in the ſhorteſt of all Times.

Draw FG ‖ AE, then the alternate Angles GFA and FAE will be equal; but ∠ EAF is = ∠ FAG, therefore FG and GA will be equal.

If therefore on G as a Center with the Radius GA, you deſcribe a Circle, it will paſs through F, and will touch



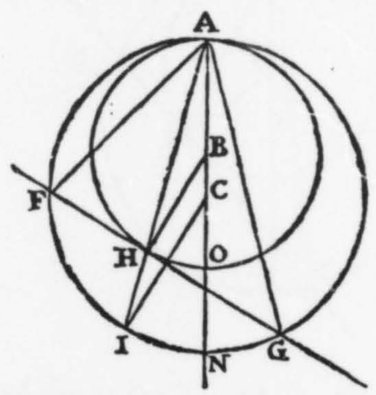
the horizontal Line AB in A, and the inclined Line CD in F; For ∠ GFC is a Right one, the Lines GF and AE being parallel to each other: Whence it's manifeſt, that all the Lines drawn from the Point A, to the inclined Plane CD, are extended beyond the Circumference, and conſe-

conſequently that the Motion thro' them muſt take up longer Time than that thro' FA requires: Wherefore, &c. *Q. E. D.*

LEMMA.

If two Circles touch one another within, the innermoſt of which ſome right Line toucheth, and the outermoſt of which the ſame right Line cuts; three Lines drawn from the Point of Contact of the two Circles, to three Points of the Tangent right Line, that is one to the Point of its Contact with the interior Circle, and the other two to the two Points where it cuts the exterior Circle, ſhall contain equal Angles, at the Point of Contact of the two Circles.

Let two Circles touch one another inwardly in the Point A; let B be the Center of the leſſer, and C that of the

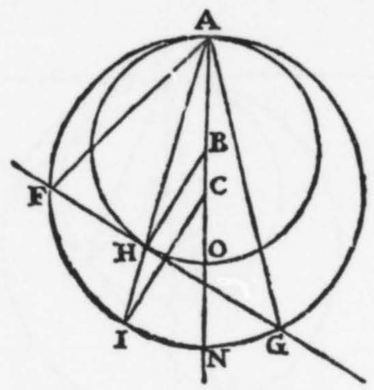


greater: And let any right Line, as FG touch the inner Circle in the Point H, and cut the outermoſt in the Points F and G; and draw the three Lines AF, AH, and AG; then, I ſay,

ſay, that the Angles by them contained, *viz.* FAH and GAH are equal.

Produce AH to the Circumference in I, and from the Centers draw BH and CI, and thro' the ſaid Centers let BC be drawn, which continued, will meet with the Point of Contact in A, and with the Circumferences of the Circles in O and N.

Then becauſe ∠'s ICN and HBO are equal, each of them being the double of the Angle IAN, the Lines BH



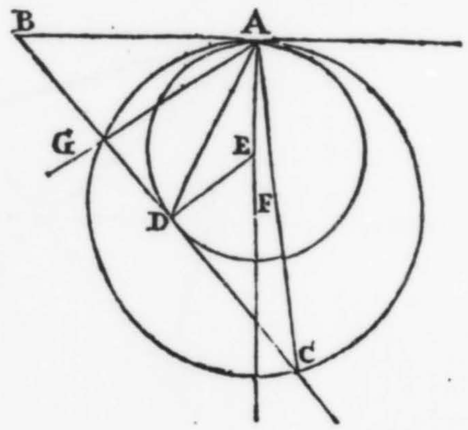
and CI are parallel. And ſince the Line BH from the Centre to the Point of Contact, is perpendicular to FG, CI will alſo be perpendicular to FG, and the Arch FI will be = the Arch IG; and conſequently the Angle FAI to the Angle IAG. *Q.E.D.*

PROP. XXXII.

PROP. XXXII. THEOR. XXI.

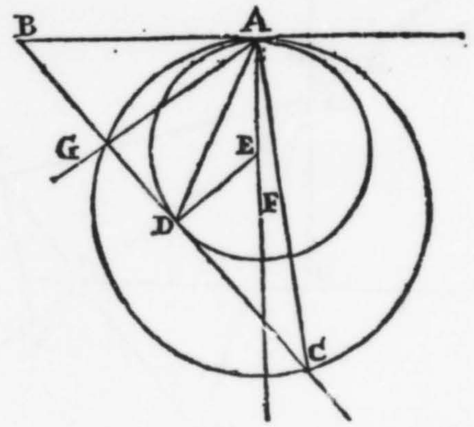
If two Points be taken in the Horizon, and any Line be inclined from one of them towards the other, out of which a right Line is drawn unto the inclin'd Line, cutting off a Part thereof = that which is included between the Points of the Horizon; the Deſcent thro' this Line ſhall be ſooner performed than along any other right Line drawn from the ſame Point to the ſaid inclin'd Line. And along other Lines which are at equal Angles on each Side this, the Deſcents are made in equal Times.

In the Horizon let there be two Points A and B, and from B incline the right Line BC; in which from the End B, take



BD = BA, and draw the Line AD; then, I ſay, that the Deſcent along AD, is more ſwiftly made than along any other Line whatſoever, drawn from the Point A to the inclined Line BC: For from the Points A and D, unto BA and BD, raiſe the Perpendiculars AE and DE, interſecting each other

in E: And becauſe in the equicrural Triangle ABD, ∠'s BAD and BDA are equal, their Complements to right Angles, *viz.* ∠'s DAE and EDA are equal; therefore if on the Center E, with the Radius EA, a Circle be deſcribed, it will paſs alſo thro' D, and will touch the Lines BA and BD, in the Points A and D: And ſince A is the End or Term of the Perpendicular AE, the Deſcent thro' AD ſhall be ſooner performed than along any other, drawn from the ſame Term A to the Line BC, beyond the Circumference of the Circle; which was firſt to be proved.



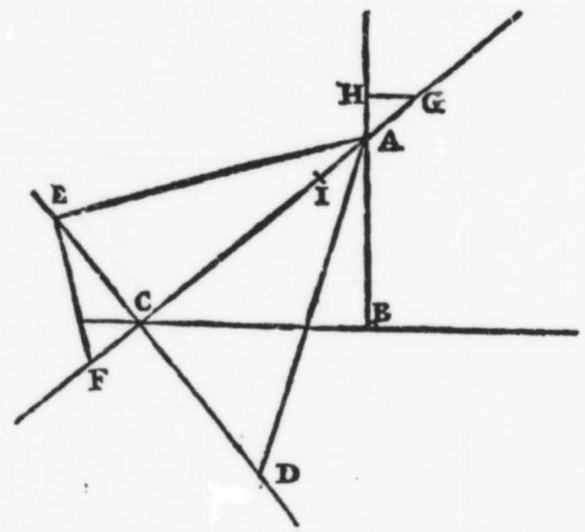
But if the Perpendicular AE being prolonged, any Center be taken in it, as F, and with the Radius FA a Circle be deſcribed, cutting the Tangent Line in G and C; drawing AG and AC, they ſhall make equal Angles with the middle Line AD, by what has been before demonſtrated, and thro' them the Motions ſhall be performed in equal Times, ſince they are terminated by A the Point above, and the Circumference of the Circle AGO.

PROP. XXXIII.

PROP. XXXIII. PROB. XII.

A Perpendicular and Plane inclin'd to it, being both of the ſame Height, and both having the ſame higheſt Term, to find a Point in the Perpendicular above the common Term, from which, if a Moveable be let go, that ſhall afterwards turn along the inclined Plane it may paſs thro' that inclined Plane in the ſame Time in which it would paſs *ex quiete* thro' the Perpendicular.

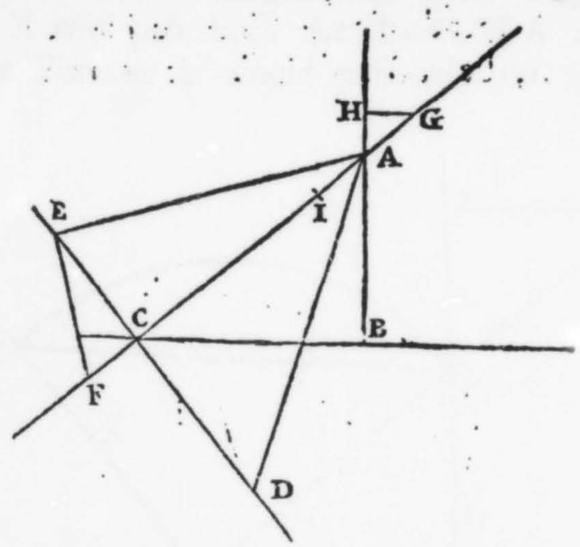
Let the Perpendicular and inclin'd Plane, whoſe Heights are the ſame, be AB and AC. It is required to find a



Point in the Perpendicular BA, produced from the Term A, from which a Moveable deſcending, may paſs the Space AC in the ſame Time wherein it would paſs the ſaid Perpendicular AB *ex quiete* in A. Draw DCE at right

right Angles to AC, and make CD = AB, and join AD; then ∠ ADC will be greater than ∠CAD (for CA is greater than AB or CD): Make ∠ DAE = ∠ ADE; make EF perpendicular to AE, and produce it till it meets the inclin'd Plane produced in F; and let AI and AG be each made = CF, and thro' G draw GH parallel to the Horizon; then I say, that H is the Point ſought. For,

Suppoſing the Time of the Fall along the Perpendicular AB to be AB, the Time of the Deſcent along AC *ex quiete* in A, will be AC; and ſince in the right-angled

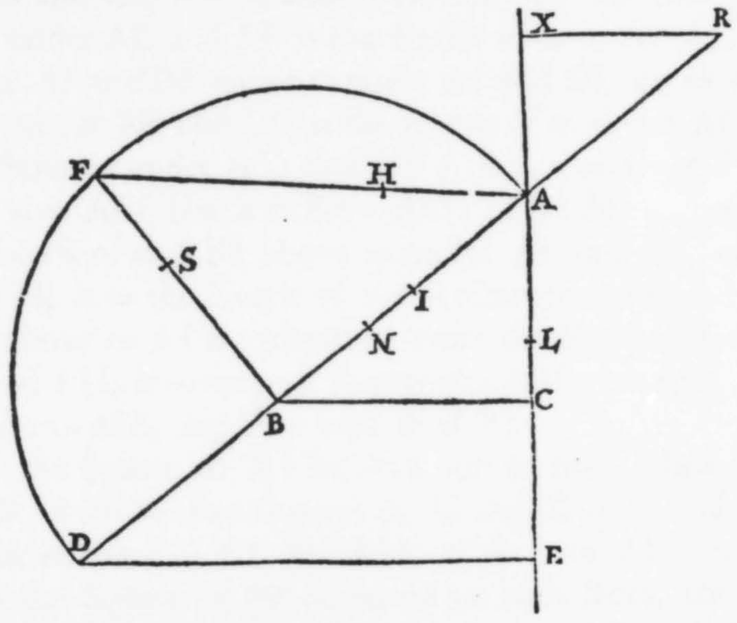


Triangle AEF, from the right Angle E, unto the Baſe AF, EC is perpendicular, AE will be a mean Proportional between FA and AC, and CE a mean between AC and CF, *i. e.* between CA and AI; and ſince the Time of AC out of A, is AC, AE will be the Time of the whole AF, and AC the Time of AI; and becauſe in the equicrural Triangle AED, the Side AE is = the Side ED, ED will be the Time thro' AF, and EC that thro' AI; therefore

fore CD, *i. e.* AB will be the Time thro' IF *ex quiete* in A, which is the ſame as if we ſay AB is the Time thro' AC out of G, or out of H. *Q. E. F.*

PROP. XXXIV. PROB. XIII.

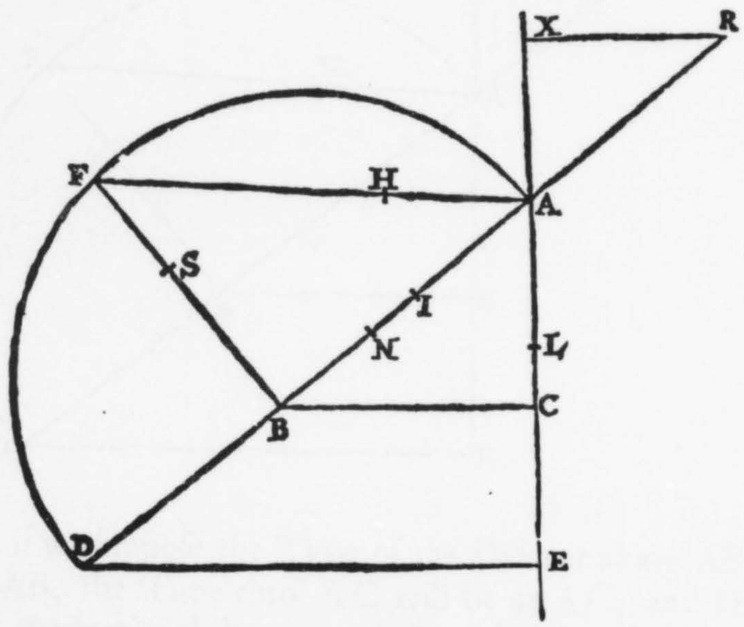
An inclin'd Plane, and a Perpendicular being given, which have the ſame Term above, to find a higher Point in the Perpendicular produced, from which a Moveable deſcending and turning along the inclin'd Plane, may paſs them both in the ſame Time as it wou'd paſs through the inclined Plane only, were it let go *ex quiete* from its higheſt Term.



Let the inclin'd Plane and Perpendicular be AB and AC, whoſe higheſt Term A is the ſame. It is required in the Perpendicular produced from A, to find a Point above,

above, from which a Moveable deſcending, and turning along the Plane AB, may paſs the aſſigned Part of the Perpendicular, together with the Plane AB, in the ſame Time as it would the ſole Plane AB out of Reſt in A.

Let BC be the horizontal Line, and make AN = AC; and make as AB to BN, ſo AL to LC: And make AI = AL, and let CE ſet off in the Perpendicular AC produced, be a third Proportional to AC and BI; then I ſay, CE is the Space ſought; ſo that the Perpendicular being extended above A, and taking the Part AX = CE, a Moveable from X will paſs both the Spaces XA and AB, in the ſame Time as it would paſs only the Space AB from A.

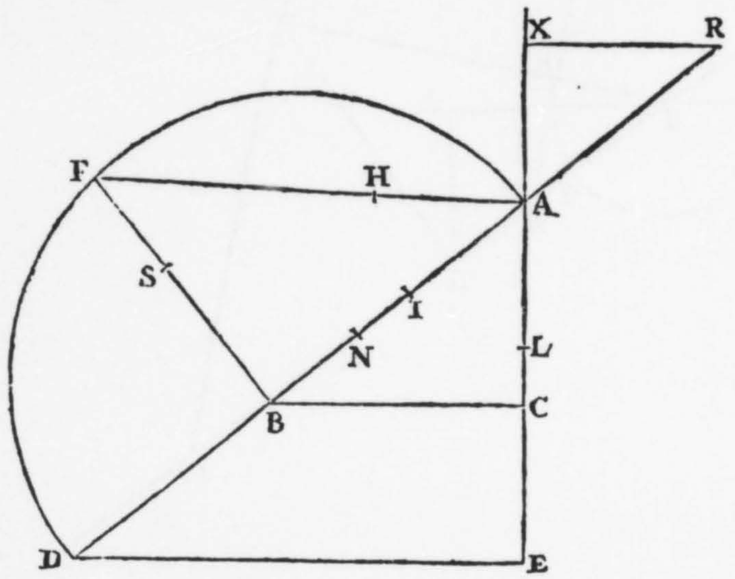


Draw the Line XR ‖ the horizontal Line BC, which let BA extended meet in R, produce AB to D, and draw ED

ED ‖ CB, and upon AD as a Diameter, deſcribe a Semicircle AFD, and from the Center B erect a Perpendicular to DA as BF, and produce it 'till it meet the Circumference in F; then 'tis manifeſt, that FB is a mean Proportional between AB and BD, and that a Line drawn from F to A will be a mean Proportional between DA and AB. Suppoſe BS = BI, and FH = FB; then becauſe it is AB : BD : : AC : CE; and becauſe BF is a mean Proportional between AB and BD; and BI a Mean between AC and CE; it will be BA : AC : : FB : BS; and becauſe it is BA : AC or AN : : FB : BS; it will be by Converſion BF : FS : : AB : BN, *i.e.* : : AL : LC; therefore ▭ under FB and CL is = ▭ under AL and SF; but this Rectangle under AL and SF is the Exceſs of ▭ under AL and FB, or AI and BF above ▭ under AI and SB, or ▭ AIB; but ▭ under FB and LC is the Exceſs of ▭ under AC and BF above ▭ under AL and BF; but ▭ under AC and BF is = ▭ ABI (for it is BA : AC : : FB : BI); therefore the Exceſs of ▭ ABI above ▭ under AI and BF, or AI and FH, is = the Exceſs of the Rectangle under AI and FH, above ▭ AIB; therefore twice the Rectangle under AI and FH, is = the two Rectangles ABI and AIB, *viz.* to twice ▭ AIB, together with □ of BI.

Let the Square of AI be common to both, then twice ▭ AIB, with the two Squares of AI and IB (*i.e.* □ of AB) will be = twice ▭ AI and FH, with □ of AI: Again, taking the Square of BF common on both Sides, the two Squares of AB and BF, that is the Square AF, will be = twice the Rectangle under AI and FH, with the two Squares of AI and FB, *i.e.* of AI and FH; but the ſame Square of AF is = twice ▭ AHF, with the two Squares of AH

AH and HF; therefore twice ▭ under AI and FH, together with □'s of AI and FH, is = twice ▭ AHF, together with □'s of AH and HF, and taking away the common Square of HF, twice ▭ under AI and FH, together with □ of AI, will be = twice ▭ AHF, together with □ of AH; and ſince FH is a common Side of all the Rectangles, the Line AH will be = the Line AI: For were it greater or leſs, ▭'s alſo FHA, and □ of HA, would be greater or leſs than ▭'s under FH and IA, and the Square of IA, contrary to what has been demonſtrated.



Now, if we ſuppoſe the Time of the Deſcent along AB, to be as AB, the Time thro' AC will be as AC, and IB the mean Proportional between AC and CE will be the Time thro' CE, or thro' XA, out of Reſt in X; and ſince AF is a mean Proportional between DA and AB, or between

between RB and BA; and ſince BF = FH, is a Mean between AB and BD, *i. e.* RA and AB, therefore from what has been before * proved, the Exceſs AH ſhall be the Time along AB out of Reſt in R, or after the Fall out of X: Since the Time along the ſaid AB, out of Reſt in A, ſhall be AB; therefore the Time along XA is IB, and along AB after RA, or after XA is AI; Therefore the Time along XAB ſhall be as AB, namely, the ſelf-ſame with the Time along AB, only out of Reſt in A, which was the Propoſition.

PROP. XXXV. PROB. XIV.

A Line inflected to a given Perpendicular, being given, to find a Part in the inflected Line, along which alone *ex quiete* a Motion may be made, in the ſame Time as it would be along the ſame with the Perpendicular.

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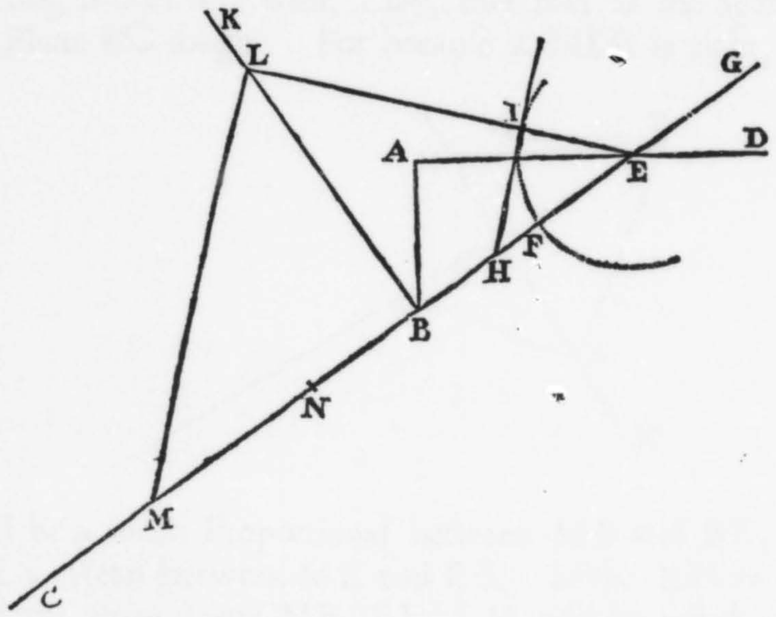
* COR. PROP. XIX.

Let the Perpendicular be AB, and the Line inflected to it BC. It is required in BC to determine a Part along which alone, out of Reſt, a Motion may be made in the ſame Time, as it would along the ſame, together with the Perpendicular AB.

Draw the horizontal Line AD, which let the inclin'd Line CB produced, meet in E; and ſuppoſe BF = BA; and on the Center E, with the Radius EF, deſcribe the Circle

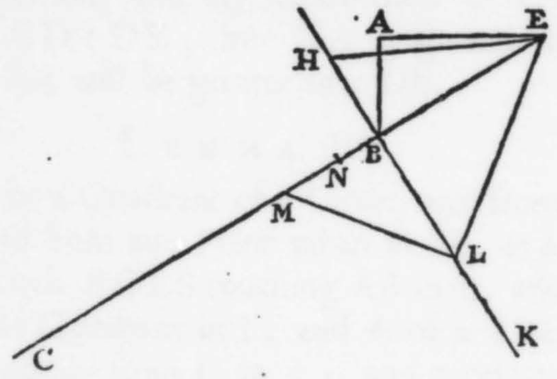


FIG, and produce FE to the Circumference in G; and cut the Line BF in H, ſo that as BG is to BF, ſo may BH be to HF; and let HI touch the Circle in I: then to BC from B erect the Perpendicular BK, which let the Line EIL meet in L; laſtly, to EL from L, raiſe the Perpendicular LM, meeting BC in M; then, I ſay, that along the

Line BM, from Reſt in B, a Motion is made in the ſame Time as it would be out of Reſt in A, along both AB and BM. Make EN = EL: And ſince it is GB : BF :: BH : HF : it will be by Permutation GB : BH :: BF : FH; and by Diviſion, GH : HB :: BH : HF. Wherefore ▭ GHF will be = HB *q*: But the ſame Rectangle is alſo = HI *q*; therefore BH is = HI: And ſince in the four-ſided Figure ILBH, the Sides HB and HI are equal, and ∠'s B and I right, the Side BL will alſo be = LI; but EI is = EF, therefore the whole Line LE or NE is = the two Lines LB and EF; take away the common Line EF, and the Remainder FN will be = LB; but FB was ſuppoſed = BA, therefore LB is = the two Lines AB and BN. Again, if you take the Time along AB to be AB, then the Time along EB will be = EB; but the Time along the whole Line EM will be EN, *viz.* a Mean between ME and EB; wherefore the Time of the Deſcent of the remaining Part BM, after it has fallen thro' EB or AB, will be the ſaid BN: But it has been ſuppoſed, that the Time along AB is AB; therefore the Time of the Fall along both AB and BM, is ABN. And becauſe the Time along EB, out of Reſt in E, is EB; the Time along BM, out of Reſt in B, will be a mean Proportional between BE and BM; but that is BL; therefore the Time along both AB and BM, out of Reſt in A, is ABN; but the Time along only BM, out of Reſt in B, is BL; but BL has been proved to be = AB and BN; therefore the Propoſition is manifeſt.

More expeditiouſly thus;

Let BC be the inclin'd Plane, and BA the Perpendicular (as before); produce CB till it meet the horizontal Line AE in E; upon EC, and from the Point B, erect a Perpendicular, which extend both Ways; then take BH = the Exceſs of BE above BA; make ∠ HEL = ∠ BHE; and let EL produced meet BK in L; and unto EL, and from the Point L erect the Perpendicular LM, meeting BC in M; then, I ſay, that BM is the Space in the Plane BC ſought. For becauſe ∠ MLE is right, BL

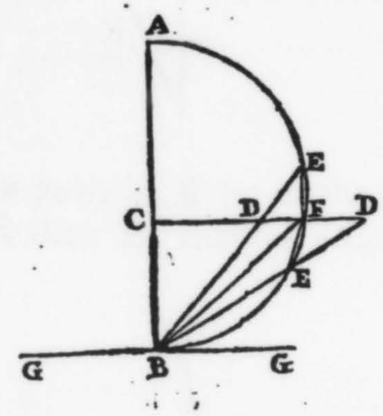


will be a mean Proportional between MB and BE, and LE a Mean between ME and EB. Make EN = LE, and the three Lines NE, EL, LH will be equal; and HB will be the Exceſs of NE above BL: but the ſame HB is alſo the Exceſs of NE above NB and BA; therefore the two Lines NB and BA are = BL: but now if we ſuppoſe EB to be the Time along EB, BL will be the Time along BM, out of Reſt in B; and BN will be the Time of the ſame after EB, or after AB; and AB will be the

the Time along AB, therefore the Times along ABM, *viz.* ABN are = the Time thro' only BM, out of Reſt in B. *Q. E. F.*

LEMMA I.

Let FC be perpendicular to the Diameter BA, and from the Term B, draw BED at Pleaſure, and join FB; then, I ſay, that FB is a mean Proportional between DB and BE. Draw a Line from E to F; and thro' B draw the Tan-



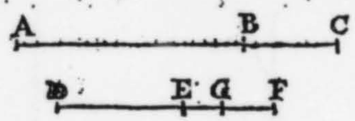
gent BG, which will be ‖ CD: Wherefore ∠ DBG will be = ∠ FDB; but ∠ EFB, in the alternate Segment, will alſo be = the ſame DBG; therefore △'s FBD and FEB are ſimilar; and therefore it will be BD : BF :: BF : BE; Therefore, *&c. Q. E. D.*

LEMMA II.

LEMMA II.

Let the Line AC be greater than DF; and let AB have a greater Proportion to BC, than DE has to EF; then, I ſay, that AB is greater than DE.

For, becauſe the *Ratio* of AB to BC is greater than that of DE to EF; therefore what *Ratio* AB has to BC, the

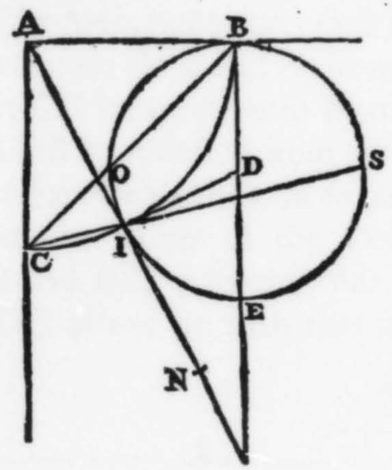


ſame ſhall DE have to a Line leſſer than EF; let it have it to EG: And becauſe it is AB : BC : : DE : EG, it will be by Compoſition, and by Converſion of Proportion, CA : AB : : GD : DE; but CA is greater than GD, therefore alſo BA will be greater than DE.

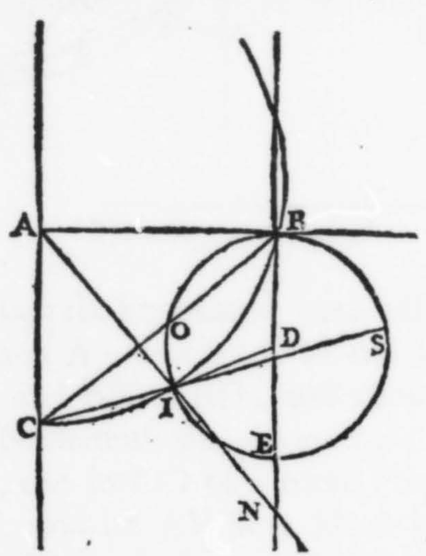
LEMMA III.

Let ACIB be a Quadrant of a Circle, and from B draw BE ‖ AC, and from any Point taken in BE, as a Center, deſcribe the Circle BOES touching AB in B, and cutting the Arch of the Quadrant in I; and draw a Line from C to B; and another from C to I; and produce it to S; then, I ſay, that the Line CI is always leſs than CO. Draw a Line from A to I, which ſhall touch the Circle BOE; for if DI be drawn it will be = DB; but ſince DB touches the Quadrant, it ſhall alſo touch DI, and be perpendicular to the Diameter AI: Wherefore alſo AI ſhall touch the Circle BOE in I: And becauſe ∠ AIC is greater than ∠ ABC, as it ſtands upon a bigger Arch, therefore alſo ∠ SIN will be greater than the ſaid ABC; wherefore the Arch IES is greater than the Arch BO; and the Line CS,

CS, nearer the Center, greater than CB: Wherefore alſo CO is greater than CI, ſince 'tis SC : CB : : OC : CI.



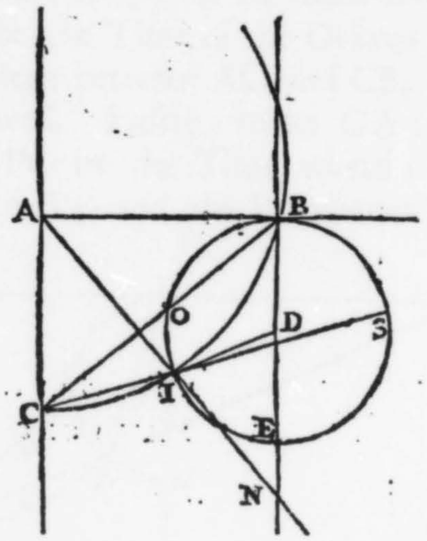
And this would be more ſo, if (as in the following Figure) BIC were leſs than the Arch of a Quadrant: For



the Perpendicular DB will then cut the Circle CIB; where-

fore

fore DI alſo will cut the ſame Circle, ſince it is = DB: And ∠ DIA will be obtuſe; and therefore AIN will alſo cut BIN: And ſince ∠ ABC is leſs than ∠ AIC, which is = SIN; and this now ſtill leſs than that which would be made at the Contact in I by the Line SI; therefore the



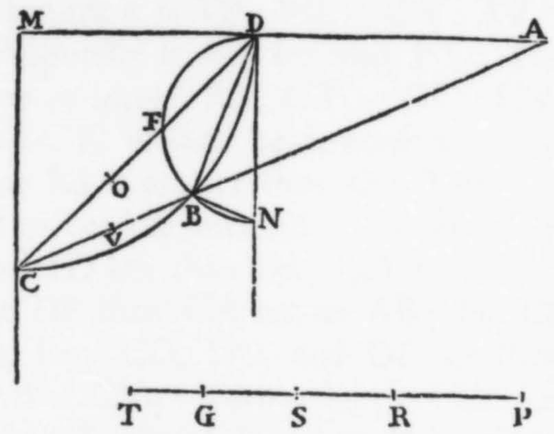
Arch SEI is far greater than the Arch BO: Whence, *&c. Q.E.D.*

PROP. XXXVI. THEOR. XXII.

If from the loweſt Point of a Circle erect to the Horizon, be elevated a Plane, ſubtending an Arch not greater than a Quadrant; from whoſe Terms, two other Planes are inflected to any Point of the Circumference; the Deſcent along both the inflected Planes will be performed in a ſhorter Time than along the former elevated Plane alone,

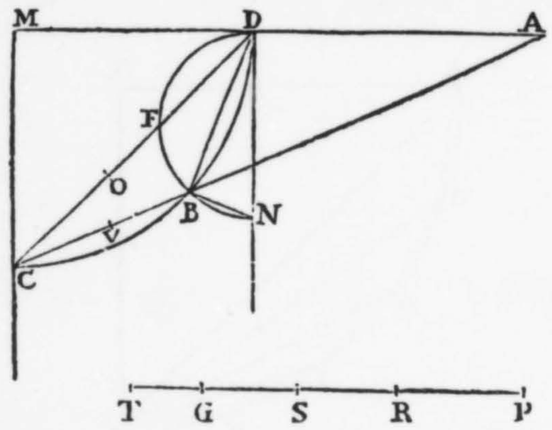
alone, or than along one only of the other two, namely, the lower.

Let CBD be the Arch, not greater than that of a Quadrant of the Circle erect unto the Horizon, from the loweſt Point C, in which let CD be an elevated Plane, and let the two Planes CB and DB be inflected from the Terms D and C, to any Point in the Arch CBD taken at Pleaſure, as B; then, I ſay, that the Times of the Deſcent along both theſe Planes DB and BC, is ſhorter than the Time of the Deſcent along DC alone, or than that along only BC *ex quiete* in B.



Thro' D draw the horizontal Line MDA, which let CB produc'd meet in A; and let DN and MC be perpendicular to MD, and BN to BD; and about the right-angled Triangle DBN circumſcribe a Semicircle as DFBN, cutting DC in F, and let DO be a mean Proportional between CD and DF, and let AV be a Mean between CA and AB; and let PS be the Time wherein the whole DC or BC ſhall be paſſed (for it is manifeſt, that they ſhall each

be paſſed in the ſame Time) and what Proportion CD has to DO, the ſame let the Time SP have to the Time PR; then the Time PR will be that wherein a Moveable from D paſſes DF; and RS that wherein it paſſes the Remainder FC: but ſince PS is alſo the Time wherein the Moveable from B, paſſes BC, if it be made BC : CD : : SP : PT; PT will be the Time of the Deſcent from A to C; ſince DC is a Mean between AC and CB, from what has been before proved. Laſtly, make CA : AV : : TP : PG, then will PG be the Time wherein the Moveable from A comes to B; and the Remainder GT the Time



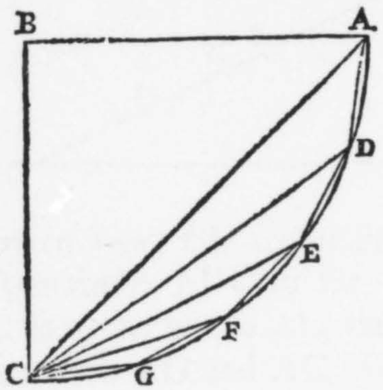
of the Motion thro' BC conſequent after the Motion from A to B. But ſince DN the Diameter of the Circle DFN is perpendicular to the Horizon, the Lines DF and DB ſhall be paſſed in equal Times. Wherefore, if it ſhall be demonſtrated, that the Moveable will ſooner paſs along BC, after the Deſcent DB, than along FC, after the Deſcent DF, we ſhall have what we wanted. But now the Moveable will, with the ſame Celerity of Time, paſs BC, coming

coming out of D along DB, as if it came out of A along AB; ſince in either Deſcent DB or AB, it requires equal Moments of Velocity: Therefore it remains to be demonſtrated, that BC after AB is paſſed in a ſhorter Time than FC after DF is. But it has been proved, that the Time wherein BC, after AB is paſſed, is GT; and the Time wherein FC, after DF is paſſed, is RS; therefore we have now to prove, that RS is greater than GT. Which is thus done; becauſe it is SP : PR : : CD : DO, therefore firſt by Converſion, and then by Inverſion of Proportion it will be RS : SP : : OC : CD. But SP : PT : : DC : CA; and becauſe it is TP : PG : : CA : AV, by Converſion of Proportion it will be alſo PT : TG : : AC : CV; therefore *ex æquali* RS : GT : : OC : CV; but OC is greater than CV, as ſhall be demonſtrated anon; therefore the Time RS is greater than the Time GT; which was required to be demonſtrated. But ſince CF is greater than CB, and FD leſs than BA, CD will have a greater Proportion to DF than CA has to AB; but CD : DF : : CO*q* : OF*q*, ſince CD, DO, and DF are Proportionals. And CA : AB : : CV*q* : VB*q*; therefore CO has a greater Proportion to OF, than CV has to VB; therefore from the foregoing Lemma, CO is greater than CV. 'Tis moreover manifeſt, that the Time along DC to the Time along DBC, is as DOC to DO, together with CV.

SCHOLIUM.

From what has been demonſtrated, may be gathered, that the ſwifteſt of all Motions from Term to Term, is not made along the ſhorteſt Line, *viz.* a right Line, but by or along the Arch of a Circle.

For in the Quadrant BAEC whoſe Side BC is perpendicular to the Horizon, let the Arch AC be divided into any Number of equal Parts, AD, DE, EF, FG, GC; and draw right Lines from C to the Points A, D, E, F, G; draw alſo the right Lines AD, DE, EF, FG, and GC: Then



'tis manifeſt, that the Motion along the two Lines AD and DC, is ſooner perform'd than that along the one Line AC, or along DC, out of Reſt in D: But out of Reſt in A, DC is ſooner paſſed over than the two AD and DC; but along the two DE and EC, out of Reſt in A, the Deſcent is likewiſe ſooner made than along CD only; therefore the Deſcent along the three Lines AD, DE, and EC is made ſooner than along the two AD and DC. And, in like Manner,

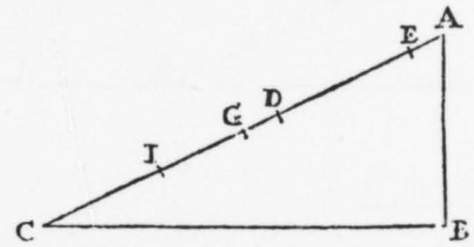
Manner, after the Deſcent along AD and DE, the Motion is ſooner made along the two EF and FC, than along FC alone; therefore the Motion is ſooner performed along the four AD, DE, EF, and FC, than along the three AD, DE, and EC. And laſtly, along the two FG and GC, after a preceding Deſcent thro' AD, DE, and EF, the Motion is ſooner accompliſhed than along FC alone; therefore along the five AD, DE, EF, FG, and GC, the Deſcent is made in a yet ſhorter Time than along the four AD, DE, EF, and FC; the nearer therefore we approach'd by inſcribed Polygons to the Circumference, the ſooner will the Motion be performed between the two aſſign'd Terms A and C.

And that which is explained in the Arch of a Quadrant, holds true likewiſe in an Arch leſs than that of a Quadrant; and the Reaſoning is the very ſame.

PROP. XXXVII. PROB. XV.

A Perpendicular and an inclined Plane of the ſame Elevation being given, to find a Part in the inclined Plane = the Perpendicular, and that may be paſſed over in the ſame Time as the ſaid Perpendicular.

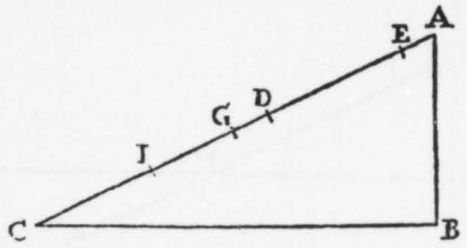
Let AB be the Perpendicular, and AC the inclined Plane.



Now we muſt find in the inclin'd Plane, a Part = the Perpendicular

pendicular AB, that after Reſt in A, may be moved thro' in a Time = the Time wherein the Perpendicular is paſſed.

Make AD = AB, and biſect the Remainder DC in I; and make AC : CI : : CI : AE; to which AE make GD equal; then it is manifeſt, that EG is = AD = AB. I ſay, moreover, that this EG is that Part of the inclined Plane which is paſſed over by a Moveable coming out of Reſt in A, in a Time = the Time wherein a Moveable falls thro' AB. For becauſe it is AC : CI : : CI : AE; or : : ID : DG; therefore by Converſion it is AC : AI : : DI : IG. And becauſe as the whole CA to the whole AI,



ſo is the Part taken away CI, to the Part taken away IG; therefore the Remainder AI is to the Remainder AG, as the whole AC is to the whole AI; therefore AI is a mean Proportional between AG and AC, and CI a Mean between CA and AE; if therefore we ſuppoſe the Time along AB to be as AB, AC ſhall be the Time along AC, and CI or ID the Time along AE; and becauſe AI is a mean Proportional between CA and AG, and CA is the Time along the whole AC; therefore AI ſhall be the Time along AG, and the Remainder IC that along the Remainder GC; but DI was the Time along AE, therefore DI and IC are the Times along both the Spaces AE and CG, therefore the

the Remainder DA ſhall be the Time along EG, and = the Time along AB. *Q. E. F.*

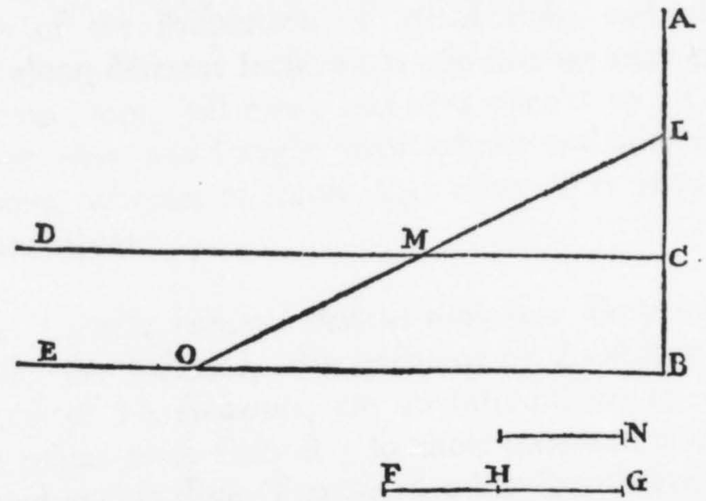
Corollary.

Hence it is manifeſt, that the Space required is that which lies betwixt the upper and lower Parts (*viz.* AE and CG) that are paſſed in equal Times.

Prop. XXXVIII. Prob. XVI.

Two horizontal Planes cut by a Perpendicular being given, to find a Point above in the Perpendicular, out of which Moveables falling and turning along the horizontal Planes, may, in Times = the Times of the Deſcents, paſs along the ſaid horizontal Planes, namely, along the upper and along the lower Spaces that have to each other any given Proportion of the leſſer to the greater.

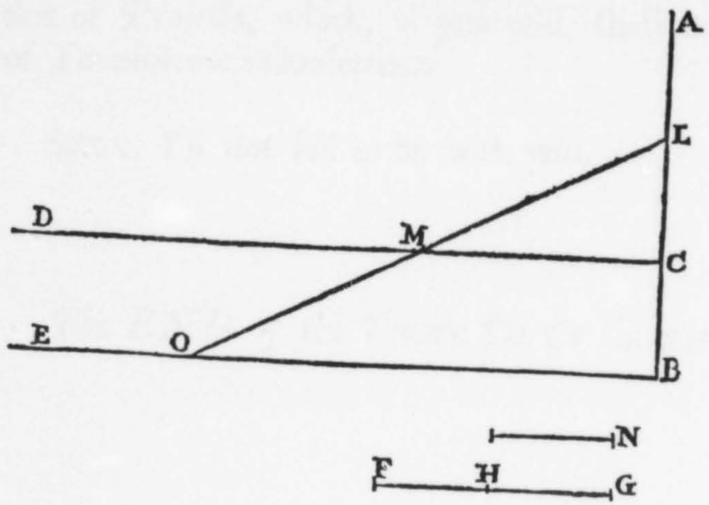
Let the horizontal Planes CD and BE be cut by the Perpendicular ACB, and let the given Proportion of the leſs



to

to the greater, be that of N to FG. It is required in the Perpendicular AB, to find a Point on high, out of which a Moveable falling, and turning along CD, may, in a Time = the Time of its Fall, paſs a Space that ſhall have unto the Space paſſed by the other Moveable falling from the ſame ſublime Point, in a Time = the Time of its Fall with a reflex Motion along the Plane BE, the ſame Proportion as the given Line N has to FG.

Make GH = the ſaid N; and make FH : HG : : BC : CL; then, I ſay, L is the Point on high required: For taking



CM double to CL, draw LM meeting the Plane BE in O; then BO will be the double of BL: And becauſe it is FH : HG : : BC : CL; it will be by Compoſition and Converſion HG: (*i. e.* N) GF : : CL : LB, *i. e.* : : CM : BO. But becauſe CM is double to LC, let the Space CM be that which by the Moveable coming from L, after the Fall LC, is paſſed along the Plane CD; and by the ſame Reaſon BO is that which is paſſed after the Fall LB, in

t

in a Time = the Time of the Fall along LB, ſince BO is double to BL; therefore the Propoſition is manifeſt.

SAGR. Moſt certainly we may grant to our *Academic*, that he hath done what, without Arrogance, he promiſed at the Beginning of this Treatiſe, *viz.* that he hath ſhewed us a *New Science* concerning *a very Old Subject*: And to ſee with what Eaſe and Clearneſs from one ſole moſt ſimple Principle he deduces the Demonſtrations of ſo many Propoſitions, I can't ſufficiently admire how this Buſineſs eſcap'd unhandled by *Archimedes*, *Apollonius*, *Euclid*, and ſo many other excellent Mathematicians and Philoſophers: Eſpecially ſince we meet with many large Volumes of Motion.

SALV. There is extant a ſmall Fragment of *Euclid* of *Motion*, but there appear no Steps of his taken towards the Diſcovery of the Proportion of *Acceleration*, and of its Varieties along different Inclinations: So that we may truly ſay, that no Door, 'till now, was ever open'd to a Contemplation new and fraught with infinite and admirable Concluſions, wherein in future Ages other Wits may exerciſe themſelves.

SAGR. I verily believe, that as thoſe few Properties of the *Circle*, for Inſtance, demonſtrated by *Euclid* in the third Book of his *Elements*, are an Introduction to innumerable others more abſtruſe; ſo thoſe produced and demonſtrated in this ſhort Treatiſe, if other ſpeculative Genius's apply to them and conſider them well, will point out the Way to infinite others ſtill more admirable: And

this I the rather imagine, on Account of the Nobleneſs of the Subject, whereby it ſo vaſtly exceeds all other Phyſical Subjects.

This Day's Conference has been very long, and laborious, wherein I have been delighted more with the bare Propoſitions than with their Demonſtrations; ſeveral of which will take more than an Hour a Piece, I dare ſay, to underſtand them well: A Task that I reſerve to myſelf to perform at Leiſure, if you pleaſe to lend me the Book, after we have ſeen the remaining Part concerning the Motion of *Projects*, which, if you will, ſhall be the Subject of To-morrow's Conference.

SALV. I'll not fail to be with you.

The END of the THIRD DAY'S CONFERENCE.

GALILÆUS

GALILÆUS

HIS

DIALOGUES.

DIALOGUE IV.

SALVIATUS, SAGREDO, *and* SIMPLICIUS.

SALV. SINCE *Simplicius* is alſo come, let us, without Delay, proceed to *Motion*; and ſee here the *Text* of our *Author*:

Of the MOTION of PROJECTS.

What Properties belong to *Equable Motion*, as alſo to *Motion naturally accelerate*, upon all whatever Inclinations of Planes, we have already conſidered: In this Contemplation we are now entering upon, I'll endeavour to ſhew you ſome of the chief Properties moſt worthy your Notice, and to eſtabliſh the ſame by ſolid Demonſtrations, belonging to a Moveable, whilſt it is carried by a Motion compounded of a twofold Lation, *viz.* the Equable and

the Naturally Accelerate: And this is that Motion which we call *The Motion of Projects*: The Generation of which Motion may be underſtood by what follows.

Conceive a Moveable projected or thrown along an horizontal Plane, all Impediments ſecluded: Now 'tis manifeſt, by what we have before ſaid at large, that its Motion will be equable and perpetual along the ſaid Plane, if the Plane be extended *in infinitum*: but if we ſuppoſe this horizontal Plane terminated and placed on high, the Moveable which I conceive endued with Gravity, being come to the End of the Plane, proceeding ſtill forward, there is added to its equable and former indelible Lation, that Propenſion downwards which it has from its Gravity, and thence a certain Motion will be produced, compounded of the equable horizontal one, and of the deſcending naturally accelerate one: Which compound Motion I call *Projection*, ſome of whoſe Properties we will demonſtrate; the firſt of which ſhall be this.

PROP. I. THEOR. I.

A *Project*, when it is carried by a Motion compounded of the horizontal Equable, and of the naturally accelerate one downwards, deſcribes by its Lation or Tract a *Semiparabolical* Line.

SAGR. It's requiſite, *Salviatus*, for my own, and, I believe, for *Simplicius's* ſake alſo, to ſtop a little; for I am not ſo far vers'd in Geometry as to have ſtudied *Apollonius* any farther than juſt to know that he treats of theſe *Parabolas*, and of the other Conic Sections; without

out underſtanding which with their Properties, I don't think one can underſtand the Demonſtrations of the other Propoſitions depending on them: And becauſe already, in the very firſt Propoſition, it is propoſed by the Author, to prove the Line deſcribed by a *Project* to be parabolical, I imagine, ſince we are to treat of theſe Lines only, that it muſt be abſolutely neceſſary to have a perfect Knowledge, if not of all the Properties which *Apollonius* has demonſtrated of thoſe Figures, yet, at leaſt, of ſuch as are neceſſary to the Science in Hand.

SALV. You think too meanly of yourſelf to make ſtrange of that, the Truth whereof, not long ſince, you made no Diſpute about; thus, when in treating of Reſiſtance, I told you that a certain Propoſition of *Apollonius* ſhou'd be underſtood, you, without the leaſt Difficulty, admitted of it.

SAGR. Perhaps I might then either accidentally conceive it, or for once might take for granted what ſerv'd my Turn in that Treatiſe: But here, where I imagine we are to hear all the Demonſtrations concerning thoſe Lines, I can't think it right to take any Fundamental for granted, for this would be going on to no Purpoſe.

SIMP. Tho' *Sagredus* may be, as I believe he is, well vers'd in what may prove neceſſary to him, yet, for my Part, I am a Stranger to the very firſt Terms; for altho' our Philoſophers have treated of the Motion of *Projects*, yet I don't remember that they have defin'd thoſe Lines which they deſcribe; any otherwiſe than to be curv'd Lines,

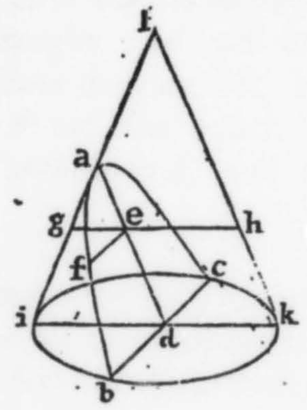
except

except the Projections be perpendicularly upwards: But of what kind thoſe Curves are they have not told us. Therfore in caſe that little Geometry which I have learnt from *Euclid*, within the Time of the foregoing Conferences, be not ſufficient to make me capable of underſtanding the following Demonſtrations, I muſt content myſelf with an implicit Belief of them.

SALV. No, you ſhan't, for the Author of this Work himſelf will inſtruct you, as heretofore he did me, when he granted me a Sight of this his Work; for finding that I at that Time was not ready in the Books of *Apollonius*, he demonſtrated to me two of the chief Properties of the *Parabola*, without any other *Precognitum*, which two only we have Need of in the preſent Treatiſe; which are both indeed proved by *Apollonius*, but after many Propoſitions proved before, which to conſider wou'd take up too much of our Time; for I would willingly ſhorten our Trouble as much as poſſible; deducing the firſt of theſe immediately from the pure and ſimple Generation of the *Parabola* itſelf; and from this without any Trouble, the Demonſtration of the other. To the firſt then: Conceive a *Right-Cone*, whoſe Baſe let be the Circle *ibkc*, and *Vertex* the Point *l*, in which, cut by a Plane parallel to the Side *lk*, there ariſes the Section *bac*, called a *Parabola*, whoſe Baſe *bc* let cut the Diameter *ik* of the Circle *ibkc* at right Angles, and let the *Axis ad* be ‖ the Side *lk*; then taking any Point *f* in the Line *bfa*, draw the right Line *fe* ‖ *bd*; I ſay, the Square of *bd* has to the Square of *fe* the ſame *Ratio* as the *Axis da* has to the *Axis ea*.

Conceive

Conceive a Plane to paſs thro' the Point *e* ‖ the Circle *ibkc*, which makes in the Cone a circular Section, whoſe Diameter let be the Line *geh*. And becauſe to the Diameter *ik* of the Circle *ibk*, the Line *bd* is perpendicular, the Square of *bd* will be = ▭ made of the Parts *id* and *dk*. And likewiſe, in the upper Circle, which is conceived to paſs thro' the Points *gfb*; the Square of the

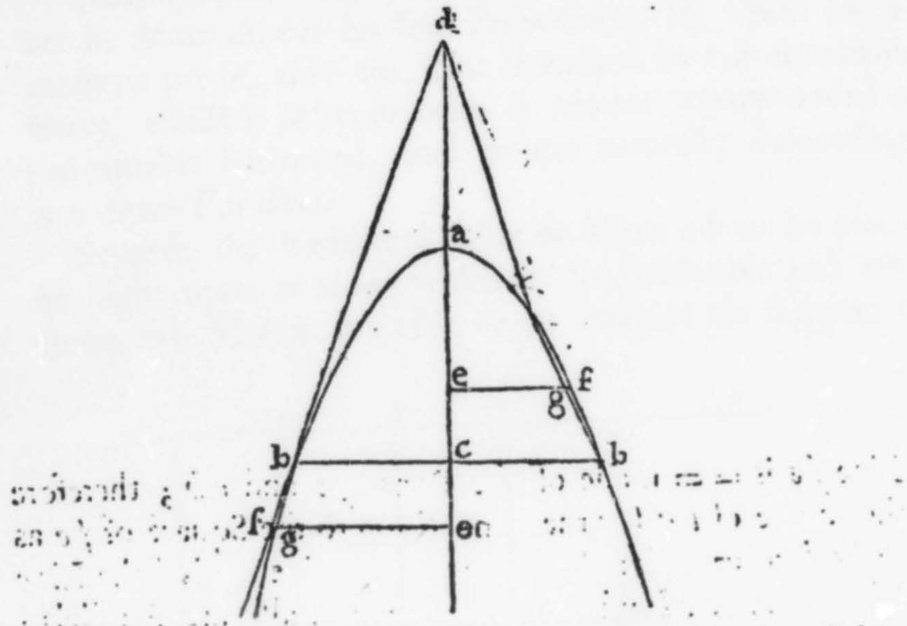


Line *fe* is = ▭ made of the Parts *ge* and *eh*; therefore the Square of *bd* has the ſame *Ratio* to the Square of *fe* as ▭ *idk* to ▭ *geh*: And becauſe the Line *ed* is ‖ the Line *bk*, the Line *eh* will be = *dk*, which alſo are Parallels; and therefore ▭ *idk* will have the ſame *Ratio* to ▭ *geh* as *id* has to *ge*; *i.e.* as *da* to *ae*; therefore ▭ *idk* to ▭ *geh*; *i.e.* □ of *bd* has the ſame *Ratio* to □ of *fe* as the *Axis da* has to the *Axis ea*. *Q. E. D.*

The other Propoſition neceſſary to the preſent Treatiſe, we will demonſtrate as follows.

Deſcribe

Deſcribe a *Parabola*, whoſe *Axis c a* produce outwards to *d*, and taking any Point *b*, thro' it conceive a Line as *bc* to be drawn ‖ to the Baſe of the *Parabola*. Suppoſe *da* = the Part *ca* of the *Axis*; then, I ſay, that a right Line drawn thro' the Points *d* and *b* will not fall within the *Parabola*, but without it, ſo as to touch it only in that very Point *b*.



If it be poſſible, let it fall within the *Parabola*, it will then cut it above, or being prolong'd it will cut it below the Point *b*. And in that Line take any Point as *g*, thro' which draw the right Line *f g e*; and becauſe the □ of *fe* is greater than □ of *ge*, the ſame Square of *fe* will have a greater *Ratio* to □ of *bc* than □ of *ge* has to the ſame Square of *bc*. And becauſe by the preceding Propoſition □ of

of *fe* is to □ of *bc*, as *ea* to *ca*, therefore *ea* has a greater *Ratio* to *ca*, than □ of *ge* has to □ of *bc*; *i.e.* than the Square of *ed* to □ of *cd*, (ſince in △ *dge*, it is as *ge* to its Parallel *bc*, ſo is *ed* to *cd*; but the Line *ea* is to the Line *ca*, *i.e.* to the I ine *ad* as four Rectangles *ead* to four Squares of *ad*, *i.e.* to □ of *cd* (which is = four Squares of *ad*); therefore four Rectangles *ead* will have a greater *Ratio* to □ of *cd* than □ of *ed* has to the ſame Square of *cd*, therefore four Rectangles *ead* will be greater than □ of *ed*, which is falſe, ſince they are leſs, becauſe the Parts *ea*, *ad* of the Line *ed* are not equal. Therefore the Line *db* touches the *Parabola* in *b*, and does not cut it. *Q. E. D.*

SIMP. Your Reaſoning is too lofty for me; you ſeem all along to preſuppoſe that all the Propoſitions of *Euclid* are as familiar as the firſt Axioms themſelves; but the Caſe is otherwiſe. And your aſſerting to me but juſt now that four Rectangles *ead* are leſs than □ of *ed*, becauſe the Parts *ea*, *ad* are unequal, doth not ſatisfy me, but leaves me in Doubt.

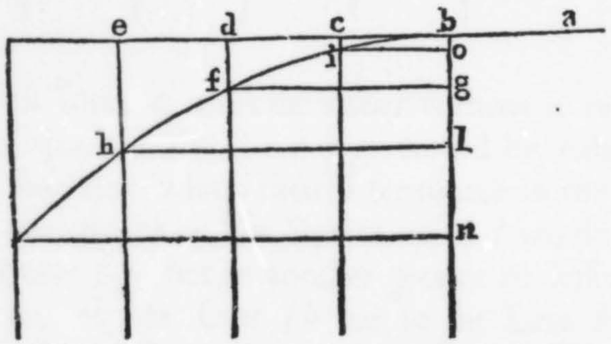
SALV. The Truth is, all Mathematicians of Note, do ſuppoſe the Reader to have the Elements of *Euclid* at his Fingers Ends; but to ſupply your Want, let it ſuffice to put you in Mind of a Propoſition in the Second Book, by which is demonſtrated, that, *When a Line is cut into equal Parts, and into unequal, the Rectangle of the unequal Parts is leſs than the Rectangle of the equal* (i. e. *than the Square of the Half*) *by the Square of the Line comprehended between the Sections.* Whence it is manifeſt, that

the Square of the Whole, which contains four Squares of the Half, is greater than four Rectangles made of the unequal Parts.

Now 'tis neceſſary that we bear in Mind theſe two Propoſitions juſt now demonſtrated, taken from the Conic Elements, in order to the better underſtanding what follows in this Tract, for of theſe two, and theſe only, the Author makes Uſe.

Now we may reaſſume the Text, and ſee in what Manner he demonſtrates his firſt Propoſition, by which he intends to prove, that the Line deſcribed by the moveable Body, whilſt it deſcends with a Motion compounded of the equable horizontal, and of the naturally deſcending, is a *Semi-Parabola*.

Suppoſe the horizontal Line or Plane *ab* to be plac'd on high, upon or along which let the Moveable paſs with an equable Motion out of *a* to *b*: imagine the Support of

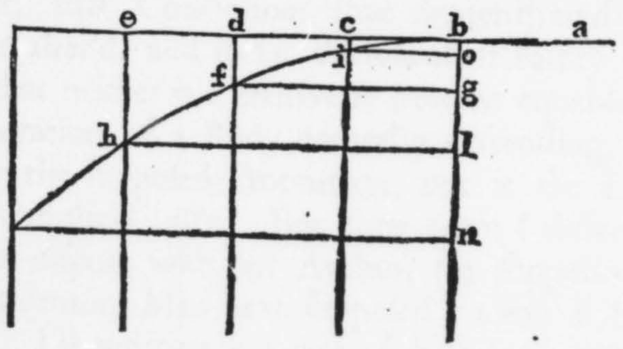


the Plane to be taken away in *b*; ſuppoſe the Moveable to acquire by its own Gravity a natural Motion downwards, according to the Perpendicular *bn*. Imagine a right Line, as *be*, placed upon the Plane *ab*, repreſenting the Flux, or Meaſure

Meaſure of Time, upon which ſet off at Pleaſure what Number of equal Parts you pleaſe, as *bc*, *cd*, *de*. And from the Points *b*, *c*, *d*, *e*, imagine Lines drawn parallel to the Perpendicular *bn*: In the firſt of which take any Part *ci*, whoſe Quadruple ſet off in the next *df* nine times, which ſet off from *e* to *h*, and ſo on in the reſt, according to the Proportion of the Squares of *cb*, *db*, *eb*, or, which is the ſame Thing, let the Lines ſet off be in a duplicate Proportion of the Lines or Times *cb*, *db*, *eb*, *&c.* And if unto the Moveable moving beyond *b* towards *c* with an equable Lation, we imagine a perpendicular Deſcent at the ſame Time to be added, according to the Quantity *ci*. In the Time *bc* the Moveable will be found in the Point or Term *i*. And proceeding farther, in the Time *db*, *viz.* in double the Time *bc* the Line of Deſcent ſhall be quadruple the firſt Line *ci*: For it has been demonſtrated in Prop. 2. of the laſt Dialogue, that the Spaces paſſed over by a heavy Body, with a Motion naturally accelerate, are in a duplicate Proportion of the Times. It conſequently follows, that the Space *eb* paſſed over in the Time *be*, will be nine times *ci*; ſo that it manifeſtly appears, that the Spaces *eb*, *df*, *ci* are one to another as the Squares of the Lines *eb*, *db*, *cb*. Now from the Points *i*, *f*, *h*, draw the right Lines *io*, *fg*, *hl*, ‖ *eb*; then the Lines *hl*, *fg*, and *io* will be = the Lines *eb*, *db*, and *cb*, each to each; as alſo *bc*, *bg*, and *bi*, to *ci*, *df*, and *eh*. And the Square of *hl* will be to the Square of *fg*, as the Line *lb* to *bg*; and the Square of *fg* to the Square of *io*, as *gb* to *bo*; therefore the Points *i*, *f*, and *h*, are in one and the ſame parabolick Line. And in like manner it may be demonſtrated, that taking any equal Particles of Time, of

what Magnitude you will, that the Places of the Moveable carried with the like compound Motion, in the ſame Times are found in the ſame parabolick Line; therefore the Propoſition is manifeſt.

SALV. This Concluſion is gathered from the Converſe of the former of the two foregoing Propoſitions: Becau he *Parabola* being, for Example, deſcribed thro' the Points *b* and *h*, if one of the two *f* and *i* was not in the deſcrib'd



parabolick Line, it muſt be either without it or within it, and conſequently the Line *f g* would be either leſſer or greater than that which ſhould terminate in the parabolick Line: And therefore the Square of *h l* would have, not to the Square *f g*, but to another greater or leſſer, the ſame Proportion as the Line *l b* has to the Line *b g*: But it hath the ſame Proportion to the Square of *f g*, therefore the Point *f* is in the parabolick Line: And ſo of all the Reſt.

SAGR.

SAGR. I can't deny but the Diſcourſe is new, ſubtile, and concluſive, ſuppoſing the tranſverſe Motion to continue always equable, and that the natural Motion downwards do likewiſe keep its Tenor, and is continually accelerate, in a duplicate Proportion of the Times; and that thoſe Motions and their Velocities mix'd with one another do not alter, diſturb, or hinder one another; ſo that the Line of the *Project* in the Continuation of the Motion degenerates into ſome other kind: Which indeed ſeems to me to be impoſſible; becauſe the Axis of our *Parabola*, which we ſay a grave Body deſcribes by its Motion, being perpendicular to the Horizon, terminates in the Center of the Earth: Since the parabolic Line doth continually depart from its Axis, no Project could ſo go on as ever to terminate in the Center; or if it ſhould tend thither, as it ſeems neceſſary it muſt, the Line of the Project muſt degenerate into another moſt different from that of a *Parabola*.

SIMP. To this Difficulty I add others alſo: One of which is this, that we ſuppoſe the horizontal Plane, which has neither Acclivity nor Declivity, to be a right Line; as if ſuch a Line was in all its Parts equally diſtant from the Earth's Center; which is not true; for by being produced from its Middle towards the Extremes, it conſtantly more and more recedes from the Center, and therefore continually aſcends; the Impoſſibility, which conſequently follows, is, that its Motion cannot be perpetual; nay, that it does not ſo much as remain equable thro' any Space, but continually grows weaker and weaker: Beſides, as I imagine, it is

impoſſible

impoſſible to avoid the Impediment of the *Medium*, but that it will deſtroy the Equability of the tranſverſe Motion, and the Law of Acceleration in falling Bodies. All which Difficulties render it highly improbable, that Things demonſtrated from ſuch uncertain Suppoſitions, ſhould afterwards hold true in Practice.

SALV. All theſe alledged Inſtances and Difficulties are ſo well grounded, that I think it impoſſible to remove them: And, for my Part, I admit them all, in the manner I believe our Author would have admitted them. And I grant, that Concluſions thus demonſtrated in the Abſtract are alter'd, and ſo far diſcloſe their Falſity in the Concrete, that neither is a tranſverſe Motion equable, nor is the Acceleration of a Body naturally deſcending, made according to the ſuppoſed Proportion, nor is the Line of a Project parabolical, *&c.* But then again I deſire, that you will not diſpute with our Author, for ſuppoſing that which other famous Men have ſuppoſed; altho' it be not ſtrictly true: I'll mention but one of theſe Men, and that is *Archimedes*, who, in his Mechanicks, and firſt Quadrature of the *Parabola* aſſumes it as a true Principle that the Iron Rod of the Ballance, or of the * *Statera Romana*, is a right Line equally diſtant in all its Points from the Center of the Earth; and that the Lines whereby the Weights are hung, are parallel to one another. Which Liberty, indeed, theſe Men excuſe, ſaying, that the Inſtruments we uſe in Practice, and the Diſtances we take, are ſo ſmall in Compariſon of our great Diſtance from the Earth's Center,

* Steel-yard.

we may very well be allow'd to take one Minute of a Degree of a great Circle for a right Line, and alſo two Perpendiculars hanging at its Ends to be parallel Lines: And if that in practical Operations there was any Regard to be had to ſuch very minute Quantities, the Architects, before all others, ought to be blam'd, who by Help of the Plumb-Line pretend to erect the higheſt Buildings between parallel Lines. To theſe I add, that we may affirm, that *Archimedes* and others have in their Contemplations ſuppoſed themſelves to be removed to an infinite Diſtance from the Center; in which Caſes their Aſſumptions were juſt, and their Concluſions abſolutely true.

Now if we would reduce to Practice Concluſions demonſtrated at a limited but immenſe Diſtance, from the Truth of ſuch Concluſions muſt be abated what is ſufficient, on Account of our Diſtance from the Earth's Center not being really infinite; but indeed this Diſtance is ſuch, as may be ſaid to be infinite, in reſpect of the Littleneſs of our practical Artifices, the greateſt of which are the Paths or Ranges of Projects, eſpecially thoſe that are thrown out of Ordnance, which, be it of what Size ſoever, will not throw a Ball four of ſuch Meaſures, as we are diſtant from the Center of the Earth near four thouſand: Wherefore ſince this Path terminates on the Superficies of the Earth, the parabolic Form cannot be ſenſibly alter'd; which we allow would be greatly trasform'd, if it ſhould terminate in the Earth's Center.

Again, as to the Irregularity proceeding from the Impediment of the *Medium*, this, we grant, is more conſiderable, and by Reaſon of its ſo manifold Varieties can't poſſibly be reduced to certain Laws: For if we ſhould only

conſider the Impediment which the Air cauſes in ſuch Motions as we have throughly examin'd into, we ſhould plainly find it to diſorder them all, and that after infinite Ways, according as the Figures, Gravities and Velocities of the Moveables are infinitely varied. For as to the Velocity, the greater it is, the greater will be the Impediment it ſhall meet with from the Air; and alſo the lighter the Moveables are, the more will they be hindered by the Air; ſo that tho' a falling heavy Body ought to be continually accelerated in a duplicate *Ratio* of the Duration of its Motion, however great the Moveable be, and let it fall from never ſo great a Height, yet ſuch will be the Impediment of the Air, that the Body will be deprived of any further Increaſe of its Velocity, and reduc'd to an uniform and equable Motion; which Equability will be ſo much the ſooner obtained, and in leſſer Heights, by how much the Moveable is the higher.

Nay, that Motion alſo, which in an horizontal Plane ought continually to be equable, all other Obſtacles being removed, ſhall, by the Impediment of the Air be altered, and at length quite ſtopped, and that the ſooner by how much the Moveable Body is the lighter: And inaſmuch as theſe Accidents of Gravities and Figures are ſubject to infinite Mutations, we can come at no certain Knowledge concerning them: Wherefore to treat of this Matter ſcientifically, 'tis requiſite to abſtract from them; and having found and demonſtrated the Concluſions, abſtracted from all Impediments, to make Uſe of them in Practice, with ſuch Limitations as Experience ſhall teach and direct us: Nor ſhall theſe Concluſions ſo abſtracted from, be of ſmall Advantage; becauſe we'll chuſe Bodies and their Figures, which

which are leſs ſubject to the Impediments of the *Medium*, ſuch are the very heavy and round ones; and their Spaces and Velocities at the greateſt ſhall not be ſo great, but that their Exorbitances may, by an eaſy Diminution, be reduced to our Purpoſe. But among thoſe Projects, which we make Uſe of, if they are of an heavy Matter, and of a round Form; nay, if they are of a lighter Matter, and have a cylindric Form, ſuch as are Arrows thrown from Bows, their Track or Path will not ſenſibly deviate from the Curve of a *Parabola*.

Nay, (to go further) among thoſe Artifices which we make Uſe of in our Practice, I can prove, by two Experiments, that their Exiguity renders their external and accidental Impediments very inconſiderable, of which that of the *Medium* is of the greateſt Moment; conſidering the Motions to be made in the Air, ſince ſuch are chiefly the Subject of our Diſcourſe; againſt which the Air exerts its Power two Ways; firſt in obſtructing the lighter Moveables more than the heavier; and in reſiſting the greater more than the leſſer Velocity of one and the ſame Moveable.

As to the firſt, we may learn by Experiment, that two Balls, equal in Magnitude, but one ten or twelve times heavier than the other; as for Inſtance, one of Lead, and the other of Oak, would by falling from an Height of 150 or 200 Yards, arrive at the Earth with very little Difference of Velocity; from whence we are plainly convinced, that the Impediment and Retardation of the Air is but very ſmall in both; if the Leaden Ball falling from the ſame Height, and at the ſame Moment of Time with the Wooden one, ſhould be but little retarded, and the Wooden one much; in ſuch Caſe the Leaden Ball, at its coming to the Ground, would leave the Wooden one a conſidera-

ble Diſtance behind it, becauſe the Leaden one is ten Times heavier; which, nevertheleſs, does not happen, for it outgoes it not indeed above an hundredth Part of the whole Height. And between a Ball of Lead and another of Stone, the Weight of this being one third, or one half that, the Difference of Time wherein they come to the Ground would be ſcarce obſervable. Now becauſe the *Impetus*, which the Leaden Ball acquires, whilſt it is falling from an Height of 200 Yards (which is ſo much as, continuing on with an equable Motion, it would in the ſame Time fall 400 Yards) is very conſiderable in reſpect of the Velocities, which we give to our Projects by Bows or other Machines (that *Impetus* which is given by Fire excepted); we may, without any conſiderable Error, conclude thoſe Propoſitions to be abſolutely true, which are demonſtrated without any Conſideration of the Alteration of the *Medium*.

But now, Secondly, we are to ſhew, that the Impediment which the ſaid Moveable receives from the Air, whilſt it moves ſwiftly, is not much greater than that which the ſame Body receives, when it moves ſlowly: And of this we are very well aſſured by the following Experiment: Let two equal Balls of Lead be hung to two Threads of equal Length, *viz.* each four or five Yards long; and theſe two Threads being faſten'd on high, let each Ball be removed from its perpendicular Situation, the one to 80 Degrees Diſtance or more, the other not to more than four or five; ſo that one being let go may deſcend, and paſſing the perpendicular Line, deſcribe very large Arches of 160, 150, 140, *&c.* Degrees, diminiſhing them by little and little; but that the other playing freely may deſcribe ſmall Arches of 10, 8, 6, *&c.* Degrees, diminiſhing them in

in like manner by little and little. Here, firſt, I aſſert, that the firſt Ball will paſs over its 160°, 150°, *&c.* in the ſame Time that the other paſſes over its 10°, 8°, *&c.* Whence it becomes manifeſt, that the Velocity of the firſt Ball will be 16, and above 18, *&c.* times greater than the Velocity of the ſecond; and in caſe the greater Velocity were to be more impeded by the Air than the leſſer, the Vibrations ought to be fewer in the greater Arches of 160 or 150 Degrees, *&c.* than in the leſſer of 10, 8, 4; nay even of 2 and of 1. But Experience contradicts this: For if two Perſons ſhall count one the greater Vibrations, the other the leſſer, they'll ſee that in counting of not only ten, but a hundred of them, there will not be the Difference of one ſingle Vibration, nay, not of the leaſt Part of one. And this Experiment confirms to us two Propoſitions at the ſame Time, namely, that the greateſt and leaſt ſingle Vibrations are made in the ſame Time: And that the Impediment and Retardation of the Air is not more in the ſwifteſt Motions than in the ſloweſt; contrary to that which before it ſeem'd that we ourſelves would in common have judged.

SAGR. Rather, becauſe it can't be denied but that the Air impedes both thoſe ſwifteſt and ſloweſt Motions, ſince they both continually grow more languid, and at laſt ceaſe moving, we ſhould ſay that theſe Retardations are made in the ſame Proportion in both Caſes. But how? Whence proceeds its making at one Time a greater Reſiſtance, at another, leſs, but that it at one Time is aſſaulted with a greater *Impetus* and Velocity, and at another Time leſs? And if this be ſo, the ſame Quantity of the Velocity of the Moveable is both the Cauſe, and alſo the Meaſure of

the Quantity of the Reſiſtance. Therefore all Motions, be they ſlow or ſwift, are retarded and impeded in the ſame Proportion; a Notion, in my Judgment, not contemptible.

SALV. We may therefore farther, in this ſecond Caſe, conclude, that the Fallacies in thoſe Concluſions which are demonſtrated, abſtracting from outward Accidents, are in our Machines but of little Moment, in reſpect of Motions made with great Velocity, of which we are chiefly ſpeaking; and of the Diſtances, which are but very ſmall in reſpect of the Semidiameter of a great Circle of the Terreſtrial Globe.

SIMP. I would gladly hear, why you ſeparate the Projects thrown by the *Impetus* of the firſt, *i. e.* as I conceive by the Force of the Gunpowder; from thoſe which are thrown from Slings, Bows, or Croſs-Bows, as touching their not being after the ſame Manner ſubject to the Alteration and Impediment of the Air.

SALV. I am induc'd thereto by the exceſſive, and, as I may ſay, ſupernatural Violence, with which ſuch Projects are driven out. And ſure, without ſpeaking hyperbolically, that Velocity with which a Ball is flung from a Musket or Piece of Ordnance, I may call ſupernatural, inaſmuch as that if ſuch a Ball deſcend naturally in the Air from ſome great Height, its Velocity, on Account of the Air's Reſiſtance, will not increaſe continually; but what we ſee happen to light Bodies falling no very great Way, I mean, their being reduced in the End to an equable Motion, will alſo happen after thouſands of Yards Deſcent to an

an Iron or a Leaden Ball: And this determinate and ultimate Velocity, may be called the greateſt which ſuch a heavy Body can naturally obtain in the Air: But this Velocity I imagine to be much leſs than that which is given to the ſame Ball flung by fired Powder: As this may be proved by the following Experiment: From an hundred Yards high, or more, fire a Musket, loaden with a Leaden Bullet, perpendicularly down againſt a Stone Pavement; and fire the ſame Musket, with the like Charge, againſt ſuch another Stone, at a Yard or two's Diſtance; and then obſerve which of theſe two Bullets is moſt flatted. Now if that fired from on high be found leſs flatted than the other, we muſt conclude, that the Air has hinder'd and diminiſhed the Velocity given it by the Force of the Powder at the Beginning of its Motion: And conſequently, that the Air would not permit it to acquire ſo great a Velocity, let it come from what Height it will: For if the Velocity impreſs'd upon it by the Powder, did not exceed that which it could acquire of itſelf by a natural Deſcent, the Blow downwards, after ſuch natural Deſcent, ought rather to be more valid than leſs. I have not indeed made this Experiment; but am inclin'd to believe, that a Cannon or Musket Ball, falling from never ſo great an Height, will not give ſo great a Stroke as ſuch another Ball gives to a Wall when fir'd at a few Yards Diſtance, *i. e.* ſo few that the ſhort Perforation or Sciſſure which it makes in the Air, is not ſufficient to obviate the Exceſs of the ſupernatural Impetuoſity impreſſed on it by the Powder. This exceſſive *Impetus* of Projects, thrown with ſuch Violence, may cauſe ſome Irregularity in the Path of the Project; by making the parabolic Lines leſs inclin'd,

or

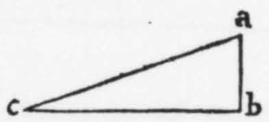
or curv'd at its Beginning than at its End; but this can be of little or no Prejudice to our Author in practical Operations: Amongſt which the Chief is the Compoſition of a Table of Ranges, which contains the Diſtances Balls are flung to at every different Elevation; and becauſe ſuch Projections or Shots are made with Mortars, by Help of a ſmall Quantity of Powder; in theſe, the *Impetus* not being ſupernatural, the Projects deſcribe their Paths exactly enough.

But let us now go on with the Treatiſe, where the Author endeavours to lead us to the Contemplation and Inveſtigation of the *Impetus* of the Moveable, whilſt it is mov'd with a Motion compounded of two Motions: And firſt of that compounded of two equable Motions; the one horizontal and the other perpendicular.

Prop. II. Theor. II.

If a Moveable be moved with a twofold equable Motion, *i. e.* an horizontal and a perpendicular one, the *Impetus* or *Momentum* of the Lation compounded of both Motions, ſhall be in *Power* both the *Momenta* of the former Motions.

For let any Moveable be moved equably with a twofold Lation, and let *a b* repreſent the perpendicular one,



and let *b c* anſwer to the horizontal one made in the ſame Time. Foraſmuch then as the Spaces *a b* and *b c* are paſſed over by equable Motions in the ſame Time, their *Momenta*

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Momenta ſhall be to each other as the ſaid *a b* and *b c*. But the Moveable which is carried by a Motion compounded of theſe two different Motions, deſcribes the Diagonal *a c*, and the *Momentum* of its Velocity ſhall be as *a c*; but *a c* in *Power* is = *ab* and *b c*, therefore the *Momentum* compounded of both the *Momenta a b* and *b c* is in *Power* = them both taken together. *Q. E. D.*

Simp. I beg you'll remove one Scruple I have as to this Matter; 'tis this, This preſent Concluſion ſeems to me to contradict another in the former Treatiſe; where it is affirmed, that the * *Impetus* of the Moveable coming from *a* to *b* is = that coming from *a* to *c*; and now you conclude, that the *Impetus* in *c* is greater than that in *b*.

Salv. The Propoſitions, *Simplicius*, are both true, but very different one to another. Here we ſpeak of one ſole Moveable, mov'd with one only Motion, but compounded of two, and thoſe both equable; but there two Moveables were ſpoken of, mov'd with Motions naturally accelerated, one along the Perpendicular *a b*, and the other along the inclin'd Plane *a c*: And, beſides, the Times there † are not ſuppoſed to be equal, but the Time along the inclin'd Plane *a c* to *b e* greater than the Time along the Perpendicular *a b*: But in the Motion we are now ſpeaking of, the Motions thro' *a b*, *b c*, *a c*, are ſuppoſed equable, and made in the very ſame Time.

Simp. Excuſe me; pray go on, I am ſatisfy'd.

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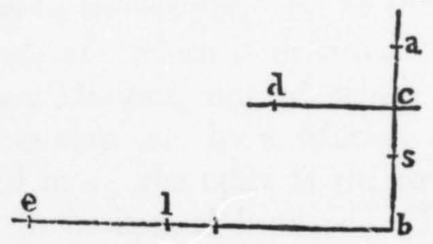
* See *Poſtulat.* before Prop. I. of Dial. III.
† Dial. III. Prop. III.

Salv.

SALV. The Author proceeds to ſhew us what happens concerning the *Impetus* of any Moveable, which is mov'd, in like manner, by a Motion compounded of two, *viz.* the one horizontal and equable, the other perpendicular, but naturally accelerated, of which, in fine, the Motion of the Project is compounded, and thence the parabolic Line is deſcribed: In every Point of which he endeavours to determine, what the *Impetus* of the Project is: For our Underſtanding whereof, the Author ſhews us the Manner or Method of regulating and meaſuring ſuch *Impetus* upon that Line along which the Motion of a Body departing from Reſt, and falling with a Motion naturally accelerate, is made, as follows.

PROP. III. THEOR. III.

Let a Motion be made along the Line *a b* from Reſt in *a*, and take in it any Point as *c*; and ſuppoſe *a c* to be



the Time or the Meaſure of the Time of the ſaid Fall thro' the Space *a c*, as alſo the Meaſure of the *Impetus* or *Momentum* in the Point *c* acquired by the Fall thro' *a c*: Now take in the ſame Line *a b*, any other Point,

Point, as *b*. In which we are to determine concerning the *Impetus* acquired by the Moveable by its Fall *a b*, in Proportion to the *Impetus* which it had in *c*, whoſe Meaſure is ſuppoſed to be *a c*.

Let *a s* be a mean Proportional between *b a* and *a c*. Now we'll demonſtrate that the *Impetus* in *b* to the *Impetus* in *c* is as the Line *s a* to *a c*. Let the horizontal Line *c d* be double *a c*, and *b e* the double of *a b*: Then, from what has been before proved, the Cadent through *a c* turning along the horizontal Line *c d*, and according to the *Impetus* acquired in *c*, carry'd with an equable Motion, paſſes the Space *c d* in the ſame Time as it paſſed thro' *a c*, with an accelerate Motion; and in like Manner *b e* is paſſed in the ſame Time as *a b*. But the Time of the Fall thro' *a b* is *a s*, therefore the horizontal Line *b e* is paſſed in the Time *a s*.

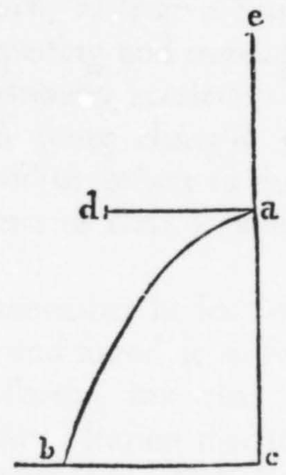
Now as the Time *s a* is to the Time *a c*, ſo let *e b* be to *b l*: And ſince the Motion thro' *b e* is equable, the Space *b l* ſhall be paſſed in the Time *a c*, according to the *Momentum* of Velocity in *b*. But in the ſame Time *a c* the Space *c d* is paſſed according to the *Momentum* of Velocity in *c*, but the *Momenta* of Velocity are one to another as the Spaces which, according to thoſe *Momenta* are paſſed in the ſame Time; therefore the *Momentum* of Velocity in *c* is to the *Momentum* of Velocity in *b* as *d c* is to *b l*. But becauſe as *d c* is to *b e*, ſo are their halfs, *viz.* *c a* and *a b*; but as *e b* is to *b l* ſo is *b a* to *a s*, therefore, *ex æquali*, as *d c* : *b l* : : *c a* : *a s*, that is, as the *Momentum* of Velocity in *c* is to the *Momentum* of Velocity in *b*, ſo is *c a* to *a s*; *i. e.* the Time along *c a* to the Time along *a b*. The Manner therefore of meaſuring the *Impetus* or

the *Momentum* of Velocity upon a Line wherein the Motion of Deſcent is made, is manifeſt; which *Impetus* is indeed ſuppoſed to encreaſe according to the Proportion of the Time.

But here, before we proceed, we muſt pre-adviſe, that ſince hereafter we are to ſpeak of a Motion compounded of an equable horizontal one, and of a naturally accelerate one downwards (for from this Compoſition reſults, and is made the Line of the *Project*, that is a *Parabola*;) it is neceſſary that we define ſome common Meaſure, according to which we may meaſure the Velocity, *Impetus*, or *Momentum* of both Motions.

And ſince the Degrees of Velocity of the equable Motion are innumerable, of which not any one at Pleaſure may be taken; but one certain one only is to be compared and conjoined to the Degree of Velocity acquired by the Motion naturally accelerate; I can think of no eaſier Way for that Choice and Determination, than by aſſuming another of the ſame Kind. But to explain myſelf the better; Let *a c* be perpendicular to the horizontal Line *c b*; let *a c* be the Height, and *c b* the Amplitude of the *Semi-parabola a b*, which is deſcrib'd by the Compoſition of the two Motions, one of which is that of the Moveable falling thro' *a c*, by a Motion naturally accelerate from Reſt in *a*; the other is the equable tranſverſe Motion along the horizontal Line *a d*. The *Impetus* acquired in *e*, by the Deſcent *a c*, is determined by the Quantity of the ſaid Height *a c*; for the *Impetus* of a Moveable falling from the ſame Height, is always one and the ſame: but in the horizontal Line, not only one, but innumerable Degrees of Velocities of equable Motions may

may be aſſigned, out of which Number, that I may ſingle or point out that for my Purpoſe, I extend the Altitude *c a* upwards, as was ſhewn before, and as is neceſſary: I'll ſettle the Height *a e*; from which if I conceive in my Mind a Moveable to fall *ex quiete* in *e*, it will appear,



that its *Impetus* acquired at the Point *a*, is the ſame wherewith I conceive the ſame Moveable turning along the horizontal Line *a d* to be carried; and its Degree of Velocity to be that, with which, in the Time of the Deſcent thro' *e a*, it will paſs a Space in the horizontal Line double to the ſaid *e a*. This Premonition I judg'd neceſſary.

You muſt know alſo, that I call the horizontal Line *c b* the Amplitude of the *Parabola*; the Axis the Altitude of the ſaid *Parabola*; and the Line *e a*, by whoſe Deſcent the horizontal *Impetus* is determined, I call the Sublimity. Theſe Things declar'd and defin'd, I proceed to the Demonſtration.

SAGR. First give me leave to take Notice, that this Thought of our Author's is like that of *Plato*'s, concerning the Determination of the different Velocities of the equable Motions, in the Revolutions of the different Celeſtial Bodies; who, peradventure, thinking that no Moveable could paſs from Reſt to any determinate Degree of Velocity, in which it ought afterwards perpetually and equably to move, unleſs it firſt paſs thro' all the other leſſer Degrees of Velocities, or, if you will, greater Degrees of Tardity, which are between the aſſign'd Degree of Velocity, and the higheſt Degree of Tardity, that is, *Reſt*; ſaid, that God, after having created the moveable Celeſtial Bodies, in order to aſſign them thoſe Velocities wherewith they were afterwards perpetually to move with an equable circular Motion, made them, departing from Reſt, to move thro' determinate Spaces with that natural Motion, and in a right Line, according to which we ſenſibly perceive our Moveables to move from a State of Reſt ſucceſſively accelerating; and adds, that having made them acquire that Degree of Velocity which it pleaſed him, ſince they were afterwards perpetually to preſerve it, he turn'd their direct Motion into a circular one; the only one fit to keep equable, always revolving, without receding from or approaching to any prefix'd Term by them ſought. This Conceit is truly worthy of *Plato*; and is the more to be praiſed, in that the Foundations or Grounds by *Plato* ſilently paſſed by, and diſcover'd by our Author, taking off the Mask or Poetic Repreſentation, do clearly ſhew it as a true Account. And I can't but believe, ſince by the Doctrine of Aſtronomy we have a competent Knowledge

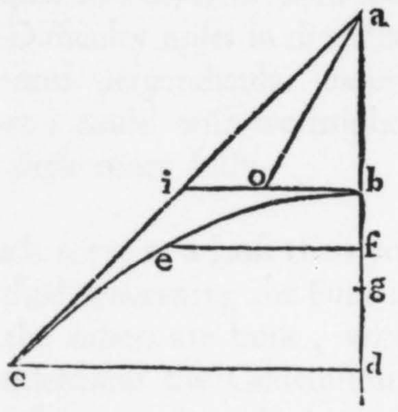
Knowledge of the Magnitudes of the Orbs of the Planets, and of their Diſtances from the Centre about which they turn, as alſo of their Velocities; that our Author (who was no Stranger to *Plato*'s Conjecture) out of Curioſity, may ſometimes have had ſome Thought of attempting to inveſtigate, whether any determinate Height could be aſſigned, from which, as from a State of Reſt, the Bodies of the Planets, departing and moving thro' certain Spaces with a right and naturally accelerate Motion, the acquir'd Velocity afterwards being chang'd into an equable one, they would be found to anſwer to the Magnitudes of their Orbs, and the Times of their Revolutions.

SALV. I now remember he has told me, that he made the Computation, and found it anſwer the Obſervations with ſufficient Exactneſs; but that he had no Mind to publiſh his Thoughts, fearing that the too many Novelties diſcover'd by him, by which he had already drawn the *Odium* of many upon him, might raiſe a new Flame. But if any one have the like Deſire, he may of himſelf, with the Inſtructions the preſent Tract will afford him, be fully ſatisfied. But to return to our Buſineſs, which is to demonſtrate

PROP. IV.

PROP. IV. PROB. I.

How to determine the *Impetus* in every Point of a given *parabolic* Curve deſcrib'd by a *Project*. Let *bec* be a *Semi-parabola*, whoſe Amplitude is *cd*, and Altitude *db*, which produced upwards, meets the Tangent of the *Parabola c a* in *a*; Thro' the *Vertex b*, draw *bi* parallel to



the horizontal Line *cd*. Now if the Amplitude *cd* be = the whole Height *da*, *bi* ſhall be = *ba*, as alſo to *bd*. And if we put *ab* for the Meaſure of the Time of the Fall thro' *ab*, and of the *Momentum* of Velocity acquir'd in *b*, by the Deſcent *ab* from Reſt in *a*, then *dc* (*viz.* the Double of *bi*) will be the Space which it ſhall paſs in the ſame Time, by the *Impetus ab*, when turned along the horizontal Line *bi*: But a Cadent thro' *bd*, from Reſt in *b*, paſſes the Depth *bd*; in the ſame Time, therefore, a Moveable falling from Reſt in *a*, thro' *ab*, with the *Impetus ab* being turned along the horizontal Parallel *b i*, ſhall

ſhall paſs a Space = *dc*; and the Fall along *bd* ſupervening, it paſſes the Depth *bd* by theſe two Motions, one horizontal the Diſtance of *cd*, the other perpendicular the Diſtance *bd*, in the ſame Time is deſcrib'd the *Parabola bc*, whoſe *Impetus* in the Term *c* is compounded of the equable Tranſverſe, whoſe *Momentum* is *ab*, and of another *Momentum* acquired in the Fall *bd*, in the Term *d* or *c*, which *Momenta* are equal. If therefore we ſuppoſe *ab* to be the Meaſure of one of them, as for Inſtance, of the tranſverſe equable one; and *bi*, which is = *bd*, to be the Meaſure of the *Impetus* acquired in *d* or *c*; then the Subtenſe *ia* will be the Quantity of the *Momentum* compounded of both: It therefore will be the Quantity or Meaſure of the whole *Momentum* which the Project, deſcribing the *Parabola bc*, ſtrikes with in *c*. Theſe Things being premiſed, take in the *Parabola* any Point *e*, in which we are to determine the *Impetus* of the Project. Draw the horizontal Line *ef*, and take *bg* a mean Proportional between *bd* and *bf*: Then ſince *ab* or *bd* is ſuppoſed to be the Meaſure of Time, and of the *Momentum* of Velocity in the Fall *bd*, from Reſt in *b*, *bg* will be the Time or Meaſure of the Time, and of the *Impetus* in *f*, coming from *b*. If therefore *bo* be ſuppoſed = *bg*; then a Line drawn from *a* to *o* will be the Quantity of the *Impetus* in the Point *e*; for *ab* is the ſuppoſed Determiner of the Time, and of the *Impetus* in *b*, which turn'd along the horizontal Parallel, always continues the ſame: But *bo* determines the *Impetus* in *f* or *e* thro' the Deſcent *bf*, from Reſt in *b*; but *ao* is in Power = theſe two *ab* and *bo*: Therefore what was ſought is manifeſt.

SAGR. The

SAGR. The Contemplation of the Compoſition of theſe different Forces, and of the Quantity of that *Impetus* which reſults from this Mixture, is ſo new to me, that I'm confounded. I don't ſpeak of the Mixture of two equable Motions, tho' unequal to one another, one of which is made along the horizontal Line, and the other along the perpendicular one; for I eaſily perceive that thence there ariſes a Motion, equal in Power to both the compounding Motions: But my Difficulty ariſes in the Mixture of the horizontal, equable, and perpendicular naturally accelerate Motion: Wherefore I could wiſh we might together conſider this Matter a little more fully.

SIMP. I am much more at a Loſs than you are, for I'm not yet wholly ſatisfied concerning the Fundamental Propoſitions on which the others are built; what I mean is, I want better to underſtand the Concluſion of the ſecond Propoſition, *viz.* what, in the Mixture of two equable Motions, the one horizontal, the other perpendicular, is meant by the *Potentia* of their Compound.

SALV. Your Requeſt is very reaſonable, and I'll endeavour to explain this to your Satisfaction; but you muſt excuſe my repeating ſeveral Things already ſaid. And to diſcourſe determinately about Motions, and their Velocities or *Impetus's*, whether they be equable or naturally accelerate, we muſt firſt pitch upon a Meaſure to be uſed in the Dimenſions of ſuch Velocities; as alſo a Meaſure of the Time. As to the Meaſure of the Time, we have what all agree in, I mean Hours, Minutes, Seconds,

Seconds, *&c.* Now as this Meaſure is received in common by all, ſo we muſt allot one to the Velocities, that is commonly underſtood and received by all, *i. e.* which is the very ſame every where. The Author, as has been ſaid, thought the Velocity of heavy Bodies naturally deſcending fit for this Purpoſe, whoſe increaſing Velocities obſerve the ſame Laws in all Parts of the World: So that that Degree of Velocity, which (*e. g.*) a Leaden Ball of one Pound Weight departing from Reſt, hath acquired in a Deſcent perpendicularly a certain Diſtance, is always and in all Places the ſame, and therefore moſt commodious to explain the Quantity of the *Impetus* ariſing from a natural Deſcent: It remains then, that we find out a Manner of determining alſo the Quantity of the *Impetus* in an equable Motion, and that in ſuch a Manner, that all they who reaſon about it may form the very ſame Conception of both its Magnitude and Velocity; and that one mayn't imagine it ſwifter, and the other ſlower, which may occaſion afterwards, in compounding and mingling this equable Motion, imagin'd by them, with the eſtabliſh'd accelerate Motion, that different Men may form different Conceptions of different Greatneſſes of Forces.

To determine and exhibit this *Impetus* and particular Velocity, our Author thought it the beſt way to make Uſe of the *Impetus* which the Moveable acquires, in a Motion naturally accelerate, whoſe every acquired *Momentum*, turn'd into an equable Motion retaining its Velocity exactly limited, and ſuch, that in an equal Time to that wherein it did deſcend, it paſſes thro' double the Space of the Height from which it fell: But becauſe this is the main Article in the Buſineſs we are upon, it may be

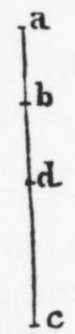
worth while, that you may ſee it the clearer, to explain it by ſome particular Example.

Re-aſſuming therefore the Velocity and *Impetus* which a heavy Body, as we have ſaid, hath acquired falling a certain Diſtance; which Velocity we will make uſe of for a Meaſure of other Velocities and *Impetus*'s upon other Occaſions; and ſuppoſing, for Example, the Time of its Fall to be four Seconds: Now to find, from this Meaſure, what is the *Impetus* of the falling Body from any greater or leſs Height, we muſt not argue and conclude the Quantity of the *Impetus* acquired in falling from this ſecond Height, from the *Ratio* the former Height had to the ſtated Diſtance, *i. e.* we muſt not ſuppoſe that a Moveable falling from four times that Height, muſt have acquired four times the Velocity; for this is falſe, becauſe in a Motion naturally accelerate, the Velocity does not increaſe or diminiſh according to the *Ratio* of the Spaces, but according to that of the Times, than which that of the Spaces is greater in a duplicate Proportion, as was before demonſtrated.

Wherefore if in a right Line we aſſign any Part for the Meaſure of the Velocity, and of the Time, as alſo of the Space run thro' in that Time, (for all theſe three Magnitudes, for Brevity's Sake, are frequently repreſented by one and the ſelf-ſame Line) to find the Quantity of the Time and Degree of Velocity which the ſame Moveable hath acquir'd in another Diſtance, which we ſhall obtain not immediately from this ſecond Diſtance, but from that Line which is a mean Proportional between thoſe two Diſtances: An Example will make this clearer.

In

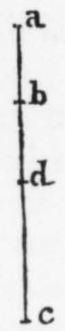
In the Line *ac*, perpendicular to the Horizon, ſuppoſe the Part *ab* to be the Space run thro' by a heavy Body naturally deſcending with an accelerate Motion; the Time of which Paſſage, ſince I may repreſent it by any Line, for Brevity's Sake ſhall be repreſented by the ſame Line *ab*, which in like manner I take for the Meaſure of the *Impetus* and Velocity acquir'd by ſuch Motion, ſo that the Part *ab* may be the Meaſure of all the Spaces that are in the Progreſs of the Diſcourſe to be conſider'd. Having now at Pleaſure eſtabliſh'd, under one ſole Magnitude *ab*, the



Meaſures of three Quantities of very different Kinds, *i.e.* of Spaces, Times, and Forces; let it be required to determine in the aſſign'd Space, and at the Height *ac*, what muſt be the Time of the Deſcent of the Moveable from *a* to *c*, and what the *Impetus* it ſhall have acquired at the Term *c*, in relation to the Time and to the *Impetus* which are meaſured by *ab*.

Both theſe Things required, will be determined by taking a mean Proportional *ad*, between the two Lines *ac* and *ab*, and affirming that the Time of the Fall thro' the whole Space *ac*, is ſuch as is the Time *ad*, in relation to the Time *ab*,

which at the Beginning was taken for the Quantity of the Time in the Deſcent *ab*: And likewiſe we will ſay that the *Impetus* or the Degree of Velocity which the falling Body will acquire at the Term *c*, in relation to the *Impetus* it had in *b*, is ſuch as is the ſame Line *ad*, in relation to



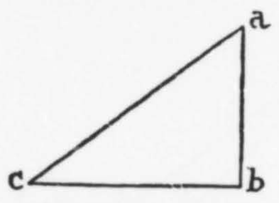
ab, ſince the Velocity increaſes in the ſame *Ratio* the Time does; of which Concluſion (altho' it was aſſumed as a *Poſtulatum*, yet) the Author was pleaſed to explain the Application above in Prop. III.

This Point being well underſtood and eſtabliſhed, we paſs to the Conſideration of the *Impetus* ariſing from two compound Motions; one whereof is compounded of a horizontal and always equable one, and of one perpendicular to the Horizon, and that alſo equable; and the other compounded of an horizontal one, in like manner always equable, and of a perpendicular one naturally accelerate.

We have already ſeen, that when both are equable, that that which ariſes from their Compoſition, is equal in *Power* to both; which, for Clearneſs Sake, we will give an Example of:

Suppoſe

Suppoſe a Moveable falling along the perpendicular Line *ab*, to have, *e. g.* three Degrees of equable *Impetus*; but if it was carried along the Line *bc* towards *c*, then the Velocity and *Impetus* to be four Degrees: So that in the ſame Time that falling it would paſs along the Perpendicular, *v. g.* three Yards, it would in the Horizontal paſs four; but in the Motion compounded of both, it will come in the ſame Time from the Point *a* to the Point *c*, proceeding always along the Diagonal *ac*, which is not ſeven Yards long, as it ſhould be, if compounded of the two



Lines *ab*, 3, and *bc*, 4; but in five Yards long, which in Power is = both 3 and 4; becauſe the Squares of 3 and 4, which are 9 and 16, added together, make 25 = the Square of *ac*, which is = the two Squares of *ab* and *bc*: Whence *ac* ſhall be as much as is the Side, or, which is the ſame Thing, the Root of the Square 25, which is 5: For a conſtant and certain Rule therefore, when we would aſſign the *Impetus* reſulting from two *Impetus*'s given, *viz.* the one Horizontal, and the other Perpendicular, but both equable, their Squares muſt be taken, and from their Sum the Square-Root muſt be extracted: And thus, in the propoſed Example, that Moveable which by the Force of its perpendicular Motion ſtrikes the Horizon with three Degrees of Power, and by its horizontal Motion ſhall

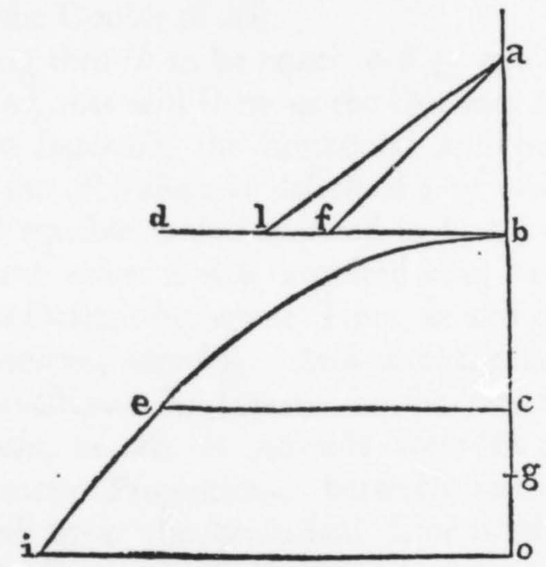
ſtrike

ſtrike in *c* with four Degrees of Power. If now both Strokes are ſtruck together, the Blow ſhall be like that made by a ſtriking Moveable, which was mov'd with five Degrees of Velocity and Power: And this Percuſſion in all Points of the Diagonal would be the ſame, becauſe the compound *Impetus*'s, without any Increaſe or Decreaſe, are always the ſame.

Now let us ſee what will happen in a Compoſition of an equable horizontal Motion, with a Motion perpendicular to the Horizon, which beginning its Fall from Reſt, is conſtantly naturally accelerated: Now it is manifeſt that the Diagonal, which is the Line of Motion compounded of theſe two, is not a right Line, but a *Semiparabolic* one, as was before demonſtrated; in which, becauſe of the continual Increaſe of the Velocity of the perpendicular Motion, the *Impetus* continually increaſes: Wherefore to determine what the *Impetus* in any aſſign'd Point of that parabolic Diagonal is, we muſt firſt aſſign the Quantity of the uniform horizontal *Impetus*, and then inveſtigate what is the *Impetus* of the falling Body in the Point aſſign'd; which can't be determined without the Conſideration of the Time ſpent from the Beginning of the Compoſition of thoſe two Motions; which Conſideration of the Time is not required in the Compoſition of the equable Motions, whoſe Velocities and *Impetus*'s are always the ſame: But here, where is inſerted into the Compoſition a Motion, which, taking its Beginning from the greateſt Tardity, conſtantly increaſes its Velocity, according to the Continuation of the Time, the Quantity of the Time muſt of Neceſſity be taken in, to ſhew us the Quantity of the Degree of Velocity in the aſſign'd Point, becauſe

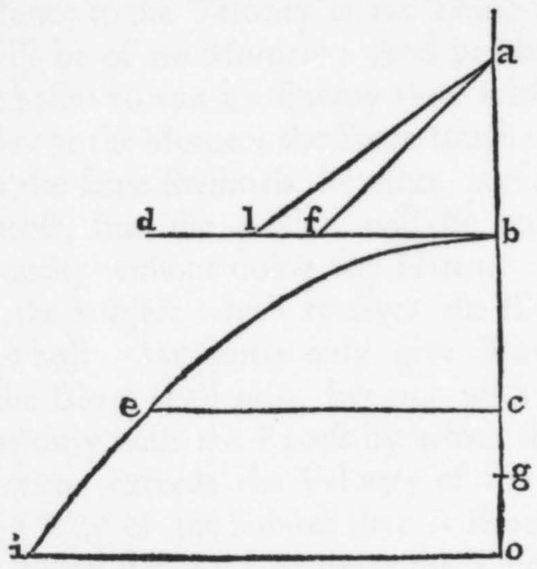
becauſe as to the Reſt, the *Impetus* compounded of theſe two, (as in uniform Motions) is equal in *Power* to both the others compounding.

But this alſo may be better explain'd by an Example: In a Line *ac* perpendicular to the Horizon, take any Part *ab*, which let repreſent the Meaſure of the Space run thro' with a natural Motion in that Perpendicular; let the ſame Line *ab* repreſent likewiſe the Meaſure of the Time, as alſo of the Degree of Velocity, or rather of the *Impetus*.



Firſt; it is manifeſt that if the *Impetus* of the Moveable in *b*, from Reſt in *a*, be turn'd along *bd*, parallel to the Horizon, with an equable Motion, the Quantity of the Velocity ſhall be ſuch, that in the Time *ab* it ſhall paſs a Space double to the Space *ab*, which let be the Line *bd*. Then ſuppoſing $bc = ba$, and drawing $ce \parallel bd$, and $= it$, thro'

thro' the Points *b* and *e* deſcribe the parabolic Line *be*; and becauſe in the Time *ab*, with the *Impetus ab*, the horizontal Line *bd* or *ce*, double to *ab*, is run thro', and in another ſuch Time the Perpendicular *bc* is paſſed with an Acquiſition of *Impetus* in *c* = the ſaid horizontal Line; Therefore the Moveable in a Time = *ab*, ſhall be carried from *b* to *e*, along the *parabolic* Line *be*, with an *Impetus* compounded of two, each of which is = the *Impetus ab*: And becauſe one of them is horizontal, and the other



perpendicular, the compound *Impetus* ſhall be in *Power* = them both, *i. e.* ſhall be the Double of one of them.

Whence ſuppoſing *bf* = *ba*, and drawing the Diagonal *af*, the *Impetus* and Percuſſion in *e* of the Moveable falling from the Height *a*, ſhall be greater than the Percuſſion

cuſſion in *b*, or than the Percuſſion of the horizontal *Impetus* along the Line *bd*, according to the *Ratio* of *af* to *ab*.

But if ſtill retaining *ba* for the Meaſure of the Space of the Fall from Reſt in *a* unto *b*, and for the Meaſure of the Time and *Impetus* of the falling Body acquired in *q*, the Altitude *bo* ſhould not be equal to, but greater than *ba*, take *bg* a mean Proportional between *ab* and *bo*; this *bg* would be the Meaſure of the Time, and of the *Impetus* in *o*, which the Moveable falling thro' *bo* hath acquired in *o*; and the Space along the horizontal Line, which is run thro' with the *Impetus ab* in the Time *ab*, would be the Double of *ab*.

Suppoſing then *lb* to be equal to *bg*, and drawing the Diagonal *al*, that will ſhew us the Quantity compounded of the two *Impetus's*, the horizontal and perpendicular, by which the *Parabola* is deſcribed; of which the horizontal and equable is that acquired in *b*, by the Deſcent *ab*; and the other is that acquired in *o*, or if you will in *i*, by the Deſcent *bo*, whoſe Time, as alſo the Quantity of its *Momentum*, was *bg*. And after the ſame Manner we may inveſtigate the *Impetus* in the extreme Term of the *Parabola*, in caſe its Altitude were leſs than *ab*, by taking a mean Proportional between them both, and ſetting it off upon the horizontal Line from *b* to *f*, and drawing the Diagonal *af*, for this *af* gives us the *Impetus* in the extreme Term of the *Parabola*.

To what has been hitherto ſaid concerning *Impetus's*, Blows, or Percuſſions of theſe Projects, I muſt add one very neceſſary Conſideration more, and that is, That it is not enough to have Regard to the Velocity only of *Projects*, in order juſtly to determine the Power and Force of

the Blow; but we muſt alſo conſider apart the State and Condition of the Subject which receives the Blow; in the Efficacy of which on many Accounts it is greatly concerned.

And firſt; there is no one but knows, that the Thing ſtruck doth ſo much ſuffer by the Velocity of the Thing ſtriking, by how much it oppoſes itſelf to it, or reſiſts it, and either totally or partially ſtops its Motion: That if the Blow happen upon ſuch a Thing as gives Way without any Reſiſtance to the Velocity of the Thing ſtriking; ſuch a Blow will be of no Moment: And in like Manner that if any one haſtes to run his Enemy thro' with a Spear, if it happens that at the Moment the Spear touches him, he gives back with the ſame Swiftneſs the other purſues, there will be no Thruſt, but the Action will be only a mere or ſimple Touch, without doing any Harm.

But if the Subject which receives the Blow be ſuch as does not wholly, but partly only give Way to the Percutient, the Blow ſhall hurt, but not with its whole *Impetus*, but only with the Exceſs by which the Velocity of the Percutient exceeds the Velocity of the Retrogreſſion and giving Way of the Subject that is ſtruck: So that if, *e. g.* the Thing ſtriking falls upon the Thing ſtruck with ten Degrees of Velocity, and this gives Way with four Degrees; the *Impetus* and Percuſſion ſhall be the ſame as if it had been made with ſix Degrees. And, laſtly, the Percuſſion will be entire, and the greateſt, on the Part of the Percutient, when the Thing ſtruck gives no Way at all, but wholly oppoſes and ſtops the whole Motion of the Percutient, if ſuch a Caſe can happen: I ſaid *on the Part of the Percutient*, becauſe when the Thing ſtruck is mov'd

mov'd with a contrary Motion towards the Percutient, the Blow and Shock at meeting ſhall be ſo much the greater, as thoſe two contrary Velocities taken together are greater than the ſingle Velocity of the Percutient.

'Tis neceſſary to remark further, that that greater or leſſer Ceſſion may ariſe, not only from the Quality of the Matter, more or leſs hard, as if it be of Iron, of Lead, or of Wooll, *&c.* but alſo from the Poſition of the Body which receives the Blow; if the Body be ſo placed that the Percutient hits it at right Angles, the Blow will be the greateſt of all: but if the Percutient ſtrikes it obliquely, the Blow will be weaker, and that ſtill more and more weak, according as it is ſtruck more and more obliquely; becauſe an Object, altho' of the ſolideſt Matter, ſo ſituate, doth not ſtop or deſtroy the whole *Impetus* and Motion of the Percutient, which ſlanting paſſes on after having continued its Motion along, at leaſt, ſome Part of the Surface of the reſiſting oppoſed Body. When therefore, the Magnitude of the *Impetus* of the *Project* at the Extremity of the parabolic Line was above determined, it ought to be underſtood of the Percuſſion received upon a Line at right Angles to the parabolic one, or to its Tangent in the aforeſaid Point: For altho' that Motion is compounded of an horizontal and a perpendicular one, yet the *Impetus* is not the greateſt, either upon the horizontal Plane, or upon that perpendicular to the Horizon, being received upon them both obliquely.

SAGR. Your mentioning theſe Blows and Percuſſions puts me in mind of a Problem, or, if you will, Queſtion in Mechanics, a Solution of which I have never met in

any Author, nor any thing to leſſen my Admiration, or in the leaſt ſatisfy my Judgment about it.——The Thing is this; I cannot conceive whence it happens, and on what Principles that Energy and immenſe Force depends, which is found in Percuſſion, whilſt with a ſimple Blow of a Hammer of not above eight or ten Pounds Weight, we ſee ſuch Reſiſtances overcome, as would not yield to the Weight of any heavy Body that, without Percuſſion, by only bearing upon it and preſſing it, hath an *Impetus*, altho' this Body be heavier than that many hundred times. I would likewiſe find a Way to meaſure the Force of this Percuſſion, which I do not think to be infinite, but believe it is limited, and thence may be compared with, and at length brought under the Laws relating to other Powers of preſſing Gravities, either of Leavers, or Screws, or other Mechanic Inſtruments, of the Multiplication of whoſe Force I am fully ſatisfied.

Salv. You are not the only one that does not ſee the Reaſon of this wonderful Effect and ſtupendous Accident. I conſider'd of it for ſome Time, but in vain, my Confuſion conſtantly increaſing; 'till at length meeting with our Academic, I received double Pleaſure; firſt in finding that he himſelf was a long Time at the ſame Loſs; and next in hearing him ſay, that after he had ſpent ſome thouſands of Hours in ſtudying and contemplating thereon, he had hit upon ſome Things very different from our firſt Conceptions relating to that Subject; Things wholly new, and on that Account worthy our Eſteem.

And

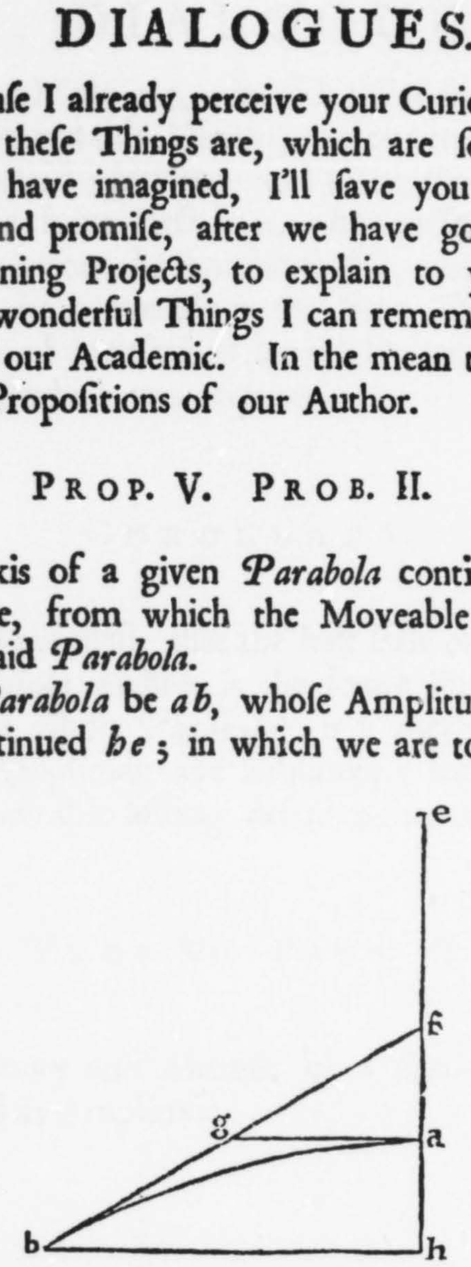
t

And becauſe I already perceive your Curioſity is mov'd to hear what theſe Things are, which are ſo far beyond what I could have imagined, I'll ſave you the Trouble of asking; and promiſe, after we have gone thro' this Tract concerning Projects, to explain to you all thoſe pleaſant but wonderful Things I can remember from the Diſcourſes of our Academic. In the mean time let us go on with the Propoſitions of our Author.

Prop. V. Prob. II.

In the Axis of a given *Parabola* continued, to find a Point above, from which the Moveable falling, ſhall deſcribe the ſaid *Parabola*.

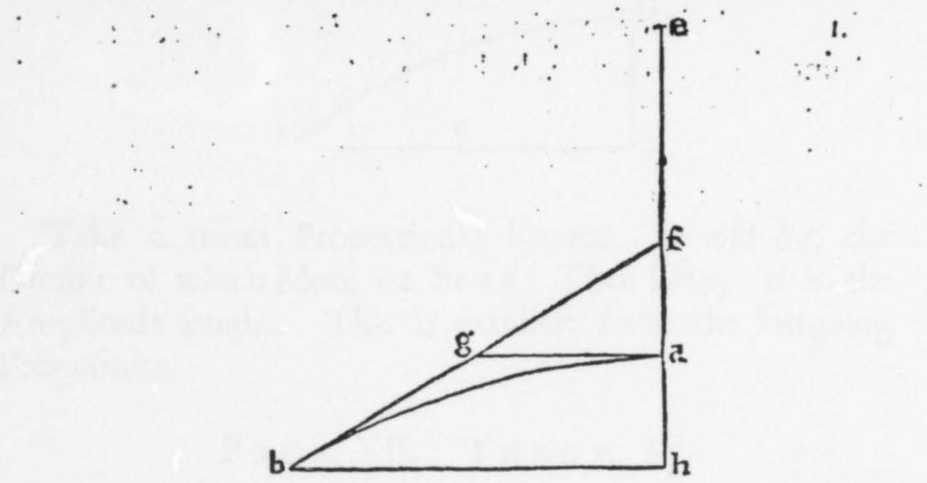
Let the *Parabola* be *ab*, whoſe Amplitude is *hb*, and its Axis continued *he*; in which we are to find a Point



on high, from which the Moveable falling, and turning the *Impetus* gained in *a*, along the horizontal Line, may deſcribe the *Parabola ab*.

Draw

Draw the horizontal Line ag, which ſhall be $\parallel bh$, and ſuppoſing $af = ah$, draw the right Line fb, which will touch the *Parabola* in b, and cut the horizontal Line ag in g; to fa and ag take ae a third Proportional, *i. e.* let ae be made of ſuch a Length as that ag may be a mean Proportional between it and af: Then I ſay, that e is the Point above ſought, from which a Moveable falling from Reſt in e, and turning the *Impetus* gain'd in a along the horizontal Line, the *Impetus* of the Deſcent in h from Reſt in a joining it, ſhall deſcribe the *Parabola* ab.



For if we ſuppoſe ea to be the Meaſure of the Deſcent from e to a, and alſo of the *Impetus* acquired in a, ag (*viz.* a Mean between ea and af) will be its Time and *Impetus* coming from f to a, or from a to h: And becauſe falling from e, in the Time ea, with the *Impetus* acquired in a, it moves in its horizontal Paſſage with an equable Motion the double of ea; being carried, therefore, with the ſame *Impetus*, it will paſs in the Time ag, the

the double the Mean ag, *viz.* bh: For the Spaces paſſed with the ſame equable Motion, are one to another as the Times of the ſaid Motions; and along the Perpendicular, with a Motion from Reſt in a, ah is paſſed in the ſame Time ga, therefore the Amplitude bh, and Altitude ha, are paſſed by the Moveable in the ſame Time; therefore the *Parabola* ab is deſcribed by the Deſcent of the falling Body from e, which was required.

Corollary.

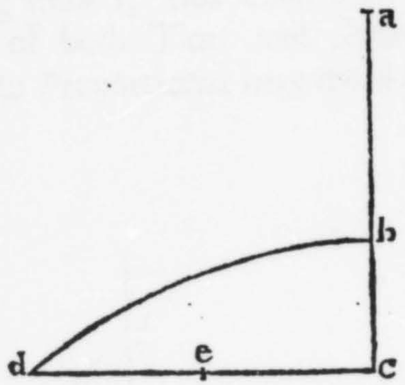
Hence it is manifeſt, that the half Baſe or Amplitude of the *Semi-parabola*, (which is the fourth Part of the Amplitude of the whole *Parabola*) is a mean Proportional between its Amplitude and Sublimity; from which Sublimity the Moveable falling, deſcribes that *Semi-parabola*.

Prop. VI. Prob. III.

The Sublimity and Altitude of a *Semi-parabola* being given, to find its Amplitude.

To

To the horizontal Line *dc*, let *ac* be perpendicular, in which let the Altitude *cb* be given, and the Sublimity *ba*: Now we are to find in the horizontal Line *cd*, the Amplitude of the *Semi-parabola*, which is deſcribed from the Sublimity *ba*, and the Altitude *bc*.



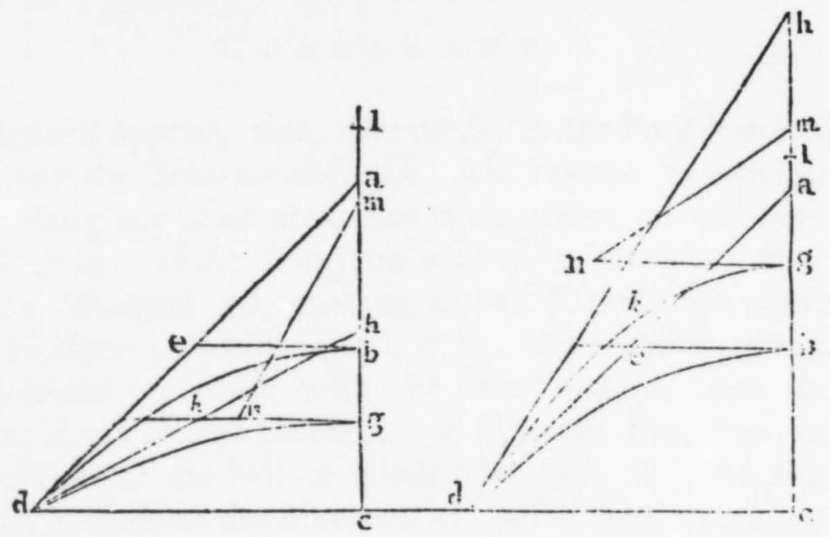
Take a mean Proportional betwee. *bc* and *ba*, the Double of which Mean let be *cd*: Then I ſay, *cd* is the Amplitude ſought. This is manifeſt from the foregoing Propoſition.

PROP. VII. THEOR. IV.

In Projects by which *Semi-parabola's* of the ſame Amplitude are deſcribed; there is leſs *Impetus* required in that Project which deſcribes that whoſe Amplitude is double its Altitude, than in any other.

Let

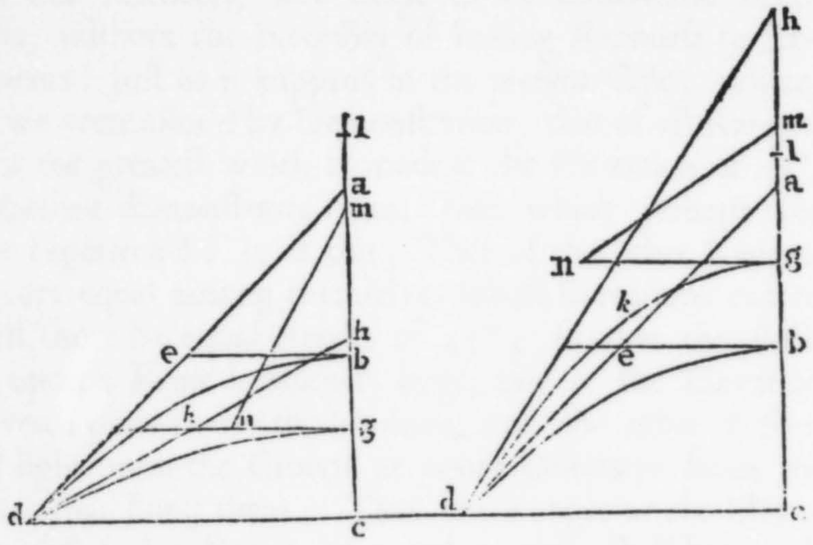
Let the *Semi-parabola* be *bd*, whoſe Amplitude *cd* is double its Altitude *cb*, and in its Axis continu'd upwards let *ba* be ſuppoſed = the Altitude *be*, and draw the Line *ad*, which will touch the *Semi-parabola* in *d*, and cut the horizontal Line *be* in *e*; and *be* will be = *bc*, or to *ba*. It is manifeſt this *Semi-parabola* is deſcrib'd by a Project, whoſe equable horizontal *Impetus* is ſuch as is in *b* of a Body falling from Reſt in *a*, and the *Impetus* of it



deſcending naturally downwards, ſuch as is that of a Thing coming to *c*, from Reſt in *b*. Whence it is manifeſt, that the *Impetus* compounded of theſe, and that ſtrikes in the Term *d*, is as the Diagonal *ae*, i. e. *in Power* = them both. Now let there be ſome other *Semi-parabola*, as *gd*, whoſe Amplitude *cd* is the ſame as before, but its Altitude *cg* leſs, or greater than the Altitude *bc*, and let *hd* touch the ſame, cutting the horizontal Line drawn thro' *g* in the Point *k*, and make *hg* : *gk* :: *gk* : *gl*;

then from what has been before demonſtrated, gl will be the Sublimity, from which the Project falling ſhall deſcribe the *Parabola* gd.

Let gm be a mean Proportional between ab and gl, gm ſhall be the Time and *Momentum* or *Impetus* in g, of the Body falling from l, (for it has been ſuppoſed that ab is the Meaſure of both Time and *Impetus*). Again, let gn be a mean Proportional between bc and cg, this



gn ſhall be the Meaſure of the Time and the *Impetus* of the Body falling from b to c. If then the Line mn be drawn, it ſhall be the Meaſure of the *Impetus* of the Project along the *Parabola* bd, ſtriking in the Term d, which *Impetus*, I ſay, is greater than the *Impetus* of the Project along the *Parabola* bd, whoſe Quantity was as ae: For becauſe gn is ſuppoſed a Mean between bc and cg, and $bc = be$, that is to bg: (for they are each of them Subduple of dc) it will be $cg : gn :: gn : gk$, and cg or bg :

$bg : gk :: ng_q : gk_q$. But as bg to gk, ſo gk is made to gl; therefore $ng : gk_q :: gk : gl$: But $gk : gl :: gk_q : gm_q$; for gm is a Mean between gk and gl; therefore the three Squares ng, gk, and gm, are continual Proportionals; and the two Extremes ng and gm taken together, *i.e.* the Square of mn, is greater than the double of the Square of gk, to which the Square of ae is double; therefore the Square of mn is greater than the Square of ae; and the Line mn greater than the Line ae. *Q.E.D.*

COROLLARY.

Hence it appears, that, *vice verſa*, in the Project out of d, along the *Semi-parabola* db, leſs *Impetus* is required than along any other of an Elevation greater or leſs than the Elevation of the *Semi-parabola* bd, which is according to the Tangent ad, making an half Right Angle above the Horizon: Which ſince it is ſo, it is manifeſt, that if Projections are made with the ſame *Impetus* from the Term d at different Elevations, the greateſt Projection or Amplitude of the half or whole *Parabola*, will be that which is made at the Elevation of half a Right Angle or 45° and the reſt made at greater or leſs Elevations will be leſs.

SAGR. The Force of neceſſary Demonſtrations is extremely both wonderful and delightful, and ſuch are Mathematical ones only: Before, I took it upon Truſt, from the Accounts I had from ſeveral Gunners, that the greateſt of all the Ranges made by a Cannon or Mortar, *i. e.* that where the Ball is flung fartheſt, is that which is made at

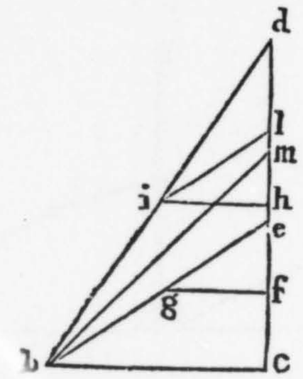
an Elevation of half a Right Angle, or of 45 Degrees, or, as they ſay, of the ſixth Point of the Square: But the underſtanding the Reaſon of this, is far beyond the ſimple Knowledge drawn from the Teſtimony of others, or even from often repeated Experiments.

SALV. You ſay very true; to which I add, that the Knowledge of one ſingle Effect acquir'd by its Cauſes, opens our Intellects, and helps us to underſtand other Effects, without the Neceſſity of having Recourſe to Experiments: juſt as it happens in the preſent Caſe; where, after we were aſſur'd by Demonſtration, that of all Ranges, that is the greateſt which is made at the Elevation of 45°, the Author demonſtrates to us, that which perhaps was never experienc'd: It is this; That of the other Ranges, thoſe are equal among themſelves whoſe Elevations exceed or fall ſhort by equal Angles of 45°; ſo that the Balls ſhot upon a Plane ſufficiently large, one at the Elevation of ſeven Points above the Horizon, and the other of five, ſhall light upon the Ground at equal Diſtances from the Mortar that flung them: Thus the Ranges at the Elevations of 8 and 4 Points, of 9 and 3, *&c.* ſhall be equal. Take the Demonſtration.

PROP. VIII. THEOR. V.

The Amplitudes of *Parabola's* deſcrib'd by Projects flung with the ſame Force at Elevations diſtant by equal Angles above and below 45°, are = each other. Let the horizontal Line *bc*, and the Perpendicular *cm*, about the Right Angle *c* of △ *mcb* be = each other, then will ∠ *mbc*

mbc be half a right one; and producing *cm* to *d*, make at the Point *b* two equal Angles, *mbe* and *mbd*, above and below the Diagonal *bm*. 'Tis to be demonſtrated, that the Amplitudes of the *Parabola's* deſcrib'd by the Projects thrown with the ſame Force from the Term *b*, at

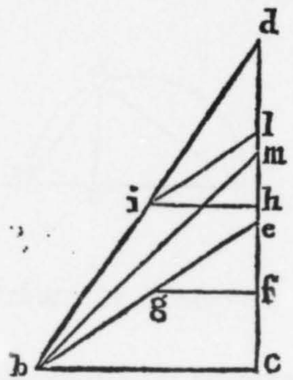


the Elevations of the Angles *ebc* and *dbc*, are equal to each other. Firſt then, becauſe the external Angle *bmc* is = the internal ones *mbd* and *dbm*, ∠ *mbc* will alſo be = them. But if inſtead of ∠ *dbm*, we put *mbe*, the ſame Angle *mbc* will be = the two, *mbe* and *bdc*; and taking away the common Angle *mbe*, the remaining one *bdc* will be = the remaining one *ebc*: Therefore △s *dcb* and *bce* are ſimilar: Biſect *dc* and *ec* equally in *h* and *f*, and draw *hi* and *fg* ‖ the Horizon *cb*, and make *dh*: *hi* :: *hi*: *hl*, then △ *ihl* will be ſimilar to △ *ihd*, to which alſo is ſimilar △ *egf*: And ſince *ih* and *gf* are equal (each being the half of *bc*) *fe*, i. e. *fc*, will be = *hl*; adding the common Line *fh*, *ch* will be = *fl*. If therefore we imagine the *Semi-parabola* to be deſcrib'd by or thro' *h* and *b*, whoſe Altitude is *hc*, and Sublimity *hl*,

its

its Amplitude will be cb, which is double to bi, the Mean between db or cb and bl; and db ſhall be a Tangent to it, the Lines ch and hd being equal.

And if again we conceive the *Parabola* to be deſcrib'd by or thro' f and b, from the Sublimity fl, with the Alti-



tude fc, a mean Proportional between which two is fg, whoſe Double is the horizontal Line cb; cb, as before, ſhall be its Amplitude, and eb a Tangent to it, ſince ef and fc are equal; but ∠'s dbc and ebc (their Elevations) are equally diſtant from a half Right Angle, or 45°. Therefore, *&c.* *Q. E. D.*

PROP. IX. THEOR. VI.

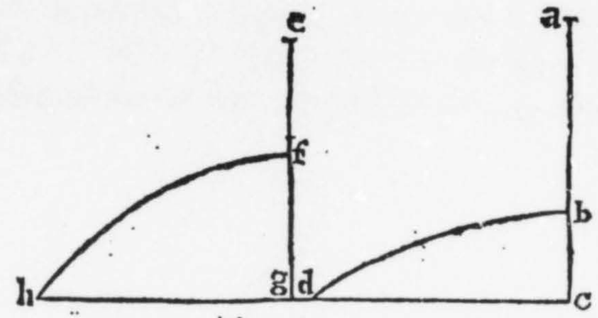
The Amplitudes of *Parabola's*, whoſe Altitudes and Sublimities are reciprocally proportional, are equal.

Let the Height gf of the *Parabola* fh have the ſame Proportion to the Height cb of the *Parabola* bd, as the Sublimity ba has to the Sublimity fe: Then I ſay, the Amplitude hg is = the Amplitude dc; for ſince the firſt

gf

†

gf has the ſame *Ratio* to the ſecond cb, as the third ba has to the fourth fe, the Rectangle gfe of the firſt and fourth will be = ▭ cba of the ſecond and third; therefore the Squares which are = theſe Rectangles will be = each other; but the Square of half gh is = ▭ gfe; and



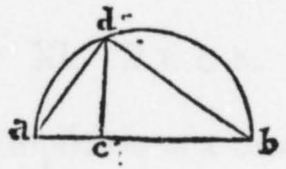
the Square of half cd is = ▭ cba; therefore theſe Squares, and their Sides, and the Doubles of their Sides: But theſe are the Amplitudes gh and cd; therefore the Propoſition is manifeſt.

LEMMA.

If a right Line be cut at Pleaſure, the Squares of the Means between the whole and two Parts are = the Square of the whole.

Let

Let *ab* be cut at Pleaſure in *c*: I ſay the Squares of the Lines, which are mean Proportionals between the Whole *ab*, and the Parts *ac* and *cb* taken together, are equal to the Square of the Whole *ab*. Upon the whole Line *ba*, deſcribe a Semicircle, and from *c* erect a Perpendicular *cd*, and draw the Lines *ad* and *bd*, and the Thing is plain.



For *da* is a mean Proportional betwen *ab* and *ac*, and *db* a Mean between *ab* and *bc*; and the Squares of the Lines *da* and *db* taken together, are = the Square of the whole Line *ab*, ∠ *adb* in the Semicircle, being a right one: therefore, *&c.*

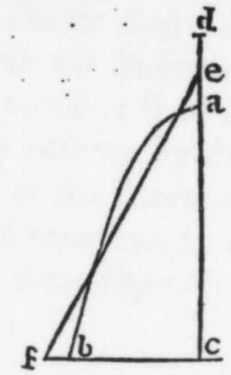
Prop. X. Theor. VII.

The *Impetus* or *Momentum* of any *Semi-parabola* is equal to the *Momentum* of any Moveable falling naturally along ſuch a Perpendicular to the Horizon, as is = the Line compounded of the Sublimity, and of the Altitude of the *Semi-parabola*.

Let

Let the *Semi-Parabola* be *ab*, whoſe Sublimity is *da*, and its Altitude *ac*; by which two is made the Perpendicular *dc*; then, I ſay, the *Impetus* of the *Semi-parabola* in *b* is = the *Momentum* of a Moveable deſcending naturally from *d* to *e*.

Suppoſe *dc* to be the Meaſure of both Time and *Impetus*; and take a mean Proportional between *cd* and *da*; to which let *ef* be equal. Again, let *ce* be a Mean between *dc* and *ca*: Now *fc* ſhall be the Meaſure of the Time and *Momentum* of the Moveable falling thro' *da*,



from Reſt in *d*, but *ce* will be the Time and *Momentum* of the Moveable falling thro' *ac*, from Reſt in *a*. And the Diagonal *ef* will be the *Momentum* compounded of both theſe; that is, that of the *Semi-parabola* in *b*; and becauſe *dc* was cut at Pleaſure in *a*, and *cf* and *ce* are Means between the whole *cd* and the Parts *da* and *ac*, their Squares taken together will be = the Square of the whole; but from the foregoing *Lemma*, the Square of *ef* is alſo = the ſame Squares; therefore the Line *ef* is = *dc*. Whence it is manifeſt, that the *Momenta* along *dc* and along the *Semi-parabola ab*, in *c* and *b* are equal. *Q. E. D.*

Corollary.

Hence it is manifeſt, that of all *Parabolas* whoſe Altitudes join'd with their Sublimities are equal, the *Impetus's* are alſo equal.

Prop XI. Prob. IV.

The *Impetus* and Amplitude of a *Semi-parabola*, being given, to find its Altitude.

Let the *Impetus* given be defined by the Perpendicular to the Horizon *a b* (Fig. 1.) and let the Amplitude in the

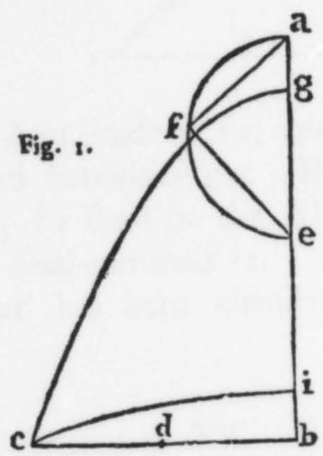


Fig. 1.

horizontal Line be *b c*. It is required to find the Altitude of the *Semi-parabola*, whoſe *Impetus* is *a b* and *Amplitude b c*.

It

It is manifeſt, from what has been already proved, that half the Amplitude *b c* will be a mean Proportional between the Altitude and Sublimity of the ſame *Semi-parabola*, whoſe *Impetus*, by the preceding Propoſition, is the ſame with the *Impetus* of the Moveable falling, from Reſt in *a*, thro' the whole *a b*. Wherefore *a b* is to be ſo cut that the Rectangle contained by its Parts may be = □ of half *b c*, which let be *b d*. Hence it appears to be neceſſary, that *d b* do not exceed the half of *a b*: For of Rectangles contained by the Parts, the greateſt is when the whole Line is biſected equally. Divide then *b a* equally in *e*, and if *b d* be = *b e* (ſee Fig. 2) the Buſineſs is done; and the Altitude of the *Semi-parabola* ſhall be *b e*, and its Sublimity *e a*: (here you may obſerve by the Way) that the Amplitude of a *Parabola* at the Elevation of a half Right Angle or 45°, as before demonſtrated, is the greateſt of all thoſe deſcribed with the ſame *Impetus*). But now ſuppoſe *b d*

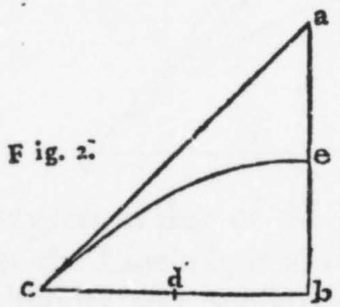
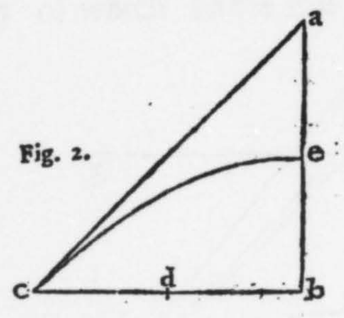


Fig. 2.

to be leſs than the half of *b a*, which *b a* is to be ſo cut that the Rectangle under its Parts be = □ of *b d*: In order to do this, biſect *a b* equally in *e*, and deſcribe the Semicircle *e f a* (ſee Fig. 1.) in the Periphery of which from *a*

lay off $af = bd$ and draw fe, to which make eg equal. Then the Rectangle bga with the Square of eg will be = the Square of ea, by the 6th Prop. of the 2d Book of *Euclid*; to which alſo the two Squares of af and fe are equal: Taking away then the equal Squares of ge and fe, there remains the Rectangle $bga = \square$ of af, *i.e.* to $\square$ of bd; and the Line bd is a mean Proportional between bg and ga; whence it is manifeſt, that of the *Semi-parabola*, whoſe Amplitude is bc, and *Impetus* ab, the Altitude is bg and Sublimity ga. But now if bi be ſet off below $= ga$; bi ſhall be the Altitude, and ia the Sublimity of the *Semi-parabola* ic.

Fig. 2.



From what has been already demonſtrated, we are taught,

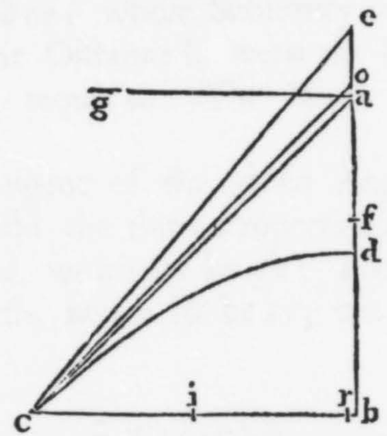
(PROP. XII. PROB. V.)

To calculate the Amplitudes of all *Semi-parabolas* deſcribed by *Projects* thrown with the ſame *Impetus*, and to make Tables thereof.

From

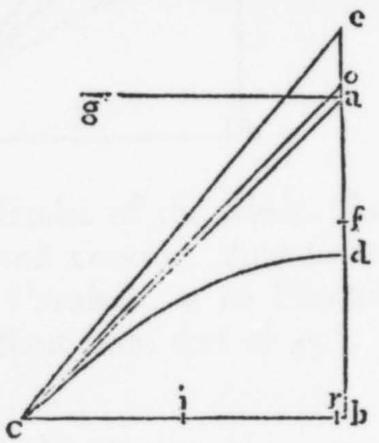
From what has been before demonſtrated, it is manifeſt that *Parabola's* are then deſcrib'd by *Projects* with the ſame *Impetus*, when their Sublimities together with their Altitudes, do make equal Perpendiculars above the Horizon. Theſe Perpendiculars therefore are to be comprehended between the ſame horizontal Parallels.

Suppoſe therefore the Perpendicular ba to be = the horizontal Line cb, and draw the Diagonal ac. Then $\angle acb$ will be half a right one or 45°. And the Perpendicular ba being divided equally in d, the *Semi-parabola* dc will be that which is deſcrib'd from the Sublimity ad together with the Altitude db: and its *Impetus* in c ſhall be as great as that of the Moveable, falling from Reſt in a thro' the Line ab, is in b: and if ag be drawn $\parallel bc$; the Altitudes and Sublimities united of all the remaining *Semi-parabola's*, whoſe *Impetus* ſhall be the ſame with that now mention'd, muſt be bounded by the Parallels ag and bc. Again, ſince it has been demonſtrated that the Amplitudes of *Semi-parabola's* whoſe Tangents are



equally

equally diſtant either above or below from an Elevation of 45°, are equal, the Calculation which we ſhall make for the Elevations above 45°, will ſerve for thoſe under 45°. We choſe moreover the Number 10000 for the greateſt Amplitude of the Projection of the *Semi-parabola* at the Elevation of 45°, ſo much therefore we muſt ſuppoſe the Line $ba = bc$ the Amplitude of the *Semi-parabola* to be. We make Choice of the Number 10000, becauſe in theſe Calculations we make uſe of a Table of Tangents: To the Tangent of 45° of which Table this Number is equal.



Now to the Buſineſs; Draw *ce* making ∠ *ecb* greater, yet acute, than ∠ *acb*; and let the *Semi-parabola* to be deſcrib'd be ſuch as *ec* may be a Tangent to, and whoſe Sublimity together with its Altitude may be = *ba*. From the Table of Tangents by Help of the given Angle *bce* take *be*; which divide equally in *f*, then find a third Proportional to *bf* and *bi*, the Half of *bc*; *i. e.* let *bi* be a mean Proportional between *bf* and the Line ſought, which Line will of Neceſſity be greater than *fa*; let it then

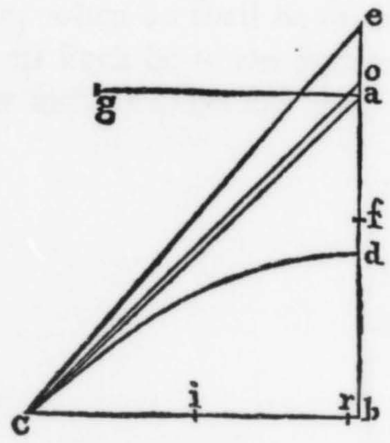
then be *fo*: Of the *Semi-parabola* therefore inſcrib'd in △ *ecb*, according to the Tangent *ce*, whoſe Amplitude is *cb*; the Altitude *bf*, and the Sublimity *fo* is found. But the whole Line *bo* riſes above the Parallels *ag* and *cb*, when it was our Buſineſs to keep it within thoſe Bounds, for ſo both it and the *Semi-parabola dc* will be deſcrib'd by the Projects flung from *c* with the ſame *Impetus*. Therefore we muſt find another like this (For innumerable greater and ſmaller like one another, may be deſcrib'd within ∠ *bce*) whoſe united Sublimity and Altitude (homologous to *ba*) ſhall be equal to *ba*. Make then *ob* : *ba* :: the Amplitude *bc* : *cr*, which *cr* will be the Amplitude of the *Semi-parabola* according to the Elevation or Angle *bce*; whoſe Sublimity together with its Altitude is = the Diſtance between the Parallels *ga* and *cb*, which was requir'd. The Work therefore is as follows.

Take the Tangent of the given Angle *bce*, to the half of which add the third Proportional to it, and to *bi* the half of *bc*, which let be *fo*: Then make *ob* : *ba* : : *bc* to a fourth, which let be *cr*, *viz.* the Amplitude ſought.

EXAMPLE.

Let ∠ *ecb* be 50°, its Tangent will be 11918; whoſe half *bf* is = 5959; the half of *bc* is 5000, the third Proportional to theſe halves is 4195; for 5959 : 5000 :: 5000 : 4195, which added to *bf*, (5959) makes 10154 for *bo*. Make again as *ob* to *ba*, *i. e.* as 10154 to 10000 ſo is *bc*, *i. e.* 10000 (for each of them is the Tangent of 45°)

45°) to a fourth which ſhall be = the required Altitude *rc* = 9848, of ſuch Parts as *bc* the greateſt Amplitude is 10000.



To theſe the Amplitudes of the whole *Parabola's* are double, *viz.* 19696 and 20000. And ſo much alſo is the Amplitude of the *Parabola*, at an Elevation of 40°, ſince this is equally diſtant from that of 45°. See Table A. *Pag.* 436.

SAGR. I am at a Loſs as to ſome Part of this Demonſtration: I don't underſtand how a mean Proportional between *bf* and *bi* muſt be neceſſarily greater than *fa*; as our Author ſays it muſt.

SALV. That ſeems to follow from hence, *viz.*

The Square of the mean of three proportional Lines is = the Rectangle of the other two; whence the Square of *bi* or of *bd* = it, ought to be = the Rectangle of the firſt *fb*, into the third ſought, which third muſt of Neceſſity

t

ceſſity be greater than *fa*, becauſe the Rectangle of *bf* in *fa*, is leſs than the Square of *bd*, and the Defect is = the Square of *df*, as *Euclid* demonſtrates in * one of his Propoſitions of his ſecond Book.

It muſt be obſerved further, that the Point *f*, which divides the Tangent *eb* equally, will often fall otherwiſe, above the Point *a*, and once in the very Point *a*; In which Caſes it is evident of itſelf, that a third Proportional to the Half of the Tangent and to *bi* (which gives the Sublimity) is wholly above the Point *a*.

But our Author took a Caſe wherein it was not manifeſt that the ſaid third Proportional is always greater than *fa*, and which therefore ſet off, above the Point *f*, extends beyond the Parallel *ag*.

Now let us go on.

It will not be uſeleſs by Help of this Table to calculate another of the Altitudes of the ſame *Semi-parabola's* of Projects thrown with the ſame *Impetus*.

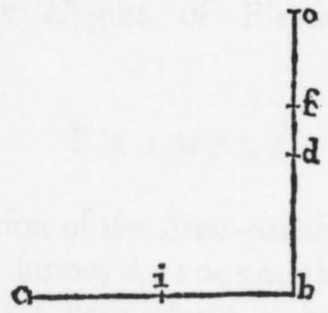
PROP. XIII. PROB. VI.

From the given Amplitudes of *Semi-parabola's* in Table A, the *Impetus* with which every one is deſcrib'd, being the ſame, to find the Altitudes of thoſe *Parabola's*.

---

* Prop. V.

Let the given Amplitude be *bc*; and let the Meaſure of the *Impetus* which is ſuppos'd always the ſame, be *ob*, *viz.* the Aggregate of the Altitude and Sublimity: Now the Altitude itſelf is to be ſeparately found; which will then be done, when *bo* ſhall be ſo divided that the Rectangle under its Parts be = the Square of half the Amplitude *bc*. Let ſuch Diviſion fall in *f*, and let both *ob* and



bc be cut equally in *d* and *i*: then is the Square of *ib* = ▭ *bfo*: But the Square of *do* is = the ſame Rectangle together with the Square of *fd*: If therefore from the Square of *do* be taken the Square of *bi* which is = ▭ *bfo*, there will remain the Square of *fd*, whoſe Side *df* + the Line *bd*, will give the Altitude ſought *bf*.

As thus;

From half the Square of *bo* given, ſubtract the Square of *bi* alſo given; take the Square Root of the Remainder, which add to *ab* given, and it gives the Altitude *bf* ſought.

EXAMPLE.

EXAMPLE.

The Altitude of a *Parabola* deſcrib'd at the Elevation of 55° is to be found.

The Amplitude from the fore-made Table A is 9396; its Half is 4698; the Square of which is 22071204; Take this from the Square of half *bo*, which is always the ſame; *viz.* 25000000, and the Remainder is 2928796, whoſe Square Root is 1710 nearly; this added to 5000, *viz.* to the Half of *bo*, gives 67101, and ſo much is the Altitude *bf*. See Table B.

It may be of uſe to add a third Table, C, of the Altitudes and Sublimities of *Semi-parabola's* whoſe Amplitudes ſhall be the ſame.

SAGR. This will be a great Pleaſure to me, ſince by its Help I may come to ſee the Difference of the *Impetus's* and Forces which are requir'd to throw a Project to the ſame Diſtance, or Range; which Difference I believe is very conſiderable according to the different Elevations; ſo that if any one at the Elevations of 3 or 4, or 87 or 88 Degrees, would fling a Ball as far as it was flung at the Elevation of 45 Degrees (at which it has been demonſtrated the leaſt *Impetus* is requir'd) I believe it would require a vaſtly greater Force.

SALV. You are right: and you will find that to accompliſh this Buſineſs thoroughly at all Elevations, we ſhall be oblig'd to take large Steps towards an infinite *Impetus*.

Prop. XIV. Prob. VII.

To find the Altitudes and Sublimities of *Semi-parabola's*, whose Amplitudes shall be equal to all the Degrees of Elevation.

This is easily done. For supposing the Amplitude of the *Semi-parabola* to be always 10000 Parts, the Half of the Tangent of any Degree of Elevation will give the Altitude.

Example.

Let the Elevation of the *Semi-parabola* be 30°, and the Amplitude, as is suppos'd, 10000, the Altitude will be 2887, for so much very nearly is the Half of the Tangent: And having found the Altitude, the Sublimity is gotten as follows; Forasmuch as it has been demonstrated that Half the Amplitude of the *Semi-parabola* is a mean Proportional between the Altitude and Sublimity; and the Altitude is already found, and Half of the Amplitude being always the same, *viz.* 5000 Parts; If the Square of this be the given Altitude, you will have the Sublimity sought. As in the Example: The Altitude found was 2887: The Square of 5000, is 25,000,000; which divided by 2887, gives 8659 very nearly for the Sublimity sought.

Salv. Here now we see first, that the aforemention'd Conjecture is true, *viz.* that at different Elevations, the more they recede from the middlemost or 45°, whether

A

The Amplitudes of Semi-parabolas described by the same Impetus

Gr		Gr
45	10000	
46	9994	44
47	9976	43
48	9945	42
49	9902	41
50	9848	40
51	9782	39
52	9704	38
53	9612	37
54	9511	36
55	9396	35
56	9271	34
57	9136	33
58	8989	32
59	8829	31
60	8659	30
61	8481	29
62	8290	28
63	8090	27
64	7880	26
65	7660	25
66	7431	24
67	7191	23
68	6944	22
69	6692	21
70	6428	20
71	6157	19
72	5878	18
73	5592	17
74	5300	16
75	5000	15
76	4694	14
77	4383	13
78	4067	12
79	3746	11
80	3420	10
81	3090	9
82	2756	8
83	2419	7
84	2079	6
85	1736	5
86	1391	4
87	1044	3
88	698	2
89	349	1

B

The Altitudes of Semi-parabolas whose Impetus's are the same.

Gr		Gr	
1	3	46	5173
2	19	47	5346
3	28	48	5523
4	50	49	5698
5	76	50	5868
6	108	51	6038
7	150	52	6207
8	194	53	6379
9	245	54	6546
10	302	55	6710
11	365	56	6873
12	432	57	7032
13	500	58	7186
14	585	59	7348
15	670	60	7502
16	760	61	7649
17	855	62	7786
18	955	63	7939
19	1060	64	8078
20	1170	65	8214
21	1285	66	8346
22	1402	67	8474
23	1527	68	8597
24	1085	69	8715
25	1786	70	8830
26	1922	71	8940
27	2061	72	9045
28	2204	73	9144
29	2351	74	9240
30	2499	75	9330
31	2653	76	9415
32	2810	77	9493
33	2967	78	9567
34	3128	79	9636
35	3289	80	9698
36	3456	81	9755
37	3621	82	9806
38	3793	83	9851
39	3962	84	9890
40	4132	85	9924
41	4302	86	9951
42	4477	87	9972
43	4654	88	9987
44	4827	89	9998
45	5000	90	10000

C Page 436.

The Altitudes & Sublimities of Semi-parabolas whose Amplitudes are the same Viz. 10,000 Parts calculated to every Degree of Elevation.

Gr	Altit.	Subli.	Gr	Altit.	Subli.
1	87	286533	46	5177	4828
2	175	142450	47	5363	4662
3	262	95802	48	5553	4502
4	349	71531	49	5752	4345
5	437	57142	50	5959	4190
6	525	47573	51	6174	4048
7	614	40716	52	6395	3906
8	702	35587	53	6635	3765
9	792	31565	54	6882	3632
10	881	28367	55	7141	3500
11	972	25720	56	7413	3372
12	1063	23518	57	7699	3247
13	1154	21701	58	8002	3123
14	1246	20056	59	8332	3004
15	1339	18663	60	8000	2887
16	1434	17405	61	9020	2771
17	1529	16355	62	9403	2658
18	1624	15380	63	9813	2547
19	1722	14522	64	10251	2438
20	1820	13736	65	10722	2331
21	1919	13024	66	11230	2226
22	2020	12376	67	11770	2122
23	2123	11778	68	12345	2020
24	2226	11230	69	13025	1919
25	2332	10722	70	13237	1819
26	2439	10253	71	14521	1721
27	2547	9814	72	15388	1624
28	2658	9404	73	16354	1528
29	2772	9020	74	17437	1433
30	2887	8659	75	18660	1339
31	3008	8336	76	20054	1246
32	3124	8.01	77	21657	1154
33	3247	7699	78	23523	1062
34	3373	7413	79	25723	972
35	3501	7141	80	28356	881
36	3633	6882	81	31509	792
37	3768	6635	82	35577	702
38	3906	6395	83	40272	613
39	4049	6174	84	47572	525
40	4196	5959	85	57150	437
41	4346	5752	86	71503	349
42	4502	5553	87	95405	262
43	4662	5362	88	143181	174
44	4828	5177	89	286409	87
45	5000	5000	90	Infinite	

whether they be higher or lower, the greater *Impetus* and Violence is requir'd, to throw a Project to the ſame Diſtance: Becauſe, ſince that *Impetus* lies in a Mixture of two Motions, horizontal, equable, and perpendicular naturally accelerate, and the Meaſure of that *Impetus* is the Aggregate of the Altitude and Sublimity; from the Table it is ſeen that ſuch an Aggregate is leaſt at the Elevation of 45°, where alſo the Altitude and Sublimity are = each other; that is each 5000, and their Aggregate 10000. But view another greater Elevation, as for Example's Sake 50°, we ſhall find the Altitude to be 5959, and the Sublimity 4196, which added together, make 10155. And ſo much alſo we ſhall find the *Impetus* to be at 40°, theſe Elevations being equally diſtant from the middle one or 45°: Where ſecondly we may note, that it is true that equal *Impetus's* are ſought from Pairs of Elevations equally diſtant from the middle one, with this pleaſant Variation, *viz.* that the Altitudes and Sublimities of the Elevations above 45° anſwer alternately to the Sublimities and Altitudes of the Elevations under 45°: So that, ſince in the propos'd Example at the Elevation of 50°, the Altitude is 5959 and Sublimity 4196: at the Elevation of 40°, contrary-wiſe, it falls out, that the Altitude is 4196, and Sublimity 5959: and the ſame happens in all the reſt, without the leaſt Difference, ſave only for avoiding of Tedioufneſs in Calculation, we have had no Regard to Fractions, which in ſo great Sums are of no Moment, but may be ſafely rejected.

SAGR. I am obſerving, that of the two *Impetus's* horizontal and perpendicular, in Projections, the higher they

are

are, the leſs they require of the Horizontal, and the more of the Perpendicular: And on the contrary, at leſſer Elevations there is required a great Force of horizontal *Impetus* becauſe the Project muſt be carried at a ſmall Height. But altho' I underſtand very well, that at the Elevation of 90°, all the Force imaginable is not ſufficient to drive the Project one ſingle Inch out of the Perpendicular, but that it muſt neceſſarily fall in the ſame Place it was thrown from; yet I dare not, with the ſame Certainty, affirm, that alſo at no Elevation, *i. e.* in an horizontal Poſition, the Project cannot by any leſs than an infinite Force, be flung to any Diſtance. So that, *e. g.* a Culverin would not be able to fling an Iron Ball horizontally, or, as they ſay, from a Point blank, *i. e.* at no Elevation: I ſay, that in ſuch Caſe, I'm in ſome doubt; but that I do not poſitively deny the Fact, another Accident no leſs ſtrange forbidding me, of which I have a Demonſtration neceſſarily concluſive. The Accident is this: 'Tis impoſſible to ſtretch a Rope right out or directly, and to keep it parallel to the Horizon; but it will always be arch'd or curv'd; nor is any Power whatſoever ſufficient to keep it ſtrait.

SALV. So then, *Sagredus*, you ceaſe admiring in this Caſe concerning the wonderful Effect in the Rope, becauſe you have a Demonſtration of it. But if this Thing be well weigh'd, we may perhaps find ſome Agreement between the Accidents of that Project and this Rope. The Curvity of the Line of the horizontal Project ſeems to ariſe from two Powers, one whereof (which is the Power of the Projicient) forces it horizontally, but the other (which is its own Gravity, draws it perpendicularly downwards;

downwards; But now in the ſtretching of the Rope, beſide their Forces who draw it horizontally, there is moreover the Gravity of the Rope, which naturally inclines it to tend downward: Now theſe two Generations are very nearly alike. And if to the Gravity of that Rope you allow ſuch Power and Energy as can reſiſt, and overcome the Force, be it never ſo great, which would draw it out directly, why won't you allow it to the Gravity of the Ball or Bullet?

Beſides I muſt tell you, that which at the ſame Time will both amaze and delight you, 'tis this, that the Rope thus ſtretch'd more or leſs, bends it ſelf into Lines very nearly parabolical, and the Likeneſs is ſuch, that if on a plain Superficies and perpendicular to the Horizon you deſcribe a parabolic Line, and turn it up-ſide-down, and to the Extremities of the Baſe of the deſcrib'd *Parabola* you hold a Chain or Cord, by ſlackening it more or leſs, you'll ſee it bend and fit itſelf to the ſame *Parabola*; and this Fitting ſhall be ſo much the more exact, by how much the deſcrib'd *Parabola* is leſs curv'd, *i. e.* more diſtended; ſo that in *Parabola's* deſcrib'd at Elevations leſs than 45°, the Chain agrees with the *Parabola* almoſt to an Hair.

SAGR. By Help then of ſuch a ſmall Chain curiouſly made, many parabolic Lines may in an Inſtant be deſcrib'd on a plain Superficies.

SALV. They may; and that with no ſmall Advantage: as hereafter I ſhall ſhew you.

SIMP.

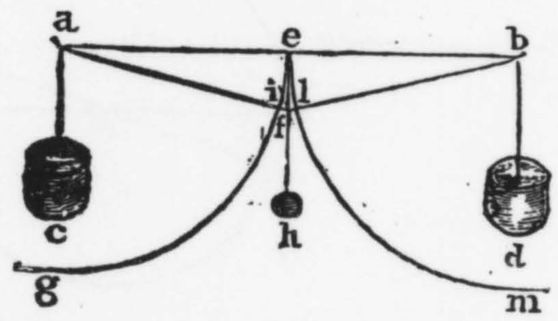
SIMP. But before we go on, I would at leaſt have that Propoſition demonſtrated of which you ſay you have a Demonſtration neceſſarily concluſive, *viz.* that it is impoſſible for a Rope to be ſtretch'd directly, and keep parallel to the Horizon.

SAGR. I'll try if I can recollect the Demonſtration; in order to underſtand which, *Simplicius*, you muſt take for granted, that which in all Mechanical Inſtruments is prov'd to be true, not only by Experience, but Demonſtration: and 'tis this; that the Velocity of the Mover, tho' this Mover has but little Power, may overcome the Reſiſtance, altho' vaſtly great, of the Thing reſiſting, (which muſt be mov'd ſlowly) whenever the Velocity of the Mover hath a greater Proportion to the Tardity of the Reſiſter, than the Reſiſtance of that which is to be moved, has to the Power of the Mover.

SIMP. This I know well already, 'tis demonſtrated by *Ariſtotle* in his mechanic Queſtions; and tis manifeſtly ſeen in the *Leaver* and *Stilliard*, in which the *Counterpoiſe* which weighs not above four Pounds, will raiſe a Weight of 400, provided the Diſtance of the ſaid *Counterpoiſe* from the Center upon which the Beam turns, be more than 100 times greater than the Diſtance of the ſaid Center from the Point, at which the great Weight hangs; And this comes to paſs, becauſe the *Counterpoiſe*, whilſt it deſcends, paſſeth a Space above an hundred times greater than the Space which the great Weight aſcends in the ſame Time, which is the ſame Thing as to ſay, that that little *Counterpoiſe*

Counterpoiſe moveth with a Velocity above an hundred Times greater than the Velocity of the great Weight.

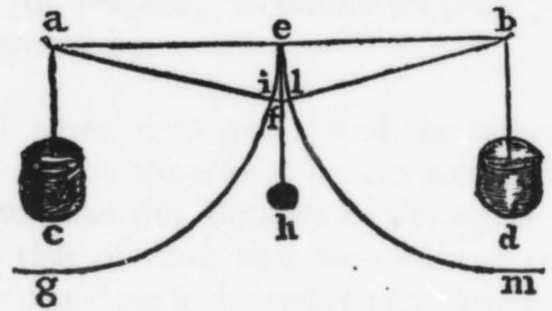
SAGR. You reaſon very juſtly, and make no Scruple of granting that howſoever ſmall the Power of the Mover be, yet it will o'ercome any whatever great Reſiſtance at all Times when that ſhall exceed more in Velocity than this does in Power and Gravity. But let us come to the Caſe of the Rope: Let the following Figure be drawn;



Imagine the Line *ab* paſſing thro' two fixed or immoveable Points *a* and *b*, to ſuſtain, hang'd at its Extreams, as you ſee, two great Weights *c* and *d*, which drawing it with great Force keep it ſtraight ſtretch'd, as a ſimple Line without any Gravity. Then I ſay if to its Middle, as the Point *e*, you hang a Weight never ſo ſmall, as is *h*, the Line *ab* will yield and incline towards the Point *f*, and by conſequence lengthening, will conſtrain the two great Weights *c* and *d* to aſcend: Which I demonſtrate as follows;

About the two Points *a* and *b*, as Centers, deſcribe two Quadrants *eig*, *elm*: and ſince the two Semi-diameters

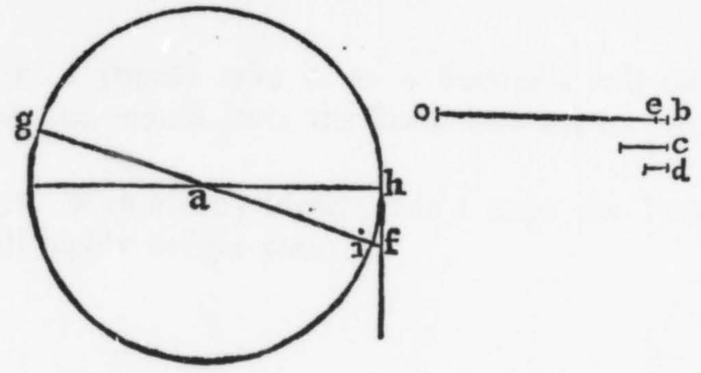
a i and *b l* are = the two *a e* and *e b*, the Exceſſes *f i*, *f l* will be Quantities by which the Parts *a f* and *f b* exceed *a e* and *e b*, and conſequently they determine the Aſcent of the Weights *c* and *d*; always therefore when the Weight *b* has a Power to deſcend to *f*; which might then be when the Line *e f*, which is the Quantity of the Deſcent of that Weight *b*, has a greater *Ratio* to the Line *f i*, which determines the Aſcent of the two Weights *c* and *d*, than the Gravity of both thoſe Weights, has to the Gra-



vity of the Weight *b*. But now this will neceſſarily happen, be the Gravity of the Weights *c* and *d* never ſo great, and that of *b* never ſo ſmall. For the Exceſs of the Weights *c* and *d*, above the Weight *b*, is not ſo much, but that the Exceſs of the Tangent *e f*, above the Part of the Secant *f i*, is more in Proportion. Which I prove as follows.

Let there be a Circle whoſe Diameter is *g a i*: and the ſame *Ratio* as the Gravity of the Weights *c* and *d* in the foregoing Fig. have to the Gravity of *b*, let the Line *b o* have to a fourth *c*, leſs than which let be the Line *d*, ſo that *b o* ſhall have a greater Proportion to *d* than it has to *c*. Between *o b* and *d* take a third Proportional *b e*:

And as *o e* is to *e b*, ſo make the Diameter *g i* (by producing it) to *i f*; and from *f* draw the Tangent *f n*. And becauſe it is *o e* : *e b* :: *g i* : *i f*, it will be by Compoſition *o b* (*o e* + *e b*) : *b e* :: *g f* (*g i* + *i f*) : *f i*; but between *o b* and *b e*, *d* is a mean Proportional: And between *g f* and *f i*, *n f* is a Mean; therefore *n f* has the ſame Proportion to *f i*, as *o b* has to *d*, which Proportion is greater than that which the Weights *c* and *d* have to the Weight *b*. Since then the Deſcent or Velocity of the Weight *b* has a greater *Ratio* to the Aſcent or Velocity of the Weights *c*



and *d*; than the Gravity of the ſaid Weights *c* and *d* has to the Gravity of the Weight *b*; it is manifeſt, that the Weight *b* muſt deſcend, that is, that the Line *a b* muſt depart from its Straitneſs: And this, which happens to a right Line *a b* having no Gravity at all, upon hanging in *e* a Weight *b* never ſo ſmall; the very ſame will happen to the Rope, which is ſuppos'd to be made of ponderous heavy Matter without the Addition of any other Weight, becauſe the Weight of the Matter the Rope *a b* is made on may be ſaid to be added or hung to it.

SIMP. I am throughly ſatisfied; wherefore *Salviatus* may, according to his Promiſe, explain to us, what that Advantage is, which may be drawn from the fore-mention'd like Chain; and then ſhew us what Obſervations our Academic has made about *the force of Percuſſion*.

SALV. We have Buſineſs enough for this Day to contemplate of what has been already ſaid; the Time, it now growing towards Evening, would not admit us to go thro' what you propoſe; wherefore we muſt put it off till another Opportunity.

SAGR. I agree with you: And the rather, becauſe I have often heard, from ſome of our Academic's intimate Acquaintance, that this Buſineſs of *Percuſſion* is very obſcure; and that of thoſe that have hitherto treated about it, not one has done it throughly and clearly. Amongſt the Concluſions that I have heard of, I can't help mentioning one that is very extraordinary, namely, that the Force of *Percuſſion* is indeterminate if not infinite. However, we'll wait *Salviatus*'s Leiſure. But in the mean Time tell me what thoſe Things are which are wrote at the End of the Treatiſe of *Projects*.

SALV. They are certain Propoſitions relating to the Center of Gravity of Solids, which our Academic found out in his Youth: Conceiving that what *Frederick Comandine* had wrote about it, to be imperfect, he thought that theſe Propoſitions following would ſupply what was wanting in *Comandine*'s Treatiſe; He apply'd himſelf to this Study at the Requeſt of the moſt Illuſtrious Lord Marquiſs

Marquiſs *Guid' Ubaldo dal Monte*, (the greateſt Mathematician of his Time; as his ſeveral printed Works do witneſs;) and gave a Copy of what he had wrote on this Head to that noble Lord, and had indeed Thoughts to have purſu'd this Matter farther in other Solids untouch'd by *Comandine*: but when after ſome Time he met with the Book of *Signore Luca Valerio*, a moſt excellent Mathematician, and ſaw that he had ſolv'd all theſe Matters, without omitting any Thing, he left of; tho' the Methods *Signore Valerio* and he took were very different to each other.

SAGR. I ſhould take it as a Favour if 'till our next Meeting you would leave the Book with me to peruſe.

SALV. With all my Heart; and I hope the Propoſitions will highly delight you.

APPENDIX.

APPENDIX.

Wherein are Contained

PROPOSITIONS *and their* DEMONSTRATIONS,

Formerly wrote by the ſame AUTHOR about the

Center *of* Gravity *of* Solids.

POSTULATE.

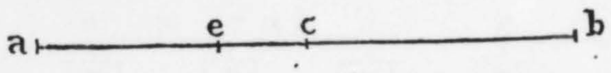
IF equal Weights alike diſpos'd at different Balances or Beams, we take for granted that if the Center of Gravity of theſe Weights compounded, that hang at one Balance, divides this Balance in any Proportion, that then the Center of Gravity of thoſe at the other Balance will divide that Balance according to the ſame Proportion.

LEMMA.

Let the Line *a b* be equally divided in *c*; whoſe half *a c* let be divided in *e*, in ſuch Manner that what *Ratio*

b e

b e has to *e a*, the ſame *a e* may have to *e c*. Then I ſay *be* is the double of *ea*. For becauſe *be* : *ea* : : *ea* : *ec*; it will be by Compoſition and Permutation *ba* : *ac* :: *ae* : *ec*.



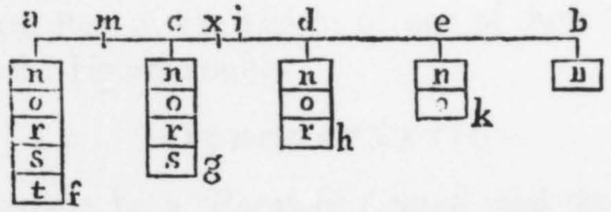
But it is as *ae* to *ec*, *i. e.* as *ba* to *ac*, ſo is *be* to *ea*. Wherefore *be* is the double of *ea*.

Theſe ſuppos'd, we may demonſtrate the following

PROPOSITION.

If Magnitudes equally exceeding one another at any Rate of Exceſs, and whoſe common Exceſs is = the leaſt of thoſe Magnitudes, are ſo diſpos'd as to hang at equal Diſtances on a Balance (or Beam); the Center of Gravity of all of them will ſo divide the Beam, that that Part of it towards the leſſer Magnitudes will be double to the remaining Part of it.

On the Beam *a b* then let there hang at equal Diſtances any Number of Magnitudes (at has been ſaid) as *f*, *g*, *h*, *k*, *n*: the leaſt of which let be *n*. And let the Center of



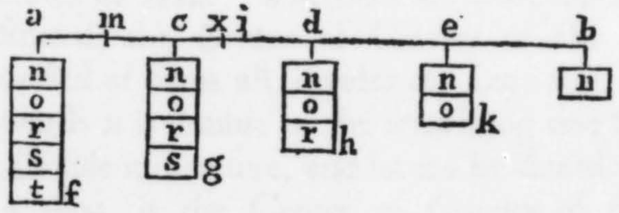
Gravity of all the Magnitudes ſo diſpos'd, be *x*. Then we are to ſhew, that the Part of the Beam *bx* towards the leſſer Magnitudes is double to the remaining Part of it, *x a*.

Let

Let the Beam be divided equally in the Point *d*, which Point will, of Courſe, fall on ſome Point of the Suſpenſions, or in the midſt of two of them; And let the remaining Diſtances of Suſpenſions, which are between *a* and *d*, be equally divided in the Points *m* and *i*. And let all the Magnitudes be divided into Parts = *n*: Now the Parts of *f* will be ſo many in Number as there are Magnitudes hanging from the Beam: but the Parts of *g* will be fewer by one, and ſo of the reſt. Let the Parts of *f* therefore be *n*, *o*, *r*, *ſ*, *t*; Thoſe of *g*; *n*, *o*, *r*, *s*; Thoſe of *h*; *n*, *o*, *r*; And thoſe of *k*; *n*, *o*; And all the Magnitudes, mark'd *n* ſhall be = the Magnitude *f*; And the Magnitudes mark'd *o* ſhall be = *g*; Thoſe mark'd *r* are = *h*; and thoſe mark'd *s*, to *k*; and the Magnitude *t* is = *n*. Therefore becauſe all the Magnitudes mark'd *n*, are = one another, they will weigh equally in the Point *d*, which divides equally the Balance or Beam *a b*; And for the ſame Reaſon, all the Magnitudes mark'd *o*, equiponderate in *i*; Thoſe mark'd *r* in *c*; And thoſe mark'd *s* equiponderate in *m*. But *t* is ſuſpended at *a*. Therefore in the Balance *a d*, at the equal Diſtances *d*, *i*, *c*, *m*, *a*, there are Magnitudes ſuſpended, exceeding one another equally, and whoſe Exceſs is = the leaſt Magnitude: And the greateſt which is compounded of all the *n*'s hangs at *d*; the leaſt which is *t*, hangs at *a*; and the reſt are orderly diſpos'd. And again there is another Balance *a b*; in which the other Magnitudes equal to the foremention'd in Number and Magnitude are diſpos'd in the ſame Order. Wherefore the Balances *a b* and *a d* are divided by the Center of all the Magnitudes in the ſame *Ratio*, one as the other. But the Center of Gravity of the ſaid Magni-

 tudes

tudes is x; wherefore x divides the Balances ba and ad in the ſame *Ratio* one as the other: So that $bx : xa :: xa : xd$



(by the foregoing Poſtulate) wherefore bx is the Double of xa, by the above mention'd *Lemma*. *Q. E. D.*

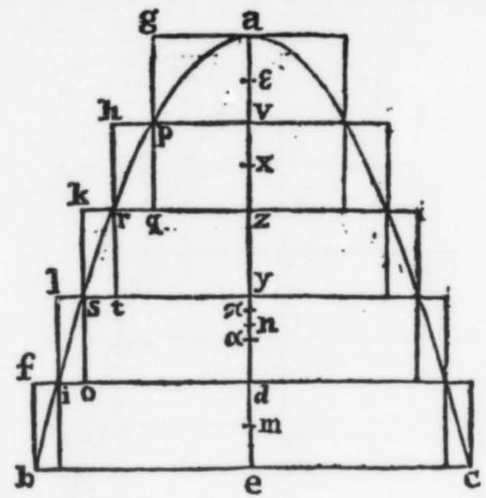
PROPOSITION.

If in a *Parabolic Conoid* a Figure be inſcrib'd, and another circumſcrib'd by Cylinders of equal Height; and the *Axis* of the ſaid *Conoid* be divided, ſo that the Part towards the *Vertex* be double the Part towards the Baſe; the Center of Gravity of the inſcrib'd Figure will be nearer to the Baſe than that Point of Diviſion will be, and the Center of Gravity of the circumſcrib'd ſhall be farther than that Point from the Baſe of the *Conoid*; and the Diſtance of either Centers from ſuch Point will be = a Line which is a Sixth Part of the Height of one of thoſe Cylinders of which the Figures conſiſt.

DEMONSTRATION.

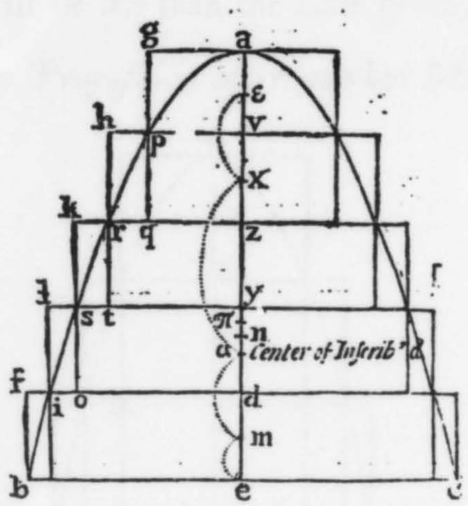
Let there be a *Parabolic Conoid*, and the Figures that have been mention'd, the one inſcrib'd, the other circumſcrib'd: And let the *Axis*, which is ae, be divided in n, ſo that an be the Double of ne. We muſt ſhew the Center of Gravity of the inſcrib'd Figure to be in the Line ne, but

the Center of the circumſcrib'd to be in an. Let the Figures ſo diſpos'd be cut by a Plane thro' the *Axis*, and let the Section of the *Parabola* be bac; and let the Section of the Cutting Plane and Baſe of the Conoid be the Line bc; and let the Sections of the Cylinders be the rect-angular Figures as deſcrib'd in the Figure before us: Then the firſt Cylinder of the Figure inſcrib'd, whoſe *Axis* is de, has the ſame *Ratio* to the Cylinder whoſe *Axis* is dy, as the Square of id has to the Square of sy, *id eſt*, as da to ay: But the



Cylinder whoſe *Axis* is dy to the Cylinder yz, is as the Square of sy to the Square of rz; *i. e.* as ya to az; and for the ſame Reaſon, the Cylinder whoſe *Axis* is zy, to that whoſe *Axis* is zv, is as za to av. Therefore the ſaid Cylinders are one to another, as the Lines da, ay; za, av: but they exceed one another equally, and the Exceſs is = the leaſt of them, ſo that az is double to av; and ay is triple to the ſame; and da quadruple; therefore the ſaid Cylinders are certain Magnitudes equally exceeding one

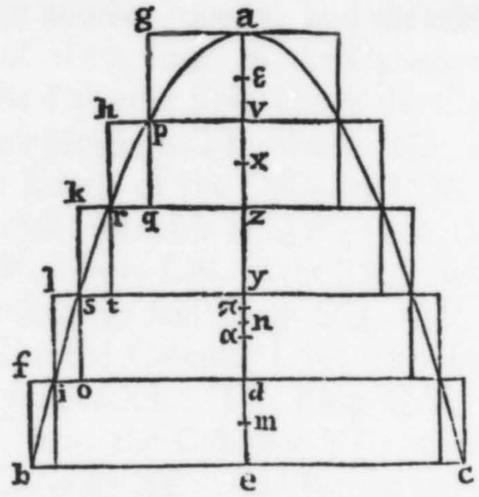
another, whoſe common Exceſs is = the leaſt of them, and the Line $x\,m$ is that in which they are ſuſpended at equal Diſtances: For each Cylinder has its Center of Gravity in the Midſt of its Axis. Wherefore by what has been above demonſtrated, the Center of Gravity of the Magnitude compounded of them all, divides the Line $x\,m$, ſo that the Part towards x is double to the remaining one (by the laſt Prop.) Divide it therefore, and let $x\,\alpha$ be double $\alpha\,m$; The Point α then, is the Center of Gravity of the inſcrib'd Figure.



Divide $a\,v$ equally in ε; εx ſhall be double of $m\,e$; but $x\,\alpha$ is double $\alpha\,m$ (by Conſtruction), wherefore $\varepsilon\alpha$ will be the double of $\alpha\,e$; wherefore $\varepsilon\,e$ will be three times $e\alpha$, but $a\,e$ is triple $e\,n$; it's manifeſt therefore that $e\,n$ is greater than $e\alpha$, and therefore α, which is the Center of the inſcrib'd Figure, happens nearer the Baſe of the *Conoid* than n; And becauſe it is as $a\,e$ to $e\,n$, ſo the Part taken away $\varepsilon\,e$, to the Part taken away $e\alpha$; And the Remainder

der to the Remainder, *id eſt*, $a\varepsilon$ to $n\alpha$, will be as $a\,e$ to $e\,n$. Therefore $\alpha\,n$ is the third Part of $a\varepsilon$, and the ſixth Part of $a\,v$.

After the ſame Manner the Cylinders of the cicumſcrib'd Figure will be demonſtrated to be equally exceeding one another, and their common Exceſs to be = the leaſt of them; And to have their Centers of Gravity at equal Diſtances in the Line $\varepsilon\,m$. If therefore $\varepsilon\,m$ be divided in π, ſo that $\varepsilon\pi$ be double of the Remainder $\pi\,m$; π will be the Center of Gravity of the whole circumſcrib'd Magnitude, and ſince $\varepsilon\pi$ is double $\pi\,m$; and $a\varepsilon$ leſs than double $e\,m$;



(they being = one another); the whole $a\,e$ ſhall be leſs than three times $e\pi$: wherefore $e\pi$ will be greater than $e\,n$; and ſince $\varepsilon\,m$ is triple $m\pi$, and $m\,e$ with twice $\varepsilon\,a$ is likewiſe three times $m\,e$; the whole $a\,e$ with $a\varepsilon$ ſhall be three times $e\pi$; but $a\,e$ is three times $e\,n$; wherefore the remaining Part $a\varepsilon$ will be three times the Remainder $\pi\,n$. Therefore $n\pi$ is a ſixth Part of $a\,v$. Which were the Things to be prov'd.

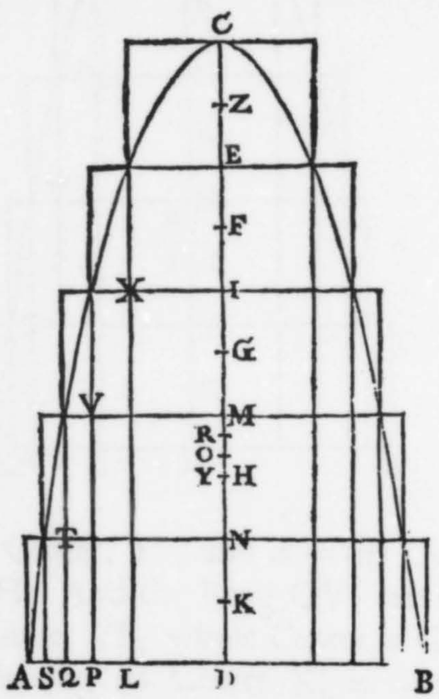
COROL-

COROLLARY.

Hence it is manifeſt, *That a Figure may be inſcrib'd in a Parabolic Conoid, and another circumſcrib'd, ſo that the Centers of their Gravities may be diſtant from the Point* n *by leſs than any given Line.*

For if a Line be taken Sextuple of the propos'd Line, and the *Axes* of the Cylinders from which the Figures are compos'd, be leſs than this aſſum'd Line; the Lines which fall between the Centers of Gravities of theſe Figures and the Point *n*, will be leſs than the Line given.

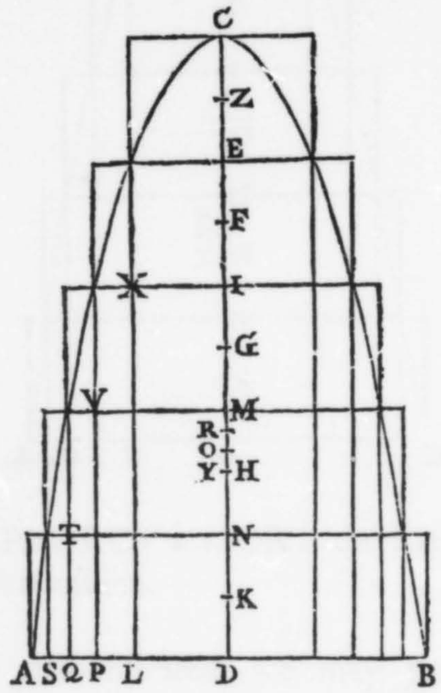
The ſame Propoſition after another Manner.



Let the *Axis* of the *Conoid* (which let be CD) be divided in O, ſo that CO may be the Double of OD; Now we

we are to prove that the Center of Gravity of the inſcrib'd Figure is in the Line OD; and that the Center of the circumſcrib'd is in CO. Let the Figures be cut by the Plane thro' the *Axis* and C, as has been ſaid. Then becauſe the Cylinders SN, TM, VI, and XE, are one to another as the Squares of the Lines SD, TN, VM, and XI, and theſe one to another as the Lines NC, CM, CI, CE; And becauſe theſe exceed each other equally, and the Exceſs is = the leaſt, *viz.* to CE; and the Cylinder TM = the Cylinder QN; And the Cylinder VI to PN; and XE to LN; therefore the Cylinders SN, QN, PN, LN exceed one another equally, and the common Exceſs is = the leaſt of them, *viz* to the Cylinder LN. But the Exceſs of the Cylinder SN, above the Cylinder QN, is the Ring whoſe Height is QT; that is ND; and Breadth, SQ. And the Exceſs of the Cylinder QN, above PN, is the Ring, whoſe Breadth is QP; but the Exceſs of the Cylinder PN, above LN, is the Ring whoſe Breadth is PL. Wherefore the ſaid Rings SQ, QP, PL, are = each other, and to the Cylinder LN; therefore the Ring ST is = the Cylinder XE: The Ring QV, which is the Double of ST, is = the Cylinder VI; which alſo is the Double of the Cylinder XE: And for the ſame Reaſon the Ring PX will be = the Cylinder TM; and the Cylinder LE = the Cylinder SN. In the Beam or Balance therefore KF, connecting the middle Points of the Right Lines EI and DN, and cut into equal Parts by the Points H and G, there are certain Magnitudes, *viz.* the Cylinders SN, TM, VI, XE; and the Center of Gravity of the firſt Cylinder is K; and of the ſecond H; of the third G; and of the fourth F. And we have another Beam MK, which

which is the Half of FK, and divided by as many Points into equal Parts, *viz.* MH, HN, NK, and on it, other Magnitudes, equal in Number and in Magnitude to thoſe on the Beam FK, and having the Centers of Gravity in the Points M, H, N, K, and diſpos'd in the ſame Order. For the Cylinder LE has its Center of Gravity in M; and is = the Cylinder SN, having its Center in K: But the



Ring PX has its Center, H; and is = the Cylinder TM; whoſe Center is H: And the Ring QV, having its Center N, is = the Cylinder VI; whoſe Center is G: And laſtly the Ring ST, having its Center K, is = the Cylinder XE, whoſe Center is F. Therefore the Center of Gravity of

t

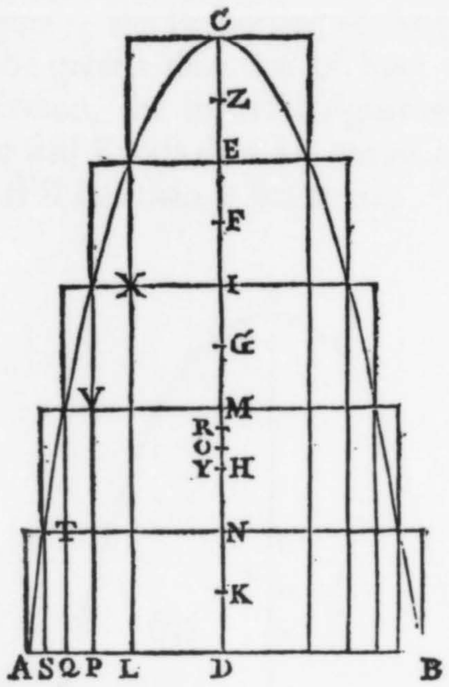
of the ſaid Magnitudes will divide the Beam in the ſame Proportion. But the Center of them is one, and therefore ſome Point common to both Beams, which let be Y. Therefore FY will be to YK as YK to YM. Therefore FY is the Double of YK; and CE being divided equally in Z, ZF will be the Double of KD; and therefore ZD the Triple of DY.

But CD is the Triple of the right Line DO: therefore the right Line DO is greater than DY; and therefore Y, the Center of the inſcrib'd Figure, is nearer to the Baſe than the Point O. And becauſe, as CD is to DO, ſo is the Part taken away ZD, to the Part taken away DY; the Remainder CZ will be to the Remainder YO, as CD to DO; to wit, YO will be a third Part of CZ; that is, a ſixth Part of CE.

By the ſame way of Reaſoning we will demonſtrate, that the Cylinders of the circumſcrib'd Figure exceed one another equally, and that the common Exceſs is = the leaſt of them, and that their Centers of Gravity are rang'd at equal Diſtances on the Beam KZ. And likewiſe, that the Rings = thoſe ſame Cylinders, are in like manner diſpos'd on another Beam KG, the Half of the ſaid KZ, and that therefore the Center of Gravity, which let be R, of the Figure circumſcrib'd, ſo divides the Beam, as that ZR is to RK, as RK is to RG. Therefore ZR ſhall be double of RK: But CZ is = the right Line KD, and not double to it. The Whole CD ſhall be leſs than triple of DR: Wherefore the right Line DR is greater than DO. That is, the Center of the circumſcrib'd Figure is farther off the Baſe, than the Point O. And becauſe ZK is triple of KR; and KD with twice ZC is triple to KD; the Whole CD

 with

with CZ will be the Triple of DR. But CD is triple to DO. Wherefore the remaining Part CZ will be triple to

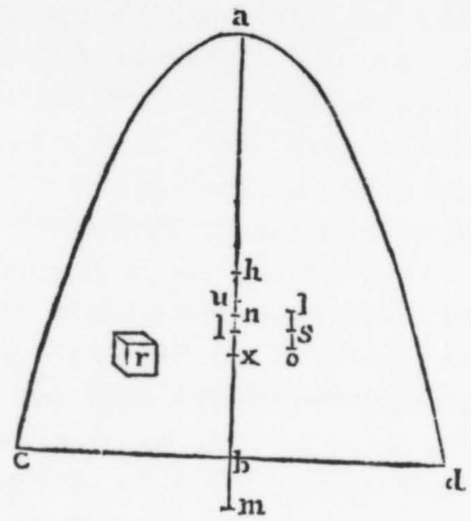


the remaining Part RO; *i. e.* OR is the ſixth Part of EC: Which is the Propoſition.

This being demonſtrated, we may prove that *The Center of Gravity of the Parabolic Conoid divides the Axis in ſuch Manner, that the Part towards the Vertex is the Double of the remaining Part towards the Baſe.*

Let

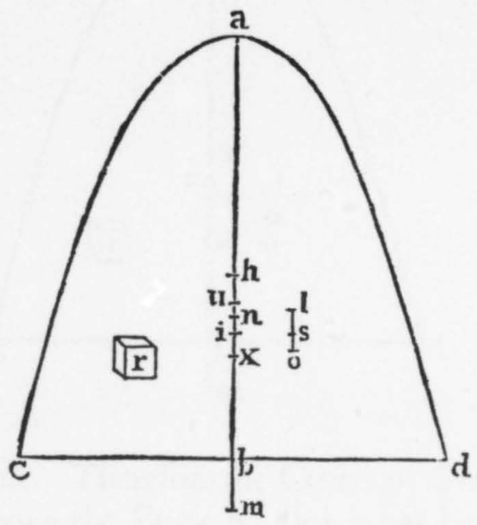
Let there be a *Parabolic Conoid*, whoſe *Axis* let be *ab*, divided in *n*, ſo that *an* is the Double of *nb*. We are to ſhew that the Point *n* is the Center of Gravity of the *Conoid*. For if *n* be not ſo, this Center muſt be either below it or above it; Suppoſe it firſt, to be below: and let it be *x*; and let the Line *lo* be drawn = *nx*; and let *lo* be divided at Pleaſure in *s*: and what Proportion *bx* and *os* both together have to *os*, let the Conoid have the ſame to the Solid *r*: And let a Figure of Cylinders having



equal Heights, be inſcrib'd in the Conoid, ſo that that which is intercepted between the Center of Gravity and the Point *n*, be leſs than *ls*; and let the Exceſs of the Conoid above it be leſs than the Solid *r*.

And that this can be done, is manifeſt. Let the inſcrib'd Figure therefore be that whoſe Center of Gravity is *i*; now *ix* will be greater than *so*: And, becauſe it is

as xb with so is to os, ſo is the *Conoid* to r; (and r is greater than the Exceſs, whereby the *Conoid* exceeds the inſcrib'd Figure); the Proportion of the *Conoid* to the ſaid Exceſs will be greater than that of both bx and os to so: And by Diviſion, the inſcrib'd Figure will have a greater *Ratio* to the ſaid Exceſs than bx has to so. But bx has to xi a *Ratio* ſtill leſs than it has to so. Therefore the in-



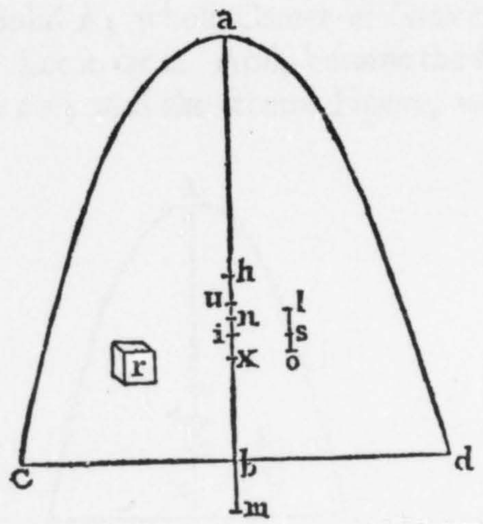
ſcrib'd Figure will have a much greater Proportion to the remaining Parts than bx has to xi. What Proportion therefore the inſcrib'd Figure has to the remaining Parts, the ſame ſome other Line will have to xi; which will neceſſarily be greater than bx. Let it therefore be mx. We therefore have x the Center of Gravity of the *Conoid:* But the Center of Gravity of the Figure inſcrib'd therein is i. Therefore the Center of Gravity of the remaining Portions by which the *Conoid* exceeds the inſcrib'd Figure, will

will be in the Line xm, and in that Point of it at which it ſhall be ſo terminated, that that Proportion which the inſcrib'd Figure has to the Exceſs of the *Conoid* above it, the very ſame it may have to xi. But it has been prov'd that this Proportion is that which mx has to xi. Therefore m will be the Center of Gravity of thoſe Portions whereby the *Conoid* exceeds the Figure inſcrib'd. Which Certainty cannot be. For if thro' m a Plane be drawn parallel to the Baſe of the *Conoid*, the ſaid Portions will be all on one and the ſame Side, and will not be divided by it. Therefore the Center of Gravity of the ſaid *Conoid* is not below the Point n. Neither is it above it. For, if it can be, let it be h. And again, as before, let the Line lo be drawn = the ſaid hn, and let it be divided at Pleaſure in s. And what Proportion bn and so both together have to sl, the ſame let the *Conoid* have to r: and let a Figure of Cylinders, as aforeſaid, be circumſcrib'd about the *Conoid*, exceeding the *Conoid* by a Quantity leſs than the Solid r: And let the Line between the Center of Gravity of the circumſcrib'd Figure and the Point n be leſs than so, the remaining Part uh will be greater than ls. And, becauſe it is, as bn and os both together is to sl, ſo is the *Conoid* to r; (and r is greater than the Exceſs by which the circumſcrib'd Figure exceeds the *Conoid*); therefore bn and so together, have a leſs *Ratio* to sl than the *Conoid* has to the ſaid Exceſs. But bu is leſs than bn and so both together; And uh is greater than sl: Therefore the *Conoid* has a much greater *Ratio* to the ſaid Portions than bu has to uh.

Therefore the ſame *Ratio* which the *Conoid* has to thoſe Portions, will a Line greater than the ſaid bu have to uh.

Let

Let it have it; and let the ſame be *mu*; and becauſe the Center of Gravity of the circumſcrib'd Figure is *u*; and that of the *Conoid* is *h*, and becauſe it is, as the *Conoid* to the remaining Portions, ſo is *mu* to *uh*, *m* will be the Center of Gravity of the remaining Portions: Which is



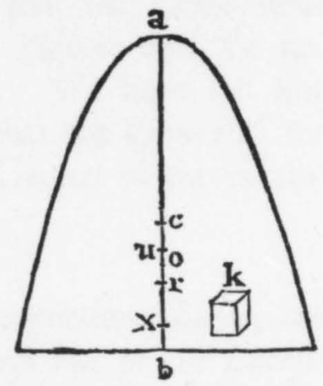
likewiſe impoſſible. Therefore the Center of Gravity of the *Conoid* is not above the Point *n*. But it has been demonſtrated that it is not below it. It remaineth therefore, that it muſt neceſſarily be in *n* it ſelf. And this may by the ſame arguing be demonſtrated of a *Conoid* cut by a Plane not perpendicular to the *Axis*.

The ſame Thing in another Manner, as appears by the following Propoſition.

The

The Center of Gravity of the Parabolic Conoid falls between the Center of the circumſcrib'd Figure and the Center of the inſcrib'd one.

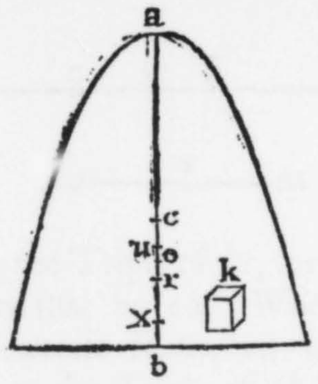
Let there be a *Conoid*, whoſe *Axis* is *ab*, and let the Center of the circumſcrib'd Figure be *c*, and that of the inſcrib'd one be *o*. I ſay, that the Center of the *Conoid* is between the Point *c* and *o*. For, if not, it will be below them, above them, or in one of them. Let it be below them, as in *r*. And, becauſe *r* is the Center of



Gravity of the whole *Conoid*; and the Center of Gravity of the inſcrib'd Figure is *o*: therefore the Center of Gravity of the remaining Portions, whereby the *Conoid* exceeds the inſcrib'd Figure, will be in the Line *or* extended on the Side of *r*, and in that Point of it in which it is ſo limited, that, the ſame *Ratio*, which the ſaid Portions have to the inſcrib'd Figure, *or* may have to the Line falling between *r* and that Point. Let this *Ratio* be that which *or* has to *rx*: Then *x* will fall either without or within the *Conoid*, or in its Baſe. To ſuppoſe it to fall

either

either without or in the Baſe, it is already manifeſt to be abſurd. Let it fall within: And becauſe *xr* is to *ro* as the inſcrib'd Figure is to the Exceſs of the *Conoid* above it; the ſame *Ratio*, which *br* has to *ro*, let the inſcrib'd Figure have to the Solid *k*, which will neceſſarily be leſs than the ſaid Exceſs. And let another Figure be inſcrib'd, which may be exceeded by the *Conoid* by a Quantity leſs than that of the Solid *k*; whoſe Center of Gravity will fall between *o* and *c*. Let it be *u*. And, becauſe the firſt Figure is to *k*, as *br* to *ro*; and the ſecond Figure, whoſe Center is *u*, is greater than the firſt, and is exceeded by the *Conoid* by a Quantity leſs than *k*: the ſame *Ratio*, which the ſecond Figure has to the Exceſs of the *Conoid* above it, will a Line greater than the ſaid *br* have to *ru*. But *r* is the Center of Gravity of the *Conoid*: and *u* of the ſecond inſcrib'd Figure. Therefore the Center of the remaining Portions will be without the *Conoid*, below *b*; which is impoſſible. And after the ſame Manner it may be demonſtrated, that the Center of Gravity of the ſame *Conoid* is not in the Line *ca*. And that it is neither of the Points *c* and

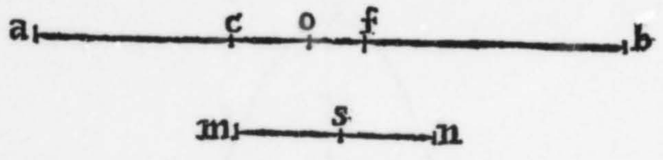


c and *o*, is manifeſt. For if you ſay it is, then, other Figures being deſcrib'd, *viz.* one inſcrib'd ſomething greater than that whoſe Center is *o*, and one circumſcrib'd, but leſs than that whoſe Center is *c*, the Center of the *Conoid* would fall without the Center of theſe Figures. Which was juſt now concluded to be impoſſible. It remaineth therefore, that it be between the Centers of the circumſcrib'd and inſcrib'd Figures. And, if it be ſo, it will neceſſarily be in that Point which divides the *Axis*, that the Part towards the *Vertex* be the Double of the remaining Part, ſince Figures may be inſcrib'd and circumſcrib'd about *n*, ſo that the Lines which fall between the Centers of thoſe Figures and the ſaid Point, be leſs than any given Line. We have led him that denies it to an Impoſſibility, that the Center of the *Conoid* ſhould not fall between the Centers of the inſcrib'd and circumſcrib'd Figures.

If there be three proportional Lines, and the ſame Proportion, which the leaſt has to the Exceſs of the greateſt above the leaſt, a certain aſſumed Line has to two Thirds of the Exceſs, whereby the greateſt exceeds the mean: And alſo if the ſame Proportion, which that compounded of the greateſt and of double the mean has to that compounded of triple the greateſt and of triple the mean, another aſſum'd Line has to the Exceſs of the greateſt above the mean; then the aſſum'd Lines both together will be a third Part of the greateſt of the three Proportionals.

Let the three proportional Lines be *ab*, *bc*, and *bf*. And the ſame Proportion, which *bf* has to *fa*, let *ms* have to two thirds of *ca*. And the ſame Proportion which that compounded of *ab* and of double *bc* has to that compounded of triple of both *ab* and *bc*, let another Line, to wit, *sn*, have to *ac*. We are to demonſtrate that *mn* is a third Part of *ab*.

Becauſe *ab*, *bc* and *bf*, are Proportionals, therefore *ac* and *cf* will be alſo in the ſame Proportion. Therefore it is, *ab* : *bc* :: *ac* : *cf*; and 3 *ab* : 3*bc* :: *ac* : *cf*. Therefore the ſame Proportion, which the Triple of *ab* with the



Triple of *bc* has to the Triple of *bc*, *ac* will have to a Line leſs than *cf*. Let that be *co*. Wherefore by Compoſition, and by converſe *Ratio*, *oa* will have the ſame Proportion to *ac*, as the Triple of *ab* with the Sextuple of *bc* has to the Triple of *ab* with the Triple of *bc*. But *ac* has the ſame Proportion to *sn*, as three times *ab* with three times *bc*, has to *ab* with double *bc*. Therefore by Equality, *oa* will have the ſame Proportion to *ns*, as three times *ab* with ſix times *bc*, has to *ab* with twice *bc*. But three times *ab* with ſix times *bc*, is the Triple of *ab* with twice *bc*. Therefore *ao* is the Triple of *sn*.

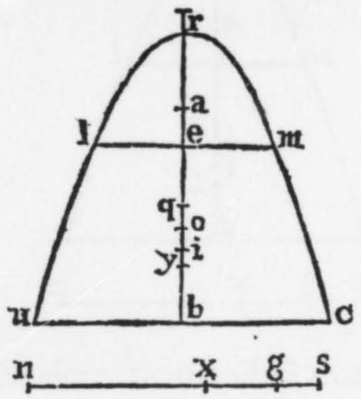
Again, becauſe *oc* is to *ca*, as three times *cb* is to three times *ab* with three times *cb*: And becauſe it is, as *ca* to *cf*, ſo is three times *ab* to three times *bc*: By Equality therefore,

therefore, in inordinate Proportion, as *oc* to *cf*, ſo will three times *ab* be to three times *ab* with three times *bc*: And, by converſe *Ratio*, as *of* to *fc*, ſo is three times *bc* to three times *ab* with three times *bc*; and, *cf* : *fb* :: *ac* : *cb* :: 3 *ac* : 3 *cb*. Therefore by Equality, in inordinate Proportion, as *of* is to *fb*, ſo is three times *ac* to three times *ab* and *bc* both together. Therefore the Whole *ob* will be to *fb*, as ſix times *ab* to three times *ab* and *bc* both together. And becauſe *fc* and *ca*, and alſo *cb* and *ba*, are in the ſame Proportion, it will be, *fc* : *ca* :: *bc* : *ba*; and by Compoſition, as *fa* is to *ac*, ſo is *ba* and *bc* both together, to *ba*; and ſo is the Triple to the Triple: Therefore, as *fa* is to *ac*, ſo is that compounded of three times *ba* and three times *bc*, to three times *ba*. Wherefore, as *fa* is to two thirds of *ac*, ſo is that compounded of three times *ba* and three times *bc* to two thirds of three times *ba*; that is, to twice *ba*. But, as *fa* is to two thirds of *ac*, ſo is *fb* to *ms*. Therefore as *fb* is to *ms*, ſo is that compounded of three times *ba* and three times *bc*, to double *ba*. But, as *ob* to *fb*, ſo was ſix times *ab* to three times *ab* and *bc* both together. Therefore by Equality, *ob* will have the ſame Proportion to *ms*, as ſix times *ab* has to double *ba*. Wherefore *ms* will be the third Part of *ob*. And it has been demonſtrated, that *sn* is a third Part of *ao*. Therefore 'tis evident, that *mn* is likewiſe a third Part of *ab*. *Q. E. D.*

The Center of Gravity of any Fruſtum of a Parabolic Conoid is in the Right Line, which is the Axis of the Fruſtum; which being divided into three equal Parts, the Center of Gravity is in the middle Part, and ſo divides

it, that the Part towards the leſſer Baſe has the ſame Proportion to the Part towards the greater Baſe, as the greater Baſe has to the leſſer.

Let the Solid, whoſe *Axis* is *be*, be cut off from the *Conoid*, whoſe *Axis* is *rb*; and let the cutting Plane be parallel to the Baſe. And let it be cut by another Plane thro' an *Axis* perpendicular to the Baſe, and let it be the Section of the *Parabola urc*. And let the Sections of this, both of the cutting Plane and of the Baſe, be the



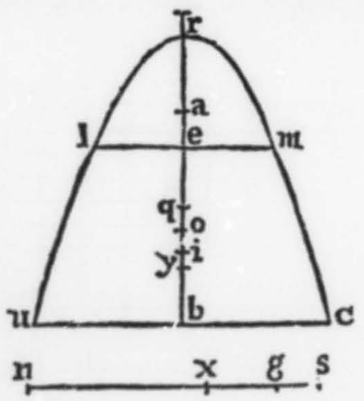
Right Lines *lm* and *uc*; *rb* will be the Diameter of Proportion, or parallel to the Diameter: *lm* and *uc* will be [Ordinates, or] ordinately apply'd. Let *eb* therefore be divided into three equal Parts, of which let *qy* be the middle one. And let this be ſo divided in the Point *i*; that, *qi* may have the ſame Proportion to *iy*, which the Baſe whoſe Diameter is *uc*, has to the Baſe whoſe Diameter is *lm*; that is, which the Square of *uc* has to the Square of *lm*. It is to be demonſtrated, that *i* is the Center of Gravity of the Fruſtum *ulmc*.

Let

Let the Line *ns* be drawn = *br*, and let *sx* be = *er*. And to the ſaid *ns* and *sx* let a third Proportional *sg* be taken. And, that Proportion, which *ng* has to *gs*, let the Line *bq* have to *io*. And it is no matter whether the Point *o* falls above or below *lm*. And becauſe in the Section *urc* the Lines *lm* and *uc* are Ordinates rightly apply'd, it will be, as the Square of *uc* to the Square of *lm*, ſo is the Line *br* to *re*. But it is, as the Square of *uc* to the Square of *lm*, ſo is *qi* to *iy*; and as *br* to *re*, ſo is *ns* to *sx*. Therefore *qi* is to *iy*, as *ns* to *sx*. Wherefore, as *qy* is to *yi*, ſo will both *ns* and *sx* be to *sx*; and as *eb* is to *yi*, ſo will that compounded of three times *ns* and three times *sx* be to *sx*. But as *eb* is to *by*, ſo is that compounded of three times *ns* and *sx* both together, to that compounded of *ns* and *sx*. Therefore, as *eb* is to *bi*, ſo is that compounded of three times *ns* and three times *sx*, to that compounded of *ns* and twice *sx*. There are therefore three proportional Lines *ns*, *sx*, and *gs*; and, the ſame Proportion which *sg* has to *gn*, a certain aſſum'd Line *oi* has to two thirds of *eb*, that is, of *nx*; and the ſame Proportion, which that compounded of *ns* and twice *sx*, has to that compounded of three times *ns* and three times *sx*, another aſſum'd Line *ib* has to *be*, that is, to *nx*. Therefore, by what has been before demonſtrated, thoſe aſſum'd Lines together will be a third Part of *ns*, that is, of *rb*. Therefore *rb* is the Triple of *bo*. Wherefore *o* will be the Center of Gravity of the *Conoid urc*. And let *a* be the Center of Gravity of the *Conoid lrm*. Therefore the Center of Gravity of the Fruſtum *ulmc* is in the Line *ob*, and in that Point which ſo terminates it, that the Line *ao* may have the ſame *Ratio* to the Line which

is

is intercepted between *o* and the ſaid Point, as the Fruſtum *ulmc* has to the Portion *lrm*. And, becauſe *ro* is two thirds of *rb*; and *ra* two thirds of *re*, the remaining Line *ao* will be two thirds of the remaining Line *eb*. And, becauſe as the Fruſtum *ulmc* is to the Portion *lrm*, ſo is *ng* to *gs*; and as *ng* is to *gs*, ſo is two thirds of *eb* to *oi*; and that the Line *ao* is equal to two thirds of *eb*: it will be, as the Fruſtum *ulmc* is to the Portion *lrm*, ſo

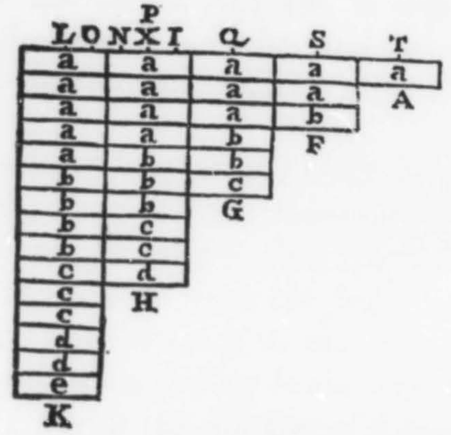


is *ao* to *oi*. Therefore 'tis manifeſt that the Point *i* is the Center of Gravity of the Fruſtum *ulmc*, and ſo divides the *Axis*, that the Part towards the leſſer Baſe, is to the Part towards the greater, as the Double of the greater Baſe together with the leſſer, is to the Double of the leſſer together with the greater. Which is the Propoſition more elegantly declar'd.

If any Number of Magnitudes ſoever, ſo diſpos'd one with another, that the ſecond ſurpaſſes the firſt by double the firſt, the third ſurpaſſes the ſecond three times the firſt, and the fourth ſurpaſſes the third four times the firſt, and ſo

ſo every following Magnitude ſurpaſſes the next before it ſo many times the firſt, according to the Number which it holds in Order: I ſay, if theſe Magnitudes be hung in Order on a Beam at equal Diſtances; the Center of Equilibrium of them all compounded, will ſo divide the Beam that the Part towards the leſſer Magnitudes be triple the remaining Part.

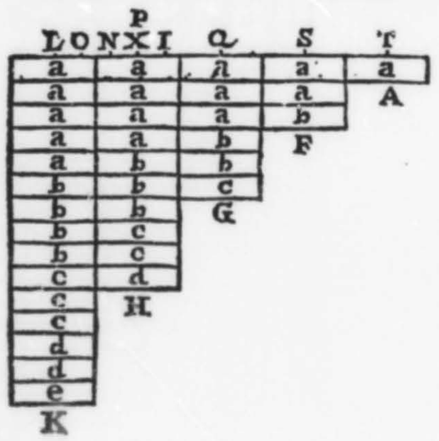
Let the Beam be LT; and let ſuch Magnitudes, as aboveſaid, hang thereon; and let them be A, F, G, H, K; of which let the firſt A be hung at T. I ſay, the Center of Equilibrium ſo divides the Beam TL, that the Part towards T is the Triple of the other Part. Let TL



be triple to LI; and SL triple LP; and QL triple to LN; and LP to LO: IP, PN, NO, and OL will be equal. And let a Magnitude in F be taken the Double of A; but in G let another be taken the Triple of the ſame; in H the Quadruple of the ſame; and ſo on in Order: And let thoſe Magnitudes, wherein *a* is, be taken: And

let

let the ſame be done in the Magnitudes F, G, H, K. For ſince the Magnitude remaining in F, *viz.* *b*, is equal to *a*; let the Double of it be taken in G, the Triple in H, *&c.* And let thoſe Magnitudes, wherein *b* is, be taken; and in the ſame Manner let thoſe, wherein *c* is, and wherein *d* and *e* alſo are, be taken. Now all thoſe, wherein *a* is, will be equal to K; but that compounded of all the *b*'s will be equal to H; that compounded of the *c*'s will be equal to G; but that compounded of all the *d*'s will be equal to F; and *e* will be equal to *a*. And be-



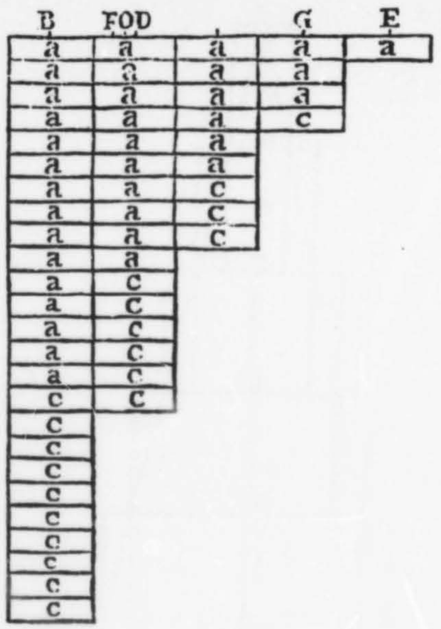
cauſe TI is double LI, I will be the Point of Equilibrium of the Magnitude compounded of all the *a*'s. And, likewiſe, ſince SP is the Double of PL, P will be the Point of Equilibrium of that compounded of all the *b*'s: And for the ſame Reaſon N will be the Point of Equilibrium of the Magnitude compounded of all the *c*'s; but *o* that of the Compound of all the *d*'s; and L that of the Magnitude *e*. There is therefore a certain Beam TL, whereon certain Magnitudes K, H, G, F, A, do hang at equal Diſtance

Diſtances. And again, there is another Beam LI, on which at equal Diſtances likewiſe do hang juſt the ſame Number of Magnitudes, and in the ſame Order as the former. For, the Compound of all the *a*'s, which hangs at I, is = K hanging at L; and the Compound of all the *b*'s, which hangs at P, is = H hanging at P; and likewiſe the Compound of the *c*'s, which hangs at N, is = G; and the Compound of the *d*'s, which hangs at O, is = F; and *e* hanging at L is = A. Wherefore the Beams are divided in the ſame *Ratio* by the Center of the compounded Magnitudes. But the Center of the Magnitude compounded of thoſe Magnitudes is one alone. Therefore a Point common to the Right Lines TL and LI will be the Center; which let be X. Therefore as TX is to XL, ſo will LX be to XI; and ſo will the whole Line TL be to LI. But TL is the triple of LI: Wherefore TX will alſo be the triple of XL. *Q. E. D.*

If any Number of Magnitudes ſoever be ſo taken, that the ſecond ſurpaſſes the firſt three times the firſt, but that the third ſurpaſſes the ſecond five times the firſt, and the fourth ſurpaſſes the third ſeven times the firſt, and ſo on that the Increaſe of every one proceeds in that next to it multiple of the firſt Magnitude according to the odd Numbers following in Order; like as the Squares of Lines equally exceeding each other, the common Exceſs being equal to the leaſt, do proceed; and if they be hung on a Beam at equal Diſtances; the Center of Equilibrium of all the Magnitudes compounded will divide the Beam, that the Part towards the leſſer Magnitudes is more than triple the remaining

Part, but, when one Diſtance is taken from it, is leſs than triple the ſaid remaining Part.

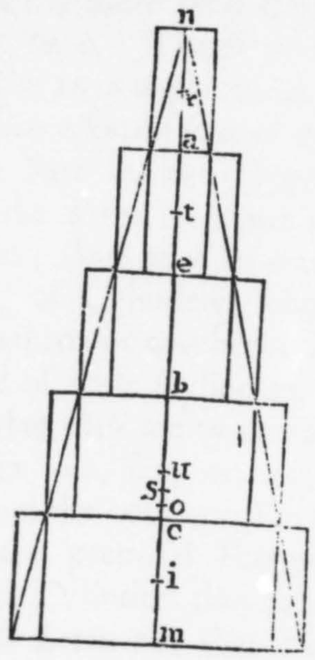
Let there be ſuch Magnitudes, as aboveſaid, on the Beam BE. From which let ſome Magnitudes, ſo diſpos'd one with another as thoſe were in the preceding Propoſition, be taken away; and let them be thoſe compounded of all the *a*'s. The remaining Magnitudes, wherein the *c*'s are, will be rang'd in the ſame Order, but wanting the



greateſt. Let ED be triple DB; and GF triple FB. D will be the Center of Equilibrium of the Compound of all the *a*'s; but F that of the Compound of all the *c*'s. Wherefore the Center of the Compound of all the *a*'s and *c*'s will fall between D and F. Let it be O. Therefore 'tis manifeſt that EO is more than the Triple of OB.; but that GO is leſs than the Triple of the ſaid OB. *Q. E. D.*

If

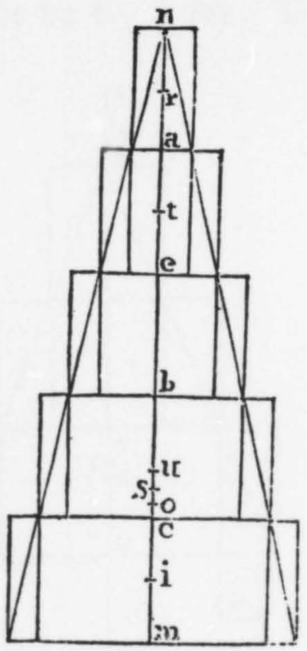
If to any Cone or Portion of a Cone, one Figure conſiſting of Cylinders having equal Heights, be inſcrib'd, and another circumſcrib'd; and alſo if its Axis be ſo divided, that the Part which is intercepted between the Point of Diviſion and the Vertex, be triple the remaining Part: the Center of Gravity of the inſcrib'd Figure will be nearer the Baſe of the Cone than that Point of Diviſion; but the Center of Gravity of the circumſcrib'd Figure will be nearer the Vertex than the ſame Point.



Let the Cone then be that, whoſe Axis is *nm*. Let it be divided in *s*, ſo that *ns* be triple the remaining Part *sm*. I ſay, that the Center of Gravity of any Figure inſcrib'd in the Cone, as aforeſaid, is in the Axis *nm*, and comes nearer to the Baſe of the Cone than the Point *s* does; but that the Center of Gravity of the circumſcrib'd Figure is likewiſe

 in

in the Axis *nm*, and nearer the Vertex than *s* is. Let therefore a Figure of Cylinders, whoſe Axes *mc*, *cb*, *be*, *ea*, are equal, be apprehended to be inſcrib'd. Therefore the firſt Cylinder, whoſe Axis is *mc*, has the ſame *Ratio* to the Cylinder whoſe Axis is *cb*, as the Baſe of the firſt is to the Baſe of the other (for their Heights are equal). But this *Ratio* is the ſame with that which the Square of *cn* has to the Square of *nb*. And in like Manner it may be prov'd that the Cylin-

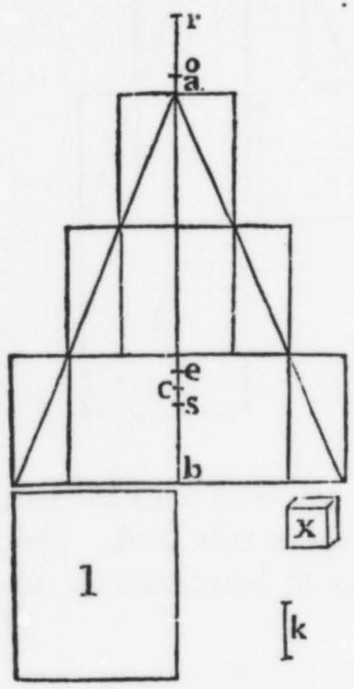


der, whoſe Axis is *cb*, has the ſame *Ratio* to the Cylinder, whoſe Axis is *be*, as the Square of *bn* has to the Square of *ne*; and that the Cylinder, whoſe Axis is *be*, has that *Ratio* to the Cylinder about the Axis *ea*, which the Square of *en* has to the Square of *na*. But *nc*, *nb*, *en*, *na*, are Lines equally exceeding each other, and their common Exceſs

Exceſs is equal to the leaſt, *viz.* to *na*. There are therefore certain Magnitudes, *viz.* inſcrib'd Cylinders, having that *Ratio* to each other in Order, which the Squares of Lines equally exceeding each other, and whoſe common Exceſs is equal to the leaſt Line, have to one another: And they are ſo diſpos'd on the Beam *ti*, that the Centers of the Gravities of every one of them are in it, and at equal Diſtances. Therefore by what has been before demonſtrated, it is evident, that the Center of Gravity of them all thus compounded ſo divides the Beam *ti*, that the Part towards *t* is more than triple the remaining Part. Let this Center be *o*. Therefore *to* is more than the Triple of *oi*. But *tn* is triple to *im*. Therefore the whole *mo* will be leſs than a fourth Part of the whole *mn*, whereof *ms* is a fourth Part by the Hypotheſis. 'Tis manifeſt therefore, that the Point *o* comes nearer the Baſe of the Cone than *s* does; (but now let there be a Figure circumſcrib'd conſiſting of Cylinders, whoſe Axes *mc*, *cb*, *be*, *ea*, *an*, are equal to one another); in like Manner it may be demonſtrated of theſe Cylinders, as it has been of the inſcrib'd ones, that they are to one another as the Squares of the Lines *mn*, *nc*, *bn*, *ne*, *an*; which exceed each other equally, and the common Exceſs is = the leaſt, *an*. Wherefore, by the premis'd Propoſition, the Center of Gravity of all the Cylinders thus diſpos'd, which let be *u*, will ſo divide the Beam *ri*, that the Part towards *r*, to wit *ru*, is more than triple the remaining Part *ui*; but that *tu* will be leſs than the Triple of the ſame. But *nt* is the Triple of *im* Therefore the whole *um* is more than a fourth Part of the whole *mn*, whereof *ms* is a fourth Part by the Hypotheſis. Therefore the Point *u* is nearer the Vertex than the Point *s*. *Q. E. D.*

A Figure conſiſting of Cylinders of equal Altitude may be circumſcrib'd about, and another inſcrib'd in a given Cone, ſo that the Line which is intercepted between the Center of Gravity of the circumſcrib'd Figure and the Center of Gravity of the inſcrib'd Figure, be leſs than any given Line.

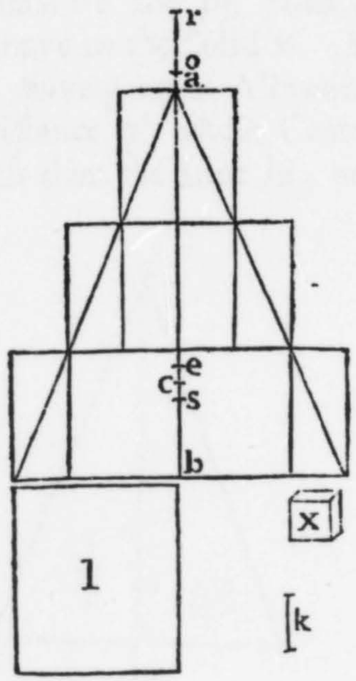
Let the given Cone be that, whoſe *Axis* is *ab*. And let the given Right Line be *k*. I ſay; Let a Cylinder *l*,



equal to that which is inſcrib'd in the Cone, be drawn by it ſelf, having its Altitude = half the *Axis* *ab*: And let *ab* be divided in *c*, ſo that *ac* be the Triple of *cb*: And, the

the *Ratio* that *ac* has to *k*, let the Cylinder *l* have to the Solid *x*. But let a Figure of Cylinders having equal Altitude be circumſcrib'd about the Cone, and let another be inſcrib'd, ſo that the circumſcrib'd Figure may exceed the inſcrib'd one by a Quantity leſs than the Solid *x*. And let *e* be the Center of Gravity of the circumſcrib'd Figure; which will fall above *c*: but let the Center of Gravity of the inſcrib'd one be *s*, falling under *c*. I ſay now, that the Line *es* is leſs than *k*. For if it be not; let *eo* be ſuppos'd equal to *ca*. Becauſe *oe* therefore has the ſame *Ratio* to *k*, that *l* has to *x*; and the inſcrib'd Figure is not leſs than the Cylinder *l*; and the Exceſs, whereby the circumſcrib'd Figure exceeds the ſaid inſcrib'd one, is leſs than the Solid *x*: Therefore the inſcrib'd Figure will have a greater *Ratio* to the ſaid Exceſs, than *oe* has to *k*: But the *Ratio* of *oe* to *k* is not leſs than that which *oe* has to *es* with *es*. Let it not be ſuppos'd leſs than *k*; therefore the inſcrib'd Figure has a greater *Ratio* to the Exceſs of the circumſcrib'd Figure above it, than *oe* has to *es*. Therefore the *Ratio*, that the inſcrib'd Figure has to the ſaid Exceſs, the ſame it will have to the Line *es*. Let ſome Line *er* be greater than the ſaid *eo*. The Center of Gravity of the inſcrib'd Figure indeed is *s*; but the Center of the circumſcrib'd one is *e*. Therefore 'tis manifeſt, that the Center of Gravity of the remaining Portions, whereby the circumſcrib'd exceeds the inſcrib'd one, is in the Line *re*, and in that Point which ſo terminates it, that, the ſame *Ratio*, which the inſcrib'd Figure has to the ſaid Portions, the Line, intercepted between *e* and that Point, has to the Line *es*. But this *Ratio re* has to *es*. Therefore the Center of Gravity of the remaining Portions, whereby the

the circumſcrib'd exceeds the inſcrib'd Figure, will be *r*. Which is impoſſible. For a Plane drawn thro' *r*, parallel to the Baſe of the Cone, does not cut the ſaid Portions.



'Tis falſe then, that the Line *e s* is not leſs than *k*. Therefore it will be leſs. And after a Manner not unlike this, the ſame may be demonſtrated to come to paſs in any Pyramid.

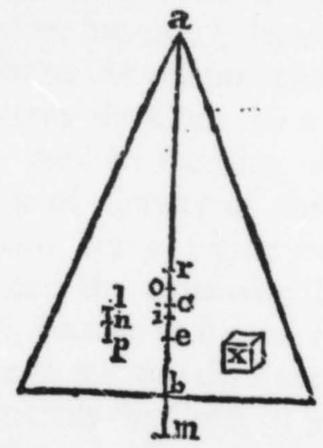
COROLLARY.

Hence it is manifeſt, that *One Figure of Cylinders of equal Altitude may be circumſcrib'd about, and another inſcrib'd*

inſcrib'd in a given Cone, ſo that the Lines, which are intercepted between their Centers of Gravity, and the Point, which divides the Axis of the Cone ſo that the Part towards the Vertex be triple the remaining Part, are leſs than any given Line.

For whereas the ſaid Point, which divides the Axis ſo as aboveſaid, is always found between the Centers of Gravity of the circumſcrib'd and inſcrib'd Figures, as has been demonſtrated; and ſince it is poſſible, that the Line, which is in the midſt between the ſame Centers, be leſs than any given Line; much leſs than the ſame given Line muſt that Line be, which is intercepted between either of thoſe Centers and the ſaid Point which divides the Axis.

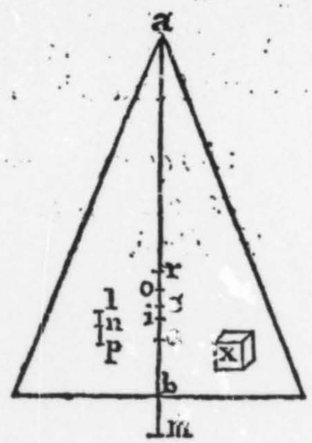
The Center of Gravity of any Cone or Pyramid divides the Axis, that the Part towards the Vertex is triple the other Part towards the Baſe.



Let there be a Cone, whoſe Axis is *ab*. And let it be ſo divided in *c*, that *ac* be the Triple of the remaining

 Part

Part *cb*. It is to be demonſtrated, that *c* is the Center of Gravity of the Cone. For if it is not, the Center of the Cone will be either above, or below the Point *c*. Firſt let it be below; and let it be *e*: And let the Line *lp* be drawn equal to *ce*; which let be divided at Pleaſure in *n*. And that *Ratio*, which *be* and *pn* both together have to *pn*, let the Cone have to the Solid *x*. And let a ſolid Figure of Cylinders, having equal Altitude, be inſcrib'd in the Cone, the Diſtance of whoſe Center of Gravity from the Point *c* is leſs than the Line *ln*; and let the Ex-



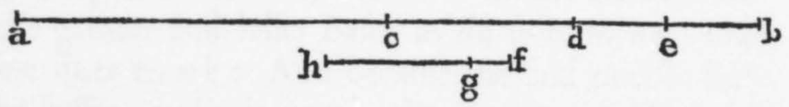
ceſs of the Cone above it, be leſs than the Solid *x*. For that theſe Things may be done, is manifeſt from what has been demonſtrated. Now let ſuch a Figure be inſcrib'd as was requir'd, whoſe Center of Gravity let *i* be. Therefore, ſince *lp* is equal to *ce*, and *ic* is leſs than *ln*, the Line *ie* will be greater than *np*: And, becauſe *be* and *np* both together is to *np*, as the Cone to *x*; and the Exceſs of the Cone above the inſcrib'd Figure is leſs than the Solid

Solid *x*; thererefore the Cone will have a greater *Ratio* to the ſaid Exceſs than both *be* and *np* to *np*: And by Diviſion, the inſcrib'd Figure will have a greater *Ratio* to the Exceſs of the Cone above it, than *be* to *np*: But *be* has ſtill a leſs *Ratio* to *ei*, than it has to *np*; ſince *ie* is greater than *np*; therefore the inſcrib'd Figure has a much greater *Ratio* to the Exceſs of the Cone above it, than *be* has to *ei*. Therefore that *Ratio*, which the inſcrib'd Figure has to the ſaid Exceſs, ſome Line, greater than *be*, will have to *ei*. Let that be *me*. Then becauſe *me* is to *ei*, as the inſcrib'd Figure is to the Exceſs of the Cone above it, and becauſe *e* is the Center of Gravity of the Cone, and *i* is the Center of Gravity of the inſcrib'd Figure; therefore *m* will be the Center of Gravity of the remaining Portions, whereby the Cone exceeds the Figure inſcrib'd in it. Which is impoſſible. Therefore the Center of Gravity of the Cone is not below the Point *c*. Nor is it above it. For, if it can be, let it be *r*; and again let the Line *lp*, cut at pleaſure in *n*, be aſſum'd: And, that *Ratio*, which *bc* and *np* both together has to *nl*, let the Cone have to *x*; And let a Figure be in like manner circumſcrib'd about the Cone, which exceeds the Cone by a Quantity that's leſs than the Solid *x*: And let the Line, that is intercepted between the Center of Gravity of that Figure and *c*, be leſs than *np*: Now let a Figure be circumſcrib'd, whoſe Center let *o* be: the remaining Part *or* will be greater than *nl*. And, becauſe as *bc* and *pn* both together is to *nl*, ſo is the Cone to *x*; and the Exceſs, whereby the circumſcrib'd Figure exceeds the Cone, is leſs than *x*; and *bo* is leſs than *bc* and *pn* both together; and *or* greater than *ln*: The Cone therefore will have a much greater *Ratio* to the remaining Portions, by which the cir-

cumſcrib'd Figure exceeds it, than *bo* has to *or*. Let *mo* have that *Ratio* to *or*: *mo* will be greater than *bc*: And *m* will be the Center of Gravity of the Portions by which the circumſcrib'd Figure exceeds the Cone. Which is abſurd. Therefore the Center of Gravity of the Cone is not above the Point *c*: But it is not below it; as has been prov'd. Therefore it will be *c* it ſelf. And intirely after the ſame Manner the ſame may be demonſtrated in any Pyramid whatſoever.

If four Lines are continual Proportionals; and the ſame Ratio, which the leaſt of them has to the Exceſs of the greateſt above it, ſome aſſum'd Line has to ¾ of the Exceſs of the greateſt above the ſecond; and the ſame Ratio, which a Line equal to all theſe (viz. the greateſt, the Double of the ſecond, and the Triple of the third) has to a Line equal to the Quadruple of the greateſt, the Quadruple of the ſecond, and the Quadruple of the third, ſome other aſſum'd Line has to the Exceſs of the greateſt above the ſecond; theſe two aſſum'd Lines together will be a fourth Part of the greateſt of the Proportionals.

For let the four proportional Lines be *ab*, *bc*, *bd*, *be*. And, the ſame *Ratio*, which *be* has to *ea*, let *fg* have to ¾ of *ac*. And the *Ratio*, which a Line equal to *ab*, and



twice *bc*, and thrice *bd* has to a Line equal to the Quadruple of the Lines *ab*, *bc*, *bd*, let *hg* have to *ac*: It is to be demonſtrated, that *hf* is a fourth Part of *ab*.

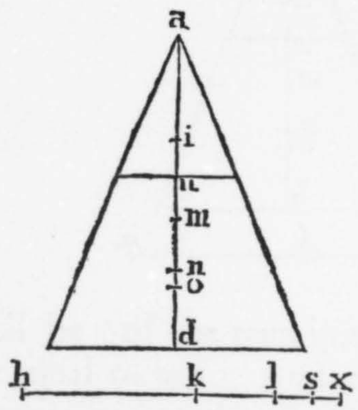
Becauſe

Becauſe then *ab*, *bc*, *bd*, *be*, are Proportionals; *ac*, *cd*, *de*, will be alſo in the ſame Proportion: And as the Quadruple of the Lines *ab*, *bc*, *bd*, is to *ab* with twice *bc*, and thrice *bd*; ſo is the Quadruple of *ac*, *cd*, *de*; that is, the Quadruple of *ae* to *ac* with twice *cd* and thrice *de*; and ſo is *ac* to *hg*. Therefore as the Triple of *ae* is to *ac* with twice *cd* and thrice *de*, ſo is ¾ of *ac* to *hg*: But, as the Triple of *ae* is to the Triple of *eb*, ſo is ¾ of *ac* to *gf*; therefore, by the Converſe of the 24th of the 5th of *Euclid*, as the Triple of *ae* is to *ac* with twice *cd* and thrice *db*, ſo is ¾ of *ac* to *hf*. And, as the Quadruple of *ae* is to *ac* with twice *cd* and thrice *db*, that is, to *ab* with *cb* and *bd*; ſo is *ac* to *hf*. And, by Permutation, as the Quadruple of *ae* is to *ac*; ſo is *ab* with *cb* and *bd* to *hf*. But, as *ac* is to *ae*, ſo is *ab* to *ab* with *cb* and *bd*. Therefore by Equality, in inordinate Proportion, as the Quadruple of *ae* is to *ae*, ſo is *ab* to *hf*. Wherefore 'tis evident, that *hf* is a fourth Part of *ab*. *Q. E. D.*

The Center of Gravity of any Fruſtum of a Pyramid or Cone whatſoever, cut by a Plane parallel to the Baſe, is in the Axis, and ſo divides it, that the Part towards the leſſer Baſe is to the remaining Part, as the Triple of the greater Baſe, with the Double of the Space of the middle between the greater and the leſſer Baſe, together with the leſſer Baſe, is to the Triple of the leſſer Baſe, with the ſame Double of the middle Space, and with the greater Baſe.

Let

Let the Fruſtum, whoſe Axis is *ud*, be cut from the Cone or Pyramid, whoſe Axis is *ad*, by a Plane parallel to the Baſe. And that *Ratio*, which the Triple of the greateſt Baſe, with the Double of the Middle, and with the leaſt Baſe, has to the Triple of the leaſt, with the Double of the Middle, and with the greateſt, let *uo* have to *od*. It is to be prov'd, that *o* is the Center of Gravity of the Fruſtum. Let *um* be a fourth Part of *ud*.



Let a Line *hx* be drawn equal to *ad*. And let *kx* be equal to *au*. And let *xl* be a third, and *xs* a fourth Proportional to *hx* and *xk*. And that *Ratio* which *hs* has to *sx*, let *md* have to a Line aſſum'd from *o* towards *a*; which let be *on*. And becauſe the greater Baſe bears the ſame Proportion to that, which is in the Middle between the greater and leſſer Baſe, as *da* does to *au*; that is, as *hx* does to *xk*: And becauſe the ſaid middle Baſe is to the leſſer, as *kx* is to *xl*: the greater, middle, and leſſer Baſes, will be in the ſame Proportion with the Lines *hx*, *xk*, and *xl*.

Wherefore

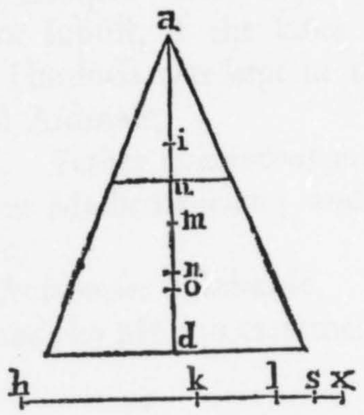
t

Wherefore as the Triple of the greater Baſe with the Double of the middle one and with the leaſt, is to the Triple of the leaſt with the Double of the middle and with the greateſt; that is, as *uo* is to *od*; ſo is the Triple of *hx* with the Double of *xk* and with *xl*, to the Triple of *xl* with the Double of *xk* and with *xh*: And by Compoſition, and Converſion, *od* will be to *du*, as *hx* with the Double of *xk* and the Triple of *xl*, is to the Quadruple of theſe, *hx*, *xk*, *xl*.

There are therefore four proportional Lines, *hx*, *xk*, *xl*, *xs*: And that *Ratio*, which *xs* has to *sh*, the aſſum'd Line *no* has to $\frac{3}{4}$ of *du*, *viz.* to *dm*; that is, to $\frac{3}{4}$ of *hk*: And the ſame *Ratio*, which *hx* with the Double of *xk* and the Triple of *xl* has to the Quadruple of theſe, *hx*, *xk*, *xl*; another aſſum'd Line *od* has to *du*; that is, to *hk*. Therefore (by what has been demonſtrated) *dn* will be a fourth Part of *hx*; that is, of *ad*. Wherefore the Point *n* will be the Center of Gravity of the Cone or Pyramid whoſe Axis is *ad*. Let the Center of Gravity of the Pyramid or Cone, whoſe Axis is *au*, be *i*. It is then manifeſt, that the Center of Gravity of the Fruſtum is in the Line *in* prolonged from *n*, and in that Point of it, which with the Point *n* intercepts a Line, to which *in* has that *Ratio*, which the Fruſtum cut off has to the Pyramid or Cone, whoſe Axis is *au*. It remains therefore to be prov'd, that *in* has the ſame *Ratio* to *no*, that the Fruſtum has to the Cone, whoſe Axis is *au*. But as the Cone, whoſe Axis is *da*, is to the Cone, whoſe Axis is *au*; ſo is the Cube of *da* to the Cube of *au*; that is, the Cube of *hx* to the Cube of *xk*. But this is the ſame Proportion that *hx* has to *xs*. Wherefore by Diviſion,

as

as *hs* is to *sx*, ſo will the Fruſtum, whoſe Axis is *du*, be to the Cone or Pyramid, whoſe Axis is *ua*. But, as *hs* is to *sx*, ſo alſo is *md* to *on*. Wherefore the Fruſtum is to the Pyramid, whoſe Axis is *au*, as *md* is to *no*. And becauſe *an* is ¾ of *ad*; and *ai* is ¾ of *au*: the remaining



Part *in* will be ¾ of the remaining Part *ud*. Wherefore *in* will be equal to *md*. And it has been demonſtrated, that *md* is to *no*, as the Fruſtum is to the Cone *au*. 'Tis manifeſt therefore, that *in* has alſo this ſame Proportion to *no*. Wherefore the Propoſition is evident.

F I N I S.

A

TABLE

OF THE

Moſt Notable Things.

A.

AIR. HAS poſitive Gravity, Page 116. How to determine that Gravity, p. 117. *Air* compreſs'd, and detained by Force, gravitates in *Vacuo*, p. 120. How to weigh it in *Vacuo*, *ibid.*

ANIMALS. In the Water larger than thoſe on the Land, and why, p. 196.

APPENDIX. Containing ſome Theorems and their Demonſtrations concerning the Center of Gravity of Solids, p. 447.

ARSENAL OF VENICE. Affords a large Field to Philoſophize in, p. 1.

B.

BONE. An Example of the Bone of an Animal, extended to triple the natural Length, how much its Thickneſs ought to be increaſed to ſuſtain itſelf, p. 193.
Bones of Animals inlarged vaſtly beyond their natural Size would not ſubſiſt, if the ſame Proportion of Thickneſs and Hardneſs was kept in them, as is in thoſe of natural Animals, p. 193.
BURNING-GLASSES. Father *Buonaventura Cavalieri* the Jeſuit, a famous Mathematician; and his Book, *Lo Specchio Uſtorio*, p. 61:
The Specula of *Archimedes* admirable, *ibid.*
BURNINGS. Are made by Motion extremely ſwift, p. 62.

C.

CENTER OF GRAVITY. *Lucas Valerius*, the *Archimedes* of our Age, hath written admirably *De Centro Gravitatis Solidorum*, p. 45.
CIRCLE. An admirable Problem of *Ariſtotle* concerning two concentric Circles, revolving together; and its true Solution, p. 30.
The Difference between a finite Circle and an infinite one, p. 59.
A Circle is a Polygon, whoſe Sides are infinite, non-quantitative, and indiviſible, p. 75.
A Circle is a Mean-proportional between two Polygons, one of which is circumſcrib'd about it, and the other Iſoperimetral to it, p. 86.
CONDEN-

CONDENSATION. According to the Author's Opinion, proceeds from a Conſtipation of Parts non-quantitative and indiviſible, p. 76.
CONTINUUM. Compos'd of Indiviſibles, p. 46 & 71.
CYLINDER. Cylinders, or Threads of any Matter, to determine to what Length they may be extended, beyond which if they are extended, they'll break by their own Weight, p. 25.
The Surfaces of equal Cylinders, their Baſes abated, are to one another in a ſubduplicate Proportion of their Lengths, p. 79.
Upright Cylinders, whoſe Surfaces are equal, are as their Heights, reciprocally taken, p. 82.
Corn-Sacks made of the ſame Stuff with a round Board at the Bottom, but of different Heights; which are the more capacious, p. 84.

E.

EFFECT *and* CAUSE. A Poſitive Effect has a poſitive Cauſe, p. 18.

F.

FALLING BODY. A heavy Body falling from a Height, hath at its Arrival to the Ground, got ſuch an Impetus, as, in all Probability, is ſufficient to drive it back to the ſame Height it fell from, p. 137.
FISHES. Admirably preſerve an Equilibrium in the Water, p. 102, And the Cauſe why, p. 196.
FLUIDS. Fluid Bodies are ſuch, becauſe they are reſolved into their firſt indiviſible Particles, p. 60.

FRACTION. If a Staff of Wood, be fixed in a Wall at right Angles, at ſuch a Length and Thickneſs as juſt to ſupport itſelf, that if it was a Hair's Breadth longer, it would break by its own Weight; there can't poſſibly be ſuch another, p. 5.
A Column, or very large Pillar of Marble broken by its own Weight; and why, p. 6.
How a Cord or Rope reſiſts Fraction, p. 11 & 29.
A Cylinder or Priſm of any Matter, hung perpendicularly, how it reſiſts *Fraction*, p. 16.

G.

GOLD-WIRE. Gold, wherewith Silver is gilt, may be drawn out and thinn'd immenſely, p. 78.
GRAVITY. The Difference of Gravity of Moveables, tho' never ſo great, cauſes no Alteration in their Velocities, p. 122.

I.

INFINITY. Is found in Unity, p. 59.

L.

LIGHT. How to find by an Experiment whether its Velocity be inſtantaneous or not, p. 63.
LINE. A Right one being given, divided any how into two unequal Parts, to deſcribe a Circle, ſo that any two Right-Lines being drawn from the Ends of the given Line to any Point in the Circumference, may have the ſame Ratio amongſt themſelves, as the Parts of the divided Line have, p. 65.

M.

MACHINES. Great material Machines, tho' made in the ſame Proportion with leſſer ones conſiſting of the ſame Matter, are leſs robuſt and able to reſiſt external Violence, than the leſſer, p. 4.
MECHANIC INSTRUMENT. A handy one invented by a humorous Perſon to let himſelf down from a great Height by Help of a Cord, without gawling his Hands, p. 14.
MEDIUM. The Reſiſtance of the *Medium* being taken away, all Matter, tho' of different Gravity, would move with like Velocity, p. 107.
The Velocity of heavy Bodies naturally deſcending towards the Center, goes on continually increaſing, till at length by the Augmentation of the Reſiſtance of the *Medium* it becomes uniform, p. 110.
To inveſtigate the Proportions of the Velocities of different Moveables, in the ſame, and in different *Mediums*, p. 111.
Any Body of whatſoever Figure, Bigneſs, or Gravity, is by the Renitence of the *Medium*, tho' never ſo thin, check'd in its Acceleration, ſo that, in the Continuation of its Motion, it is reduced to Equability, p. 135.
MOTION. *Of Local Motion.* Dial. III.
—— Equable, p. 228.
—— Naturally accelerated, p. 237.
Of the Motion of Projects. Dial. IV.
MOVEABLES. The greater or leſſer Scabroſity and Poroſity in their Surfaces, a probable Cauſe of their greater or leſſer Retardation, p. 129.

Moveables deſcending along Chords ſubtending any Arches of a Circle, paſs thro' them all, as well the greater as the leſſer ones, in equal Times, p. 139.

MUSICK. A String of a muſical Inſtrument being touch'd, moves and makes all other Strings, ſet to an Uniſon, to a Fifth, and to an Eighth with it, to ſound; and why, p. 143.

Problems concerning *Muſical* Proportions, and their Solutions, p. 143 to 157.

N.

NUMBER. As an infinite Number has infinite Square and Cube Roots, ſo it has infinite Square and Cube Numbers, p. 48.

P.

PARABOLA. Its Quadrature exhibited from a ſingle Demonſtration only, p. 216.

Several Ways of deſcribing a *Parabolic* Line, p. 220.

PENDULUM. Moveables and Pendulums deſcending along the Arches of thoſe Chords, elevated above the Horizon as far as 90 Degrees, paſs thro' the ſaid Arches in equal Times, but ſhorter than thoſe wherein the Chords are run thro', p. 140.

To inveſtigate the Length of a String, by which a Moveable is hung, from the Frequency of its Vibrations, p. 141.

Every *Pendulum* hath the Time of its Vibrations limited, ſo that it is impoſſible to make it perform the Motion in another Period, p. 142.

POINTS. How infinite Points may be aſſigned in a finite Line, p. 70.

PUMP. Water raiſed and attracted by a Pump, never riſes above 18 Cubits, p. 25.

Q.

QUANTITY. The quantitative Parts in diſcreet Quantity are neither finite, nor infinite; but anſwerable to every aſſign'd Number, p. 52.

R.

RANGES. A Table of the Ranges of Projects thrown from a Piece of Ordnance according to the different Elevations of it, p. 436.

RAREFACTION. Is the Diſtraction of infinite Indiviſibles, with the Interpoſition of infinite indiviſible Vacuities. p. 76.

Immenſe *Rarefaction* is that of a little Gun-powder into a vaſt Maſs of Fire, p. 89.

ROPE. It is impoſſible with any immenſe Force to ſtretch out a Rope into a Right Line parallel to the Horizon, p. 441.

S.

SOLIDS. Concerning their Coherence and their Power of reſiſting Fraction, being preſs'd down with their own Weight. Dial. II. throughout.

Solids can't be diminiſhed ſo much in Surface, as in Weight, keeping the Likeneſs of Figure, p. 131.

Similar Solids are to one another in a Seſquialteral Proportion of their Surfaces, p. 133.

SURFACES. The equal Surfaces of two Solids, equal Parts being continually taken away from both Parts, will at length terminate, the one in the Circumference of a Circle, the other in a Point, p. 40.

T.

TIME. An Inſtant or a Moment is ſuch a Part of quantitative Time, as a Point is of a quantitative Line, p. 75.

TUBE. A hollow one being given, to find a Solid Cylinder equal to it, p. 224.

V.

VACUUM. Partly the Cauſe of the Connection and Coherence of the Parts of Solids, p. 17. How to meaſure its Power therein diſtinct from all other concurrent Cauſes, p. 20

The moſt minute *Vacuums* diſſeminated and interpoſed between the ſmalleſt Particles of Solids probably the Cauſe of the Connection of thoſe Particles to one another, p. 28.

Ariſtotle's Argument againſt a *Vacuum* is *Argumentum ad Hominem*, p. 92.

VELOCITY. Moveables of different Gravity, but of the ſame Matter, falling from a great Height, move with equal Velocity, p. 93.

The Quantity of the Velocity of a Moveable is at once the Cauſe and the Meaſure of the Quantity of the Reſiſtance of the *Medium*, p. 110.

The Proportion of the Velocity of like and unlike Moveables in the ſame and in different *Mediums* may be aſſigned, p. 111.

The Velocity of a Musket or Cannon Ball is incomparably greater than the Velocity of other Projects, p. 388.

The different Velocity of the different Motions of the Planets according to *Plato*, was conferred upon them from a Motion in a right Line, and continued afterwards in their Revolutions in their Orbs; this might be very exactly illuſtrated from the Speculations of our Author, p. 396.

VIBRATION. The Times of the Vibrations of ſeveral Moveables hanging by longer or ſhorter Strings, are amongſt themſelves in Subduplicate Proportion of the Lengths of the Strings by which they hang, p. 140.

W.

WATER. Innumerable Particles of Water by inſinuating themſelves into Cables, draw and lift an immenſe Weight, p. 29.

A Ball of Wax prepared to make an Experiment of the different Gravities of Waters, p. 103.

Water has no Reſiſtance to Diviſion, p. 104.

Water form'd into a large Heap upon Cole-wort or Cabbage-leaves, how it ſuſtains itſelf, p. 104.

WEIGHT. A Nail twice as thick as another, drove into a Wall, ſuſtains a Weight eight times greater than the leſſer one will, p. 8.

The END.

ERRATA.

PAG. 24, *lin.* 10. *for* reſemble, *read* reaſſemble. *p.* 79, *l.* 2. 79, 16. 80, 3. 81, 9. *for* ſubduple, *r.* ſubduplicate. 82, 23. *for* AC, *r.* AB. 88, 3. *after* Permutation, *inſert*, the Triangle DOA ſhall have greater Proportion to the Sector FOA. 91, 27. *inſert the Comma after* Vacuum *at* Time,. 125, 16. *for* like way, *r.* like, nay. 140, 16. *for* ſubduple, *r.* ſubduplicate. 147, 7. *for* Seſquitertian, *r.* Seſquialter. 155, 15. *for* which, *r.* with. 182, 4. *after* Proportion of, *inſert* the Cube of the Diameter AB to. 189, 26, *for* : As *r.*, Is. 205, 19. *for* AGD, *r.* ACB. 208, 5. *for* CA, *r.* BA. 2 09, 5. *for* DAO, *r.* DAB. 216, *ult. for* longeſt Line, *r.* greateſt Space. 217, 11. *for* Square, *r.* Space. 218, 6. *after* i. e. *dele* of. D°. 8. *dele* by an Exceſs = the leaſt of them, the Square of. 222, 1. *for* Cane, *r.* Lance. D°. 5. *for* Canes, *r.* Lances. 225, *at the Fig. for* H, *r.* R. 231, 15. *for* fourth, *r.* third. 235, 21. *for* EG, *r.* E to G. 244, 13. *r.* diminiſhes. 295, 296, 297, 328, 329, 331. *every where for* × *r.* +. 313, 18. *for* :: FC, *r.* :: FE. 331, 2. *for* OA, *r.* OB. D°. *l.* 7, 9. *for* Double, *r.* Duplicate. 338, *ult. for* AGO, *r.* AGC. 340, 16. *for* AC, *r.* EC. 343, 2. *for* Center, *r.* Point. 350, 21. *for* it ſhall alſo touch DI, *r.* DI ſhall alſo touch it. 379, 26. *for* bc, bg, and bi, *r.* bo, bg, and bl. 388, 14. *for* firſt, *r.* fire. 390, 20. *after* Power, *inſert* =. 409, 5. *for* q, *r.* b. 415, 13. *for* Amplitude, *r.* Altitude. 421, 8. *for* mbd, *r.* mdb. 423, 7. *before* : But, *inſert* will be equal :. 425, 5. *for* d to e, *r.* d to c. 434, *penult. for* ab, *r.* db. 443, *at the Fig. for* h, *r.* n. 461, 8. *for* Certainty, *r.* certainly.